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The Cycle Satisfactory Roommates Problem

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Abstract. The most efficient treatment for kidney failure is called as transplanta -tion. Recently, many interesting algorithmic problems have arisen from various countries, in the case of kidney exchange schemes, whereby live donors are matched with recipients according to compatibility and other considerations. There are attempts to organize exchanges between patient-donor pairs. One such problem can be modeled and solved by Irving. In the same way we have discussed the problem by using another technique called Cycle satisfactory matching, and proposed an algorithm for finding the solution, which gives the same matching what the existing technique gives.

Keywords: Kidney transplantation, Optimal matching, Preference Value, Satisfactory value matrix, Satisfactory Level, Cycle Satisfactory matching

AMS Mathematics Subject Classification (2010): 05C70, 91B68, 93A30

1. Introduction

The stable roommates problem (SR) was introduced by Gale and Shapley [3] in which, there is a single set of participants in even size. It illustrates a real problem used in colleges to match students to share double rooms. In this setting each person strictly ranks all others in to a preference list, and we are looking for a matching such that no two persons prefer one another to their assigned partners. Such a matching is called stable matching, and it admits no blocking pair. The most notable difference between SM and SR is that an instance of SR need not admit a stable matching. A solution for the stable roommates' problem was proposed by Irving [6] as an algorithm that either finds a stable matching or concludes that no stable matching exists. Later Tan [11] used this algorithm to give a good characterization; Subsequently Gusfield [4] presented an analysis of the structure of SR instances, and exploited this structure to solve variants of the problem. The monograph of Gusfield and Irving [5] includes a comprehensive treatment of these results.

Kidney exchange is still a controversial issue, it is often the case that an individual who requires a kidney. In this setting, cycles should be as short as possible for logistical, medical reasons which mean that not all compatible donors are equally good for a particular recipient. This situation can be modeled by each recipient having an ordered list of preferences over the compatible donors. By representing each donor–recipient pair (d_i, r_i) as a participant p_i, we can construct a stable roommates instance to represent the set of available participants. The objective is to allocate donors to recipients in some optimum way, and if we wish to keep cycle lengths to two, as mentioned earlier, then we are seeking a matching of participants in the derived roommates' instance.

In the kidney exchange context, a full discussion of the background is given by A.E Roth et al [8], Cechlárová et al [1,2] have posed the question as to whether there is a polynomial-time algorithm to determine whether a cycle stable matching exists, and if so to find such a matching. Here we show that the decision problem is NP-complete. We restrict our attention to what might be termed "classical" stable roommates, where there is an even number of participants and all pairs are acceptable, so that any stable matching must match all participants. Clearly the NP-completeness of the special case implies the same conclusion for the general problem.

In this work we are finding the cycle satisfactory matching, in our point of view it is better than cycle stable matching because, if we are having more parameters in kidney transplantation, then we can examine all the parameters by using satisfaction concept. So that finally we can get a better result in all aspects, which is very helpful to find the cycle satisfactory matching.

2. Cycle Satisfactory Roommates Problem (CSFRP)

A cycle satisfactory matching is a set of n participants with n even, and for each participant having a totally ordered preference list containing all the other participants. This is also like same that of classical satisfactory roommates' problem, where there is an even number of participants and all pairs are acceptable, so that any satisfactory matching must match all participants. In this setting avoid one to one mapping to get cycle satisfactory roommates'. Clearly some of the cycle satisfactory matching problems are unsolvable, which must be like in the case of satisfactory roommates' problem. In this paper we applied our own technique to solve the problem, which is given by R.W.Irving [9]. For satisfactory roommates' problem and its related concepts refer [10]

3. Cycle Satisfactory Roommates Algorithm (CSFRA)

The main steps of the algorithm are listed below.

Step1: Get the preference lists of all members

Step 2: Form a satisfactory Value Matrix (SVM)

Step 3: Apply Hungarian method to find optimal (satisfactory) matching for all members such that the total assignment value should be maximized. That indicates optimal satisfactory level.

Step 4: In the SVM choose two better satisfactory level results in each row

Step 5: Create a multidimensional array with indexing as the member list and better satisfactory level as array values. Eg array list

Step 6: Using a looping function with initial array index parameter as 1 to move through all the indexing

Step 7: Check if the visiting array index in loop is not the same as the initiated one if same loop to next index.

Step 8: Print the array index value

Step 9: Initiate the visiting array index to 1

Step 10: Loop though the satisfactory values of that index

Step 11: If the value is same as the initiated one continue with step 10

Step 12: If value not found continue with the step 6 with parameter value as the current index value of the step 10.

Example 1. Consider the problem instance of cycle satisfactory matching based on order of preference [9].

1	3 5 2 6 4
2	1 4 6 3 5
3	6 2 5 1 4
4	2 6 5 3 1
5	6 3 1 4 2
6	1 4 3 2 5

$$SVM = \begin{array}{c c} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \begin{pmatrix} - & \frac{8}{5} & \frac{7}{5} & \frac{2}{5} & \frac{7}{5} & \frac{7}{5} \\ \frac{8}{5} & - & \frac{6}{5} & \frac{9}{5} & \frac{2}{5} & \frac{5}{5} \\ \frac{7}{5} & \frac{6}{5} & - & \frac{3}{5} & \frac{7}{5} & \frac{8}{5} \\ \frac{2}{5} & \frac{9}{5} & \frac{3}{5} & - & \frac{5}{5} & \frac{8}{5} \\ \frac{7}{5} & \frac{2}{5} & \frac{7}{5} & \frac{5}{5} & - & \frac{6}{5} \\ \frac{7}{5} & \frac{5}{5} & \frac{8}{5} & \frac{8}{5} & \frac{6}{5} & - \end{pmatrix} \end{array}$$

Reduced preference list

1	5 2
2	1 4
3	6 5
4	2 6
5	3 1
6	4 3

Cycle satisfactory matching for the above instance is 1→5→3→6→4→2→1.
By using Irvings Algorithm [6] in the given instance

1→ ~~3~~ 5
2→ 1
3→ 6
4→ 2
5→ ~~6~~ 3
6→ ~~1~~ 4

We get the cycle stable matching as 1→5→3→6→4→2→1
From the above Instance we got same Cycle stable matching and Cycle satisfactory matching.

Example 2. Consider the problem instance of cycle satisfactory matching based on order of preference [9].

1	3 2 5 4 6
2	1 4 5 6 3
3	5 4 2 6 1
4	3 2 6 1 5
5	1 6 4 3 2
6	5 4 2 1 3

The above Instance gives no cycle satisfactory matching as well as cycle stable matching.

4. Conclusion

In this work we find the cycle satisfactory matching by using our own method. In our point of view it is an alternate method for the previous one, and our result is matched with the existing result, that was given by Irving [9].

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