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Emotion Classification through Musical Features using Fuzzy Relational Maps (FRMs)

A. Victor Devadoss [1] **and** ***S. Aseervatham*** [2]

[1]PG & Research Department of Mathematics, Loyola College,
Chennai – 600034, TamilNadu, India. E-mail: hanivictor@ymail.com

[2]PG & Research Department of Mathematics, Loyola College,
Chennai – 600034, TamilNadu, India. E-mail: aseerambro.math@gmail.com

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Abstract. In this paper we study to find out the vital reason for evoking emotions from different musical features selection using Fuzzy Relational Maps (FRMs). Classification of the musical features for the song segment or any composition in bringing emotion is a challenging problem. Since human perception for taking decision comes under subjective nature, musical arrangement of the song varies from one emotion to other. For each emotion class, this approach deals with the necessary concentration on musical features. Results are shown to illustrate the effectiveness of the approach. In the first section introduction part is given. We study the fundamentals of Fuzzy Relational Maps (FRMs) in section two. Third Section deals with the adaptation of FRM to the problem. The final section derives the conclusion based on our study.

Keywords: Fuzzy relational maps, Limit cycle, fixed point, melody, timbre.

AMS Mathematics Subject Classification (2010): 94D05

1. Introduction

Music has received a great deal of attention in communication of emotions during the last years. The role of individual musical features in predicting the emotions suggested or invoked by the music have been investigated in a large number of empirical studies. More specifically, Musical features contributing to emotions have been premeditated systematically since Kate Hevner's pioneering work. Contemporary summaries of the musical features connected to particular emotions in music have been offered by Gabrielsson and Lindstrom research (2010) and Juslin and Laukka (2003). The structural features of music are systematically varied to see their effect on the ratings of emotions in the sense of causal.

Here the causal relationship between musical features and emotions are studied using fuzzy approach. These causal relationships are often analyzed by fuzzy logic. We employ one of the fuzzy models called Fuzzy Relational Maps in our emotion classification system. In the year 2000, this method was introduced by Dr. W.B. Vasantha and Yasmin Sultana. In FRMs we divide the very casual associations into two disjoint units, like for example the relation between the symptom and disease or relation between causes and

effects and so on. In this study, we focused on the direct relationship between basic perceptual musical properties and emotions.

2. Methodology of fuzzy relational maps

We need a domain space and a range space to define a FRM which are disjoint in the sense of concepts. Moreover Within the elements in the domain and the range space we assume that there is no intermediate relations exist. In general the number of elements in the range space need not be equal to the number of elements in the domain space.

Fuzzy Relational Maps (FRMs) 2.1 Here the elements of the domain space and the range space are taken from the real vector space of dimension n and m (m in general need not be equal to n) respectively.

Let $D_1,\ldots,D_n$ be the nodes in the domain space where $D_i=\{x_1,\ldots,x_n) / x_i = 0 \text{ or } 1\}$ for $i = 1,\ldots,n$. If $x_i = 1$, it means that the node D_i is in the ON stage and if $x_i = 0$ it means that the node D_i is in the OFF state. Similarly the nodes in the range space are $R_1,\ldots,R_m$, where $R_j = \{x_1,x_2,\ldots,x_m)/ x_j = 0 \text{ or } 1\}$ for $j = 1,\ldots,m$. If $x_j = 1$ it means that the node R_j is in the ON state and if $x_j = 0$ it means that the node R_j is in the OFF state.

Edge Weight of FRM 2.2 A FRM is a directed graph or a map from D to R with concepts like policies or events etc as nodes and causalities as edges. The relations between spaces D and R will be represented in causal. Let D_i and R_j denote that the two nodes of an FRM. The directed edge from D_i to R_j denote the causality relations of D_i on R_j. FRM edges are weighted with the numbers in the set $\{0,\pm1\}$. Let e_{ij} be the weight of the edge $DiRj$, $e_{ij} \in \{0,\pm1\}$.

$$e_{ij} = \begin{cases} 1 & ; \text{ if increase (decrease) in } D_i \text{ implies increase (decrease) in } R_j \\ 0 & ; \text{ if } D_i \text{ does not have any effect on } R_j \\ -1 & ; \text{ if increase (decrease) in } D_i \text{ implies decrease (increase) in } R_j \end{cases}$$

Here the case of increase in D_i implies decrease in R_j (vice versa) is not considered in our problem.

Fuzzy Nodes and Simple FRM 2.3 If FRM nodes are fuzzy sets then they are called fuzzy nodes. Simple FRMs have the edge weights as $\{0, \pm1\}$.

Relational Matrix 2.4 Let $D_1,\ldots,D_n$ be the nodes of the domain space D of an FRM and $R_1,\ldots,R_m$ be the nodes of the range space R of an FRM. Let the matrix E be defined as $E = (e_{ij})$ where e_{ij} is the weight of the directed edge $D_i R_j$ (or $R_j D_i$), E is called the relational matrix of the FRM.

Instantaneous Stage Vector 2.5 Let $D_1,\ldots,D_n$ and $R_1,\ldots,R_m$ denote the nodes of the FRM. Let $A = (a_1,\ldots,a_n)$; $a_i \in \{0, \pm1\}$. A is called the instantaneous stage vector of the domain space and it denotes the ON-OFF position of the nodes at any instant. Similarly let $B = (b_1,\ldots,b_m)$; $b_j \in \{0,\pm1\}$. B is called instantaneous stage vector of the range space and it denotes the ON-OFF position of the nodes at any instant. $a_i = 0$ if a_i is OFF and $a_i = 1$ if a_i is ON for $i = 1,2,\ldots,n$. Similarly, $b_j = 0$ if b_j is OFF and $b_j = 1$ if b_j is ON, for $j = 1,2,\ldots,m$.

Cycle and Acyclic 2.6 Let $D_1,\ldots,D_n$ and $R_1,\ldots,R_m$ denote the nodes of the FRM. Let D_i R_j (or R_j D_i) be the edges of an FRM, j=1,2,…,m and i = 1,2,…,n. Let the edges form a directed cycle. An FRM is said to be a cycle if it possesses a directed cycle. An FRM is said to be acyclic if it does not possesses any directed cycle.

FRM with Feedback 2.7 An FRM with cycles is said to be an FRM with feedback.

Dynamical System 2.8 When there is a feedback in the FRM, (i.e., when the causal relation flow through a cycle in a revolutionary manner), the FRM is called a dynamical system.

Hidden Pattern 2.9 Let D_i R_j (R_jD_i), $1 \le j \le m$, $1 \le i \le n$. When R_i (or D_j) is switched on and if causality flows through edges of the cycle and if it again causes R_i (or D_j), we say that the dynamical system goes round and round. This is true for any node R_j (or D_i) for $1 \le j \le m$ (or $1 \le i \le n$). The equilibrium stage of this dynamical system is called the hidden pattern.

Fixed Point 2.10 If the equilibrium state of a dynamical system is a unique stage vector, then it is called a fixed point. Consider an FRM with $R_1,R_2,\ldots, R_m$ and $D_1,D_2,\ldots D_n$ as nodes. For example, let us start the dynamical system by switching on R_1 (or D_1). Let us assume that the FRM settles down with R_1 and R_m (or D_1 and D_n) i.e. the stage vector remains as (1, 0,…,0,1) in R (or 1,0,0,…,0,1) in D). This state vector is called the fixed point.

Limit Cycle 2.11 If the FRM settles down with a state vector is repeating in the form $A_1\rightarrow A_2\rightarrow A_3\rightarrow\ldots\rightarrow A_i\rightarrow A_1$ (or $B_1\rightarrow B_2\rightarrow \ldots\rightarrow B_i\rightarrow B_1$) then this equilibrium is called a limit cycle. Now we give the methods of determining the hidden pattern.

Method of Finding Hidden Pattern 2.12
Let R_1, $R_2,\ldots,R_m$ and $D_1,D_2,\ldots,D_n$ be the nodes of a FRM with feedback. Let E be the relational matrix. Let us find a hidden pattern when D_i is switched on i.e. when an input is given as vector A_1 = (1,0,…,0) in D_1, the data should pass through the relational matrix E. This is done by multiplying A_1 with the relational matrix E. Let A_1E = ($r_1,r_2,\ldots,r_m$). After thresholding and updating the resultant vector we get $A_1E\in R$. Now let $B = A_1E$, we pass on B into E^T and obtain BE^T. We update and threshold the vector BE^T so that BE^T $\in D$. This procedure is repeated till we get a limit cycle or a fixed point.

3. Description of problem and finding hidden pattern using FRM
We take the following attributes in the case of emotion classification for musical terms as domain space elements of FRM.

Emotional Terms considered for the analysis			
D_1 – Happy / Joy	D_2 – Sad / Gloom	D_3 – Energy / Activity / Excitement	
D_4 – Potency	D_5 – Tension	D_6 – Relaxation / Calm / Softness / Peace	
D_7 – Solemnity / Dignity	D_8 – Anger	D_9 – Fear	
D_{10} – Tenderness / Love	D_{11} – Boredom	D_{12} – Disgust	D_{13} – Surprise

Table 1: Emotion Terms

The following musical features are collected through expert opinions, survey and theoretical studies for the basic corresponding emotions described in this problem.

We take the following attributes in the case of musical features in Western Classical Music as range space elements of FRM.

R_1 – Mode: Major - Melody is the linear/horizontal presentation of pitch. In melody, Major scale is made up of seven notes: the eighth duplicates the first at double its frequency. The sequence of intervals between the notes of a major scale is: whole, whole, half, whole, whole, whole, half.

R_2 – Mode: Minor - There are three variations of the minor scale: natural, harmonic and melodic. A natural minor scale is that the 3rd, 6th and 7th note positions of the major scale are lowered by one half-tone / semitone. A harmonic minor scale is that the 7th note position of the natural minor scale is raised by one half-tone / semitone. For ascending scale notes, A melodic minor scale is that the 6th and 7th note positions of the natural minor scale are raised by one half-tone / semitone.

R_3 – Melodic Range: Wide – It is the distance between the lowest and highest pitches greater than an octave. Melodic range may be defined as the distance between the lowest and highest pitches of a melody. If there are a large number of notes between the lowest and highest pitches, the melody is said to have a wide range.

R_4 – Melodic Range: Narrow - If only a few notes separate the lowest and highest pitches, the melody is said to have a narrow range.

R_5 – Melodic Direction: Ascending - Melodic motion is the quality of movement of a melody. The movement of the melody often is in ascending direction.

R_6 – Melodic Direction: Descending - The movement of the melody often is in descending direction.

R_7 – Pitch Level: High – Distribution of high notes (in terms of frequency) in a composition.

R_8 – Pitch Level: Low - Distribution of low notes (in terms of frequency) in a composition.

R_9 – Pitch Variation: Large - The property of sound that varies with large variation in the frequency of vibration.

R_{10} – Pitch Variation: Small - The property of sound that varies with small variation in the frequency of vibration.

R_{11} – Rhythm: Regular - Rhythm is the distribution or arrangement of the tones according to their various time-values. Rhythmic arrangement is regular when the comparatively longer tones occupy the accented beats.

R_{12} – Rhythm: Irregular - Rhythmic arrangement is regular when shorter tones occupy the accents.

R_{13} – Tempo: Fast - the speed of the beat. Moderato, allegro (faste 'happy') and presto (very fast) are the classification for fast.

R_{14} – Tempo: Slow – Largo (labored slowly), adagio (slow) and andante (steady 'walking ') are the classification for slow.

R_{15} – Loud: Louder - Loudness are expressed mostly in the Italian language. Crescendo (gradually louder), mezzo forte (moderately loud), forte (loud), fortissimo (very loud) are the classification of loudness.

R_{16} – Loud: Softer – Pianissimo (very soft), decrescendo (gradually soft), piano (soft), diminuendo (gradually softer), mezzo piano (moderately soft) are the classification of softness.

R_{17} – Harmony: Consonance – The effect of certain triads in a scale is said to be "restful and stable". This condition of any chord, interval or tone in music is known as consonance. A harmonic combination that is stable, usually in thirds.

R_{18} – Harmony: Dissonance – The effect of other triads is said to be "active and unstable". This condition of any chord, interval or tone in music is known as dissonance. A harmonic combination that is unstable, often including seconds or sevenths.

R_{19} – Timbre: Few harmonics / Soft timbre - Sound quality of a sound source is timbre or tone color in music. Music timbre contributes greatly to the effect of mood in music.

R_{20} – Timbre: Many Harmonics / Sharp timbre. Timbre is the characteristic that allows us to distinguish between one instrument and another.

R_{21} – Articulation: Staccato - Playing notes shorter and separating them from each other is called staccato.

R_{22} – Articulation: Legato - Playing notes longer and connecting them to each other is called legato.

R_{23} – Envelope: Sharp - Refers to the type of attack and decay of tones which is rapid attack and decay. That is fast tone attack and fast decay.

R_{24} – Envelope: Round – Slow tone attack and slow decay

The relational graph for the emotions and musical features is shown below.

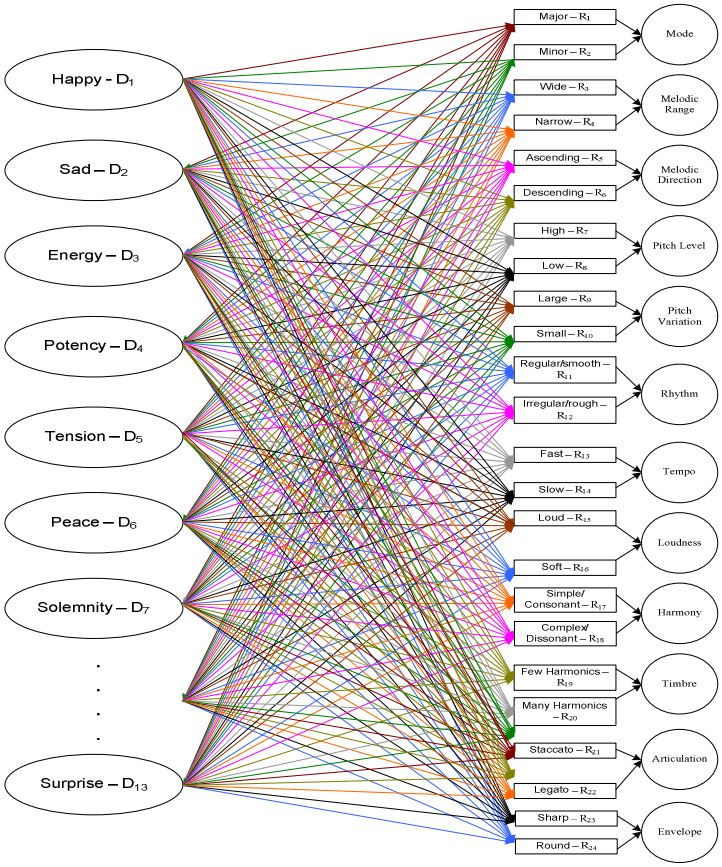

Figure 1: Relational graph

Now using the expert's opinion who is a music composer, we have constructed the following relational matrix.

$$E = \begin{pmatrix}
1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&0&1&1&0&1&0\\
0&1&0&1&0&1&0&1&0&0&0&0&0&1&0&1&0&1&1&0&0&1&0&1\\
0&0&0&0&0&0&1&1&1&0&0&0&1&0&1&0&0&1&0&1&1&0&1&0\\
0&0&0&0&0&0&1&0&0&0&0&0&1&0&1&0&0&0&0&1&0&0&0&1\\
0&1&0&0&1&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0\\
1&0&0&1&0&0&1&1&0&0&1&0&0&1&0&1&1&0&0&0&0&1&0&0\\
1&1&0&0&0&0&0&1&0&0&1&0&0&1&1&0&1&0&0&0&0&1&1&0\\
0&1&0&0&1&0&1&0&0&1&0&1&1&0&1&0&0&1&0&1&1&1&1&0\\
0&0&0&0&1&0&1&0&0&1&0&0&1&1&0&1&0&1&1&1&1&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&1&0&0&1&0&1\\
0&0&0&0&0&1&0&1&0&1&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&1&0&0&0&1\\
0&0&0&0&1&0&1&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0
\end{pmatrix}$$

In order to study the effect of each attribute on the dynamical system E, the hidden pattern of the dynamical system E (relational matrix) for each state vector is to be found.

Suppose node D_1 is in the ON state and all other nodes are in off state, the causes of the state vector X_1 = (1 0 0 0 0 0 0 0 0 0 0 0 0) on the dynamical system E is given as follows:

X_1E = (1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 0 1 0)
↪ (1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 0 1 0) = Y_1

Y_1E^T = (12 0 7 4 2 4 5 7 6 0 0 1 6)
↪ (1 0 0 0 0 0 0 0 0 0 0 0 0) = X_2 (=X_1)

X_2E = (1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 0 1 0)
↪ (1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 0 1 0) = Y_2 (=Y_1)

The resultant vector is a fixed point given by the binary pair {(1 0 0 0 0 0 0 0 0 0 0 0 0), (1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 0 1 0) }. Similarly we do this procedure when other attribute is in ON state. The fixed points are shown in the following table.

Input	Output
D_1	(1 0 0 0 0 0 0 0 0 0 0 0 0) , (1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 0 1 0)
D_2	(0 1 0 0 0 0 0 0 0 0 0 0 0) , (0 1 0 1 0 1 0 1 0 0 0 0 0 1 0 1 0 1 1 0 0 1 0 1)
D_3	(0 0 1 0 0 0 0 0 0 0 0 0 0) , (0 0 0 0 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 1 1 0 1 0)
D_4	(0 0 0 1 0 0 0 0 0 0 0 0 0) , (0 0 0 0 0 0 1 0 0 0 0 0 1 0 1 0 0 0 0 1 0 0 0 1)
D_5	(0 0 0 0 1 0 0 1 0 0 0 0 0) , (0 1 0 0 1 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0)
D_6	(0 0 0 0 0 1 0 0 0 0 0 0 0) , (1 0 0 1 0 0 1 1 0 0 1 0 0 1 0 1 1 0 0 0 0 1 0 0)
D_7	(0 0 0 0 0 0 1 0 0 0 0 0 0) , (1 1 0 1 0 0 0 1 0 0 1 0 0 1 1 0 1 0 0 0 0 1 1 0)
D_8	(0 0 0 0 0 0 0 1 0 0 0 0 0) , (0 1 0 0 1 0 1 0 0 1 0 1 1 0 1 0 0 1 0 1 1 1 1 0)
D_9	(0 0 0 0 0 0 0 0 1 0 0 0 0) , (0 0 0 0 1 0 1 0 0 1 0 0 1 1 0 1 0 1 1 1 1 0 1 1)
D_{10}	(0 1 0 0 0 0 0 0 0 1 0 0 0) , (0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 1 0 0 1 0 1)
D_{11}	(0 0 0 0 0 0 0 0 0 0 1 0 0) , (0 0 0 0 0 1 0 1 0 1 0 0 0 1 0 0 0 0 1 0 0 0 0 1)
D_{12}	(0 0 0 0 0 0 0 0 1 0 0 1 0) , (0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 1 0 0 0 1)
D_{13}	(1 0 0 0 0 0 0 0 0 0 0 0 1) , (0 0 0 0 1 0 1 0 1 0 0 0 1 0 0 0 0 0 0 1 0 0 1 0)

Table 2: Fixed points

4. Conclusion

We derived the recommendations for evoking emotion through musical structure as follows:

- The combination of Major mode with wide range of melodic portion in ascending direction, high pitch notes, large pitch variation, regular rhythmic pattern with fast tempo, loudness, consonance, many harmonies in timbre, and staccato, sharp envelope will be concentrated while making happy/joyful musical excerpts.

- To bring out sad emotions in musical excerpts, we should have the musical features such as Minor mode with narrow melodic range in descending direction, low pitch notes, slow tempo, soft loudness, dissonance harmony, few harmonies in timbre, legato articulation and round envelope.

- When musical excerpts have the qualities of high or low pitch notes, large pitch variation, fast tempo, loud, dissonance, many harmonies, staccato, sharp envelope, it is easier to promote energetic feelings.

- The combination of high pitch level, fast tempo, loudness, many harmonies and round envelope features can be concentrated while making potential activity musical excerpts.

- While making emotions such as tension and anger we need to focus on the combination of minor mode, ascending, loudness, dissonance in harmony.

- Peaceful emotion can be got through the combination of major mode with narrow melodic range in high or low pitch notes, regular rhythmic pattern with slow tempo, softer loudness, consonance, legato articulation.

- To have solemnity/dignity feelings through musical excerpt, the combination of major or minor mode, narrow range, low pitch level, regular rhythmic pattern with slow tempo, loudness, consonance, legato, sharp envelope will be pondered.

- The combination of minor mode, ascending, high pitch level, small variation, irregular with fast tempo, loud, dissonance, many harmonies, staccato or legato, sharp will bring out anger.

- To create fear through music, we should give attention to the combination of ascending direction, high pitch level, small variation, fast or slow tempo, softness, dissonance, few or many harmonies, staccato, sharp or round envelope.

- While evoking emotions such as sad and love, we need to ponder on the combination of slow tempo, softer loudness, few harmonies in timbre, legato articulation, round envelope.

- Boredom feelings can be expressed through the combination of descending direction of notes, low pitch level, small variation, slow tempo, few harmonies, round envelope in music.

- To evoke fear and disgust feelings, we should have small pitch variation, slow tempo, many harmonies, round envelope in musical features.

- The combination of ascending melodic direction, high pitch notes with large pitch variation, fast tempo, many harmonies, and sharp envelope will bring out happy and surprising feelings.

These are the necessary combination of attributes to be concentrated on musical excerpts to evoke the respective emotions.

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REFERENCES

1. Bart Kosko, *Neural Networks and Fuzzy Systems*, Prentice – Hall, Englewood Cliffs, New Jersey, 1992.
2. Byeong-jun Han, Seungmin Rho, Sanghoon Jun and Eenjun Hwang, Music emotion classification and context-based music recommendation, *Multimedia Tools and Applications*, 47 (03) (2010) 433-460.

3. D.Ghosh and A.Pal, Use of fuzzy relational maps and intuitionistic fuzzy sets to analyze health problem of agricultural labourers, *Annals of Pure and Applied Mathematics*, 5(1) (2013) 1-10.
4. John A. Sloboda, Music Structure and Emotional Response: Some Empirical Findings, *Psychology of Music*, 19 (1991) 110-120.
5. T.Pathinathan, K.Thirusangu and Mary John, On causes for school dropouts – a fuzzy approach, *Acta Ciencia Indica*. XXXI M, 4 (2005) 1279-1299.
6. P.K.Giri, M.K.Maiti and M.Maiti, Solid transportation problem under budget constraint using fuzzy measure, *Annals of Pure and Applied Mathematics*, 7(1) (2014) 41-46.
7. S.Beveridge and D.Knox, An exploration of the effect of structural and acoustical features on perceived musical emotion, *Proceedings of the 4th Audio Mostly Conference on Interaction with sound, Scotland*, 98, 2009.
8. Sylvain Le Groux and F.M.J.Paul and Verschure, Subjective emotional responses to musical structure, expression and timbre features: a synthetic approach, *9th ISCMMR*, June, (2012) 160-176.
9. T.Eerola, Modeling listeners' emotional responses to music, *Topics in Cognitive Science* 4 (2012) 607-624.
10. J.D.Richard, K.R.Scherer and H.Hill Goldsmith, Handbook of Affective Sciences, *Series in Affective Sciences*, Oxford University Press, 2003.
11. W.B.Vasantha Kandasamy, Florentin smarandache and ilanthenral, *Elementary Fuzzy Matrix Theory and Fuzzy Models for Social Scientists*, Printed in United States of America, 2007.
12. W.B.Vasantha Kandasamy and S.Yasmin, FRM to analyse the employee–employer relationship, *Journal of Bihar Mathematical Society*, 21(2001) 25-34.
13. A.Victor Devadoss and S.Aseervatham, A comparative study between properties of carnatic ragas and emotions of devotional songs using bidirectional associative memories, *International Journal of Engineering and Research Technology*, 2(6) (2013) 1380-1385.
14. W.B.Vasantha Kandasamy, F.Smarandache and K.Amal, *Super Fuzzy Matrices and Super Fuzzy Models for Social Scientists*, InfoLearnQuest, Printed in the United States of America, 2008.
15. Zadeh L A., Fuzzy sets, *Information and Control*, 8 (1965) 338-353.
16. Zentner, M. and Eerola, T., Self-report measures and models, *Handbook of Music and Emotion: Theory, Research, Applications*. Juslin, P. & Sloboda, J. (eds.)., 2011.
17. http://www.potsdam.edu/academics/Crane/MusicTheory/Musical-Terms.