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A Fuzzy Goal Programming Approach for Food Product Distribution of Small and Medium Enterprises

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Abstract. This paper addresses the frozen food products demand in Small and Medium Enterprise (SME) in Malaysia which is an emerging industry now. Frozen food products are in high demand but not all demands can be met due to certain limiting factors. Also these demands are assumed to be imprecise in nature. This study is undertaken to develop a fuzzy goal programming model in order to satisfy the customers' demands to the fullest of a SME company producing frozen foods considering majorly three objectives. These are achieving the total distribution of five products of frozen foods to three different locations, maximizing total profits and minimizing the total manufacturing costs using LINDO 11.0 as the optimizer solver. The distribution of all five products of frozen foods is satisfied at Ampang and Kaula Lumpur and not in Kulai for two products. Achieving a satisfactory profit level with the linear membership function, μ_{16} representing imprecise nature of this goal are achieved partially. The number of goals to be considered can also be increased based on the desirability of the decision maker in relation to their aspiration level..

Keywords: Frozen foods; food product distribution; fuzzy goal programming; small and medium enterprise; goal programming

AMS Mathematics Subject Classification (2010): 90C70, 90C29, 90C99

1. Introduction

Small and Medium Enterprise's (SME's) is defined as that manufacturing industry having full-time employees employed numbering not more than 150 people. The total number of registered SME's are 33113 based on the 2005 Associations and Enterprise Census, which contributes 29 per cent to the number in the manufacturing sector, 31 per cent to value-added and 44 per cent to total employment according to Ninth Malaysian Plan 2006-2010, Economic Planning Unit, Prime Minister Department. Merzifonluogly and Geunes, 2006 stated that to fulfill the demand at minimum cost, product manufacturing planning is required and further determined the optimum level of demand, supply and inventories for every planning schedule using the heuristic dual method. Hausman et al., 1998 considered various inventory systems which were represented by the multivariate normal distribution to satisfy 'the early bird gets the worm' rule further applied heuristic approach to study the probability of demands being optimally met. Jomalnia and Soukhakian, 2009 used the non-linear hybrid fuzzy goal programming approach with different goal priorities to aggregate production planning. Cunha and Mutarelli, 2007 proposed a spreadsheet based optimization model for the integrated problem of producing and distributing a weekend news magazines in Brazil reportedly reducing the costs by 7 per cent. Goswami et al., 2014 presented a multi-objective transportation problem whose transportation cost is varying due to capacity of 2-vehicles as well as transport quantities which is solved using fuzzy programming. Giri et al., 2014 formulated a fixed charge solid transportation problems under a budget constraint at each destination assuming transporting units to be crisp in nature. Further, crisp model is solved using Generalized Reduced Gradient (GRG) method.

Goal programming extends linear programming to problems which involve multiple objectives. Hassan and Ayop, 2012 proposed that it is necessary to specify aspiration levels for the objectives and aims to reduce the deviations from aspiration levels. Goal programming popularity is increasing day by day as it is useful in decision making policies which aim at optimizing resources available such as food product distribution of small and medium enterprises. Hassan and Loon, 2012 discussed the utility function for fund allocation of a university library, Hassan and Mohammad Basir, 2009 used scheduling political campaign visits, Hassan et al., 2012 used nutrient management for chilli plantation. In the case of a problem with nonequivalent goals the weight or priority of the goal is reflected through its deviation variables. Often, in real world problems the aspiration levels and/or priority factors of the DM, and sometimes even the weights to be assigned to the goals, are imprecise in nature. In such situations, Zadeh, 1965 introduced fuzzy set theory.

The use of fuzzy set theory in GP was first considered by Narasimhan, 1980, Hannan, 1981, 1982; Narasimhan, 1981; Ignizio, 1982. Rubin and Narsimha, 1984; Tiwari et al., 1985, 1986 have investigated various aspects of decision problem using FGP. An extensive review of these papers is given by Tiwari et al. in 1985. The main difference between fuzzy goal programming (FGP) and GP is that the GP requires the definite aspiration values set by DM for each objective that he/she wishes to achieve, whereas in FGP all these aspiration levels are specified in an imprecise manner. Hannan, 1981 assigns aspiration values for the membership functions of the fuzzy goals (which restricts the membership function from full achievement, i.e., unity) and uses the additive property to aggregate the deviational variables of the membership functions to minimize them. Throughout this paper a fuzzy goal is considered as a goal with imprecise aspiration level.

In conventional GP the simple additive model for m goals $G_i(x)$ with deviational variables p_i, n_i is defined as:

Minimize: $\sum_{i=1}^{m}(p_i + n_i)$

Subject to $\mathrm{G}_i(x)+n_i-p_i=g_i$, (1)

$p_i \cdot n_i = 0$,

$p_i, n_i, x \geq 0$, $i=1,2,...,m$,

where g_i represents the aspiration level of the *i-th* goal. Here we use a similar model using membership function instead of deviational variables.

2. Methodology

The proposed approach is based on the fuzzy goal programming (FGP). The objective of carrying out this study is to develop a FGP model to a real life production situation for a small and medium Enterprise (SME), a frozen food enterprise, based on seafood products in the Kuala Selangor district. The products of this company under consideration are frozen cockle fills, crab balls, squid balls, shrimp balls and fish nuggets. The company has to make sure that only the demands that are profitable should be fulfilled as demand exceeds supply. Thus, demand from each location is assumed to be fuzzy in nature. Also the monthly net profit is assumed to be fuzzy in nature to the allocated budget.

Fuzzy Goal Programming Model

Now, further consider the FGP problem formulated as:

Find X

To satisfy $G_i(X) \gtrsim g_i$, $i=1,2,...,m$, (2)

Subject to $AX \leq b$,

$X \geq 0$,

where X is an *n*-vector with components $x_1, x_2, ..., x_n$ and $AX \leq b$ are system constraints in vector notation. The symbol '$\gtrsim$'refers to the fuzzification of the aspiration level (i.e., approximately greater than or equal to). The ***i-th*** fuzzy goal $G_i(X) \gtrsim g_i$ in (2) signifies that the DM is satisfied even if less than the g_i upto certain tolerance limit is attained. A linear membership function μ_i for the *i-th* fuzzy goal $G_i(X) \gtrsim g_i$ can be expressed according to Zimmermann (1976, 1978) as:

$$\mu_i = \begin{cases} 1 & \text{if } G_i(X) \geq g_i \\ \dfrac{G_i(X)-L_i}{g_i-L_i} & \text{if } L_i \leq G_i(X) \leq g_i \\ 0 & \text{if } G_i(X) \leq L_i \end{cases}, \tag{3}$$

where L_i is the lower tolerance limit for the fuzzy goal $G_i(X)$. In case of the goal $G_i(X) \lesssim g_i$, the membership function is defined as:

$$\mu_i = \begin{cases} 1 & \text{if } G_i(X) \le g_i \\ \dfrac{U_i - G_i(X)}{U_i - g_i} & \text{if } g_i \le G_i(X) \le U_i \\ 0 & \text{if } G_i(X) \ge U_i \end{cases}, \tag{4}$$

where U_i is the upper tolerance limit.

The additive model of the FGP problem (2) is formulated by adding the membership functions together as:

Maximize $\quad \mathrm{V}(\mu) = \sum_{i=1}^{m} \mu_i$

Subject to $\quad \mu_i = \dfrac{G_i(X) - L_i}{g_i - L_i}$,

$AX \le b$,

$\mu_i \le 1$,

$$X, \mu_i \ge 0, i = 1, 2, ..., m, \tag{5}$$

where $\mathrm{V}(\mu)$ is called the fuzzy achievement function or fuzzy decision function.

There are three main demand location, namely Kulai, Ampang and Kuala Lumpur. The demand from every location differs according to customer needs. The delivery costs are also different due to the varying distance. A few assumptions are made.

1. The demand of the product is always uncertain, so it is assumed to be fuzzy in nature and one sets a certain demand level as the average monthly demand of that location.
2. The delivery costs are borne equally by both supplier and buyer.
3. The gross profit is calculated as the difference between total sales and production cost of each product.
4. The monthly net profit must be at least 30% of the allocated budget. It is also assumed to be fuzzy as it is uncertain.
5. All types of the i-th food product sent to all three locations must not be nil.

3. Fuzzy goal programming model development

1) The demand of the product is always uncertain, so it is assumed to be fuzzy in nature and one sets a certain demand level. As the company wants to increase the sales, therefore, our objective is to reduce the underachieved level of the set demand and want to increase it. So,

$$x_{ij} \gtrsim D_{ij} i = 1, 2, ..., 5, \; j = 1, 2, 3 \tag{6}$$

This implies that certain lower level from the set demand is acceptable.

2) A certain monthly budget is allocated for manufacturing of all 5 products, say B. The company does not want to exceed this budget strictly. A small increase in it is acceptable. So, allocated budget is assumed to be fuzzy in nature.

$$\sum_{i=1}^{5}\sum_{j=1}^{3} c_i x_{ij} \lesssim B \tag{7}$$

This implies that certain upper level from the set budget is acceptable.

3) The net profit should be 30% or more of the budget allocated. It is also assumed to be fuzzy as it is uncertain and thus certain lower level is also acceptable. Therefore,

$$\left(\sum_{i=1}^{5} a_i x_{i1} - \alpha_K\right) + \left(\sum_{i=1}^{5} a_i x_{i2} - \alpha_A\right) + \left(\sum_{i=1}^{5} a_i x_{i3} - \alpha_L\right) \gtrsim 0.3B \tag{8}$$

The first part of the above inequality represents the net profit from Kulai, followed by Ampang and Kuala Lumpur respectively. The net profit is the difference between the gross profit and the delivery cost to every location.

and rewritten as

$$\sum_{i=1}^{5} a_i x_{i1} + \sum_{i=1}^{5} a_i x_{i2} + \sum_{i=1}^{5} a_i x_{i3} \gtrsim 0.3B + \alpha_K + \alpha_A + \alpha_L \tag{9}$$

4) The monthly supply to each location must be within the minimum and maximum demands

$$S_i^l \le \sum_{j=1}^{3} x_{ij} \le S_i^u, i = 1, 2, ..., 5 \tag{10}$$

5) Supply of each product must be at least 1

$x_{ij} \geq 1$ i= 1,2,...,5, j = 1,2,3 (11)

where,

x_{ij} = Number of food products, i delivered at location j

D_{ij} = Demand of ith food product from location j

B = Monthly allocated budget

C_{ij} = Manufacturing cost of product i at location j

α_K = Delivery cost at Kulai

α_A = Delivery cost at Ampang

α_L = Delivery cost at Kuala Lumpur

a_i = Profit of profit i per kg

S_i^l = Minimum demand of product i

S_i^u = Maximum demand of product i

The list of data are listed in Table 1, Table 2, and Table 3.

I	Food product	Cost per kg	Sales per kg
1	Kerang beku	4.50	7.00
2	Bebola ketam	6.00	8.00
3	Bebola sotong	6.00	8.00
4	Bebola udang	6.00	8.00
5	Nuget ikan	6.00	8.00

Table 1: Costs and selling price for each food product

Food product	Supply		Demand		
	Minimum	Maximum	Kulai	Ampang	Kuala Lumpur
Kerang beku	2000	3000	2000	700	500
Bebola ketam	400	500	300	200	200
Bebola sotong	400	500	300	200	200
Bebola udang	400	500	300	200	200
Nuget ikan	400	500	300	200	200

Table 2: Supply and demand of products

Location	Kulai	Ampang	Kuala Lumpur
Delivery	400	200	200

Table 3: Delivery costs

Every month, the company allocates RM 28, 000 as a budget to produce frozen foods. Equation (6), (7) and (9) using the above data can be represented in the form:

$$(7.0-4.50)(x_{11}+x_{12}+x_{13})+2(x_{21}+x_{22}+x_{23}+x_{31}+x_{32}+x_{33}+x_{41}+x_{42}+x_{43}+x_{51}+x_{52}+x_{53})\gtrsim 0.3*28000+400+200+200$$

$$4.5(x_{11}+x_{12}+x_{13})+6(x_{21}+x_{22}+x_{23}+x_{31}+x_{32}+x_{33}+x_{41}+x_{42}+x_{43}+x_{51}+x_{52}+x_{53})\lesssim 28000$$

$$x_{11}\gtrsim 2000$$
$$x_{12}\gtrsim 700$$
$$x_{13}\gtrsim 500$$
$$x_{21}\gtrsim 300$$
$$x_{22}\gtrsim 200$$
$$x_{23}\gtrsim 200$$
$$x_{31}\gtrsim 300$$
$$x_{32}\gtrsim 200$$
$$x_{33}\gtrsim 200$$
$$x_{41}\gtrsim 300$$
$$x_{42}\gtrsim 200$$
$$x_{43}\gtrsim 200$$

$$x_{51} \gtrsim 300$$
$$x_{52} \gtrsim 200$$
$$x_{53} \gtrsim 200$$

Let the tolerance limits of the fuzzy goals be
((1500, 400, 400, 150, 100, 90, 160, 120, 100, 160, 100, 180, 100, 100, 150), 9000, 3000)
Thus the fuzzy goal programming problem is formulated as:

$$Maximize \sum_{i=1}^{17} \mu_i$$

Subject to

$$\mu_1 = \frac{x_{11} - 1500}{2000 - 1500}$$
$$\mu_2 = \frac{x_{12} - 400}{700 - 400}$$
$$\mu_3 = \frac{x_{13} - 400}{500 - 400}$$
$$\mu_4 = \frac{x_{21} - 150}{300 - 150}$$
$$\mu_5 = \frac{x_{22} - 100}{200 - 100}$$
$$\mu_6 = \frac{x_{23} - 90}{200 - 90}$$
$$\mu_7 = \frac{x_{31} - 160}{300 - 160}$$
$$\mu_8 = \frac{x_{32} - 120}{200 - 120}$$
$$\mu_9 = \frac{x_{33} - 100}{200 - 100}$$
$$\mu_{10} = \frac{x_{41} - 160}{300 - 160}$$
$$\mu_{11} = \frac{x_{42} - 100}{200 - 100}$$
$$\mu_{16} = \frac{\begin{array}{l} 2.5(x_{11} + x_{12} + x_{13}) + 2(x_{21} + x_{22} + x_{23} + x_{31} + x_{32} + x_{33} + x_{41} + x_{42} + \\ x_{43} + x_{51} + x_{52} + x_{53}) - 9000 \end{array}}{9200 - 9000}$$

$$\mu_{17} = \frac{30000 - (4.5(x_{11}+x_{12}+x_{13}) + 6(x_{21}+x_{22}+x_{23}+x_{31}+x_{32}+x_{33}+x_{41}+x_{42} + x_{43}+x_{51}+x_{52}+x_{53})}{30000-28000}$$

Hard constraints

$2000 \le x_{11} + x_{12} + x_{13} \le 3000$

$400 \le x_{21} + x_{22} + x_{23} \le 500$

$400 \le x_{31} + x_{32} + x_{33} \le 500$

$400 \le x_{41} + x_{42} + x_{43} \le 500$

$400 \le x_{51} + x_{52} + x_{53} \le 500$

$x_{ij} \ge 1$

where $i = 1,2,...,5$, $j = 1,2,3$

4. Results and discussion

Based on the problem formulation and set of constraints above, the fuzzy goal programming problem is then being solved by using the LINDO 11.0 optimizer solver package and the following results are obtained:

$x_{11} = 1500$, $x_{12} = 700$, $x_{13} = 500$, $x_{21} = 300$, $x_{22} = 200$, $x_{23} = 200$, $x_{31} = 250$, $x_{32} = 200$, $x_{33} = 200$, $x_{41} = 300$, $x_{42} = 200$, $x_{43} = 200$, $x_{51} = 300$, $x_{52} = 200$, $x_{53} = 200$

$\mu_1 = 0$, $\mu_2 = 1$, $\mu_3 = 1$, $\mu_4 = 0$, $\mu_5 = 0.5$, $\mu_6 = 1$, $\mu_7 = 0.643$, $\mu_8 = 1$, $\mu_9 = 1$, $\mu_{10} = 1$, $\mu_{11} = 1$, $\mu_{12} = 1$, $\mu_{13} = 1$, $\mu_{14} = 1$, $\mu_{15} = 1$, $\mu_{16} = 0.675$, $\mu_{17} = 0.039$

From the above result, we can conclude that demand of all the products is satisfied in Ampang and Kuala Lumpur but not in Kulai for products Kerang beku and Bebola ketam. Also the manufacturing cost is RM 28650, which exceed the allocated monthly budget of RM 28000 by RM650. The net profit is found to be RM 12250 which is more than RM 9200, the 30 percent of the total budget by RM3050.

It can be seen that the FGP model is a useful tool for Small and Medium Enterprises to satisfy the growing demands of their markets by determining their production planning. The number of goals to be considered can also be increased based on the desirability of the decision maker in relation to their aspired objectives.

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