Annals of Pure and Applied Mathematics
Vol. 9, No. 2, 2015, 183-189
ISSN: 2279-087X (P), 2279-0888(online)
Published on 10 March 2015
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Time to Recruitment in a Two Grade Manpower System under Two Sources of Depletion Associated with Different Renewal Processes

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Received 18 February 2014; accepted 3 March 2015

Abstract. In this paper for a two grade manpower system with two sources of depletion having three components for the breakdown threshold, analytical results for some performance measures related to time to recruitment are obtained using a univariate policy of recruitment by considering different forms of the threshold for the loss of manpower in the manpower system.

Keywords: Two grade manpower system, two sources of depletion of manpower, threshold with three components, univariate policy of recruitment and performance measures for time to recruitment

AMS Mathematics Subject Classification (2010): 60K05, 60K10, 62N05

1. Introduction

In any organization depletion of manpower occurs due to policy decisions (forming one source) and due to transferring the personnel to sister organizations (forming another source). As immediate recruitment after depletion of each personal is not advisable due to cost and time consumption, recruitment is postponed to a point of time beyond which normal activities cannot be continued due to shortage of manpower. This level of allowable manpower depletion is called threshold. Elangovan et.al [6] have initiated the study on recruitment problem for a single grade manpower system with two sources of depletion and obtained the variance of time to recruitment using univariate CUM policy of recruitment when the loss of man power in the organization due to the two sources of depletion, inter-policy decision times, inter-transfer decision times and threshold for the loss of man power in the organization are independent and identically distributed exponential random variables. Arivazhagan et.al[1] have determined the mean time to recruitment for a single manpower system with policy decisions forming the only one source of depletion when the threshold has three components namely normal threshold of depletion of manpower, threshold of frequent breaks of existing workers and threshold of backup or reservation of manpower sources. In [2], Dhivya and Srinivasan have extended the work of Elangovan et.al [7] for a two grade manpower system for different forms of the thresholds for the cumulative loss of manpower in the organization when the inter-policy decisions and inter-transfer decisions form the same ordinary renewal process. In[3] Dhivya and Srinivasan have studied this problem when the renewal processes are different. In [4], Dhivya and Srinivasan have studied their work in [2] when the policy decisions are classified into two types according to the intensity of attrition. Later in [5] Dhivya and Srinivasan have studied their work in [2] and [4] when threshold for the cumulative loss of manpower has three components. The objective of the present paper is to study the problem of time to recruitment work in [3] when the threshold has three components.

2. Model description

For i=1,2,3…, let X_{Ai} and X_{Bi} be the continuous random variables representing the amount of depletion of manpower(loss of man hours) in grades A and B respectively caused due to the ith policy decision. It is assumed that X_{Ai} and X_{Bi} are independent for each i and each form a sequence of independent and identically distributed random variables with distributions $G_A(.)$ and $G_B(.)$ and probability density functions $g_A(.)$ and $g_B(.)$ respectively. Let $\bar{X}_{Am_1}$ and $\bar{X}_{Bm_2}$ be the total depletion of manpower in the first m_1 and m_2 policy decisions in grades A and B respectively. For j=1,2,3…,let Y_{Aj} and Y_{Bj} be the continuous random variables representing the amount of depletion of manpower in grades A and B respectively caused due to the jth transfer decision. It is assumed that Y_{Aj} and Y_{Bj} are independent for each j and each form a sequence of independent and identically distributed random variables with probability density functions $h_A(.)$ and $h_B(.)$ respectively. Let $\bar{Y}_{An_1}$ and $\bar{Y}_{Bn_2}$ be the total depletion of manpower in the first n_1 and n_2 transfer decisions in grades A and B respectively. Let $\bar{X}_{m_1,n_1}$ be the cumulative depletion of manpower in the organization due to the first m_1 policy and n_1 transfer decisions in grade A. Let $\bar{Y}_{m_2,n_2}$ be the cumulative depletion of manpower in the organization due to the first m_2 policy and n_2 transfer decisions in grade B. For each i and j X_{Ai}, X_{Bi}, Y_{Aj} and Y_{Bj} are statistically independent. Let $\bar{s}_k(.)$ be the Laplace transforms of $s_k(.)$. Let Z be the threshold level for the loss of manpower in the organization. Let Z_{A1} be the normal threshold of depletion of manpower, Z_{A2} be the threshold of frequent breaks of existing workers and Z_{A3} be the threshold of backup or reservation of manpower sources for grade A. Let Z_{B1}, Z_{B2} *and* Z_{B3} be the normal threshold of depletion of manpower, threshold of frequent breaks of existing workers and threshold of backup or reservation of manpower sources for grade B respectively. Let $k(.)$ be the probability density function of Z respectively. Let the inter-policy decision times for grades A and B be independent and identically distributed exponential random variables with distribution $F(.)$ and U(.), probability density function $f(.)$ and u(.) and mean $\frac{1}{\mu_{1A}}$ and $\frac{1}{\mu_{1B}}$ $(\mu_{1A}, \mu_{1B} > 0)$ respectively . Let the inter-transfer decision times for grades A and B be independent and identically distributed exponential random variables with distribution $W(.)$ and V(.), probability density function $w(.)$ and v(.) and mean $\frac{1}{\mu_{2A}}$ and $\frac{1}{\mu_{2B}}$ $(\mu_{2A}, \mu_{2B} > 0)$ respectively. It is assumed that the two sources of depletion are independent. Let $\delta_m(.)$ be the m-fold convolution of $\delta(.)$ with itself. The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the threshold for the loss of man hours in this organization

Let T be the random variable denoting the time to recruitment with mean E(T) and variance V(T). Let $N_P(T)$ be the number of policy decisions required to make recruitment at T and $N_{Trans.}(T)$ be the number of transfer decisions required to make recruitment at T. Let $\bar{X}_{N_P(T)}$ and $\bar{Y}_{N_{Trans.}(T)}$ be the respective total loss of manpower in $N_P(T)$ and $N_{Trans.}(T)$ decisions incurred by the organization during (0,T).

3. Main results

From renewal theory [7], the survivor function of T is

$$P(T>t) = \sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\left\{\begin{matrix}[F_{m_1}(t)-F_{m_1+1}(t)][U_{m_2}(t)-U_{m_2+1}(t)][W_{n_1}(t)-W_{n_1+1}(t)][V_{n_2}(t)-V_{n_2+1}(t)]\\ P(\bar{X}_{m_1,n_1}+\bar{Y}_{m_2,n_2}<Z)\end{matrix}\right\} \quad (1)$$

where

$$P(\bar{X}_{m_1,n_1}+\bar{Y}_{m_2,n_2}<Z) = \int_0^{\infty} P(\bar{X}_{m_1,n_1}+\bar{Y}_{m_2,n_2}<z)\; k(z) \quad (2)$$

We now obtain some performance measures related to time to recruitment for different forms of Z.

Case (i) $\boldsymbol{Z = \min(Z_{A1}+Z_{A2}+Z_{A3},\ Z_{B1}+Z_{B2}+Z_{B3})}$.

Since $P(Z>z) = \{P(Z_{A1}+Z_{A2}+Z_{A3}>z)P(Z_{B1}+Z_{B2}+Z_{B3}>z)\}$, from the hypothesis and on simplification it can be shown that

$$\begin{aligned} k(z) = {} & C_1(\theta_{A1}+\theta_{B1})e^{-(\theta_{A1}+\theta_{B1})z} + C_2(\theta_{A2}+\theta_{B2})e^{-(\theta_{A2}+\theta_{B2})z} + C_3(\theta_{A3}+\theta_{B3})e^{-(\theta_{A3}+\theta_{B3})z} \\ & - C_4(\theta_{A1}+\theta_{B2})e^{-(\theta_{A1}+\theta_{B2})z} + C_5(\theta_{A1}+\theta_{B3})e^{-(\theta_{A1}+\theta_{B3})z} \\ & - C_6(\theta_{A2}+\theta_{B1})e^{-(\theta_{A2}+\theta_{B1})z} - C_7(\theta_{A2}+\theta_{B3})e^{-(\theta_{A2}+\theta_{B3})z} \\ & + C_8(\theta_{A3}+\theta_{B1})e^{-(\theta_{A3}+\theta_{B1})z} - C_9(\theta_{A3}+\theta_{B2})e^{-(\theta_{A3}+\theta_{B2})z} \end{aligned} \quad (3)$$

where

$$\begin{aligned} & C_1 = \frac{\theta_{A2}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A1})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B1})},\ C_2 = \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A2})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B2})}, \\ & C_3 = \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}}{(\theta_{A3}-\theta_{A2})(\theta_{A3}-\theta_{A1})(\theta_{B3}-\theta_{B2})(\theta_{B3}-\theta_{B1})},\ C_4 = \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A1})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B2})}, \\ & C_5 = \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A1})(\theta_{B3}-\theta_{B2})(\theta_{B3}-\theta_{B1})},\ C_6 = \frac{\theta_{A1}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A2})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B1})}, \\ & C_7 = \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A2})(\theta_{B3}-\theta_{B2})(\theta_{B3}-\theta_{B1})},\ C_8 = \frac{\theta_{A1}\theta_{A2}\theta_{B2}\theta_{B3}}{(\theta_{A3}-\theta_{A2})(\theta_{A3}-\theta_{A1})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B1})}, \\ & C_9 = \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B3}}{(\theta_{A3}-\theta_{A2})(\theta_{A3}-\theta_{A1})(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B2})} \end{aligned} \quad (4)$$

From (1), (2) and (3) we get

$$\begin{aligned} P(T>t) = {} & C_1 D_{\theta_{A1},\theta_{B1}}(t) + C_2 D_{\theta_{A2},\theta_{B2}}(t) + C_3 D_{\theta_{A3},\theta_{B3}}(t) - C_4 D_{\theta_{A1},\theta_{B2}}(t) + C_5 D_{\theta_{A1},\theta_{B3}}(t) - C_6 D_{\theta_{A2},\theta_{B1}}(t) \\ & - C_7 D_{\theta_{A2},\theta_{B3}}(t) + C_8 D_{\theta_{A3},\theta_{B1}}(t) - C_9 D_{\theta_{A3},\theta_{B2}}(t) \end{aligned} \quad (5)$$

where

$$\begin{aligned} D_{\alpha,\beta}(t) = {} & \textstyle\sum_{m_1=0}^{\infty}[F_{m_1}(t)-F_{m_1+1}(t)]\,[\bar{g}_A(\alpha+\beta)]^{m_1}\ \sum_{n_1=0}^{\infty}[W_{n_1}(t)-W_{n_1+1}(t)]\,[\bar{h}_A(\alpha+\beta)]^{n_1} \\ & \textstyle\sum_{m_2=0}^{\infty}[U_{m_2}(t)-U_{m_2+1}(t)]\,[\bar{g}_B(\alpha+\beta)]^{m_2}\ \sum_{n_2=0}^{\infty}[V_{n_2}(t)-V_{n_2+1}(t)]\,[\bar{h}_B(\alpha+\beta)]^{n_2} \end{aligned}$$

(5) can be written as $P(T>t) = \sum_{i=1}^{9} P_i(t)$ (6)

where $P_i(t)$ is the i^{th} term in the right side of (5),i= 1,2...,9.

On simplification, the first term of right side of (6) is found to be

$$P_1(t) = C_1 D_{\theta_{A1},\theta_{B1}}(t)$$
$$= C_1 \Big\{1 - [1-\bar{g}_A(\theta_{A1}+\theta_{B1})] \sum_{m_1=1}^{\infty} F_{m_1}(t)[\bar{g}_A(\theta_{A1}+\theta_{B1})]^{m_1-1} - [1-\bar{h}_A(\theta_{A1}+\theta_{B1})] \sum_{n_1=1}^{\infty} W_{n_1}(t)[\bar{h}_A(\theta_{A1}+\theta_{B1})]^{n_1-1} +$$
$$[1-\bar{g}_A(\theta_{A1}+\theta_{B1})] \sum_{m_1=1}^{\infty} F_{m_1}(t)[\bar{g}_A(\theta_{A1}+\theta_{B1})]^{m_1-1}[1-\bar{h}_A(\theta_{A1}+\theta_{B1})] \sum_{n_1=1}^{\infty} W_{n_1}(t)[\bar{h}_A(\theta_{A1}+\theta_{B1})]^{n_1-1}\Big\}$$
$$\Big\{1 - [1-\bar{g}_B(\theta_{A1}+\theta_{B1})] \sum_{m_2=1}^{\infty} U_{m_2}(t)[\bar{g}_B(\theta_{A1}+\theta_{B1})]^{m_2-1} - [1-\bar{h}_B(\theta_{A1}+\theta_{B1})] \sum_{n_2=1}^{\infty} V_{n_2}(t)[\bar{h}_B(\theta_{A1}+\theta_{B1})]^{n_2-1} +$$
$$[1-\bar{g}_B(\theta_{A1}+\theta_{B1})] \sum_{m_2=1}^{\infty} U_{m_2}(t)[\bar{g}_B(\theta_{A1}+\theta_{B1})]^{m_2-1}[1-\bar{h}_B(\theta_{A1}+\theta_{B1})] \sum_{n_2=1}^{\infty} V_{n_2}(t)[\bar{h}_B(\theta_{A1}+\theta_{B1})]^{n_2-1}\Big\} \quad (7)$$

Since $f_{m_1}(t) = \frac{\mu_{1A}{}^{m_1} e^{-\mu_{1A}t} t^{m_1-1}}{(m_1-1)!}$, $w_{n_1}(t) = \frac{\mu_{2A}{}^{n_1} e^{-\mu_{2A}t} t^{n_1-1}}{(n_1-1)!}$, $u_{m_2}(t) = \frac{\mu_{1B}{}^{m_2} e^{-\mu_{1B}t} t^{m_2-1}}{(m_2-1)!}$ and $v_{n_2}(t) = \frac{\mu_{2B}{}^{n_2} e^{-\mu_{2B}t} t^{n_2-1}}{(n_2-1)!}$ by hypothesis ,we find that

$$P_1(t) = C_1 e^{-[\mu_{1A}[1-\bar{g}_A(\theta_{A1}+\theta_{B1})]+\mu_{2A}[1-\bar{h}_A(\theta_{A1}+\theta_{B1})]+\mu_{1B}[1-\bar{g}_B(\theta_{A1}+\theta_{B1})]+\mu_{2B}[1-\bar{h}_B(\theta_{A1}+\theta_{B1})]]t} \quad (8)$$

Since $E(T^r) = r\int_0^{\infty} t^{r-1}P(T>t)\,dt$, $r \ge 1$, from (6) ,(7) and (8) we get

$$E(T) = C_1E_{\theta_{A1},\theta_{B1}} + C_2E_{\theta_{A2},\theta_{B2}} + C_3E_{\theta_{A2},\theta_{B2}} - C_4E_{\theta_{A1},\theta_{B2}} + C_5E_{\theta_{A1},\theta_{B2}} - C_6E_{\theta_{A2},\theta_{B1}} - C_7E_{\theta_{A2},\theta_{B2}}$$
$$+ C_8E_{\theta_{A2},\theta_{B1}} - C_9E_{\theta_{A2},\theta_{B2}} \quad (9)$$

and

$$E(T^2) = C_1E^2{}_{\theta_{A1},\theta_{B1}} + C_2E^2{}_{\theta_{A2},\theta_{B2}} + C_3E^2{}_{\theta_{A2},\theta_{B2}} - C_4E^2{}_{\theta_{A1},\theta_{B2}} + C_5E^2{}_{\theta_{A1},\theta_{B2}} - C_6E^2{}_{\theta_{A2},\theta_{B1}} - C_7E^2{}_{\theta_{A2},\theta_{B2}}$$
$$+ C_8E^2{}_{\theta_{A2},\theta_{B1}} - C_9E^2{}_{\theta_{A2},\theta_{B2}} \quad (10)$$

(9) gives the mean time to recruitment and from (9) and (10), the variance of the time to recruitment can be computed for case (i), where $C_i, i = 1 \text{ to } 9$ are given by (4).

From (6) we obtain the following additional results related to time to recruitment:

i. Hazard rate at time t

$$= \frac{C_1\tilde{A}e^{-\tilde{A}t}+C_2\tilde{B}e^{-\tilde{B}t}+C_3\tilde{C}e^{-\tilde{C}t}-C_4\tilde{D}e^{-\tilde{D}t}+C_5\tilde{E}e^{-\tilde{E}t}-C_6\tilde{F}e^{-\tilde{F}t}-C_7\tilde{G}e^{-\tilde{G}t}+C_8\tilde{H}e^{-\tilde{H}t}-C_9\tilde{I}e^{-\tilde{I}t}}{C_1e^{-\tilde{A}t}+C_2e^{-\tilde{B}t}+C_3e^{-\tilde{C}t}-C_4e^{-\tilde{D}t}+C_5e^{-\tilde{E}t}-C_6e^{-\tilde{F}t}-C_7e^{-\tilde{G}t}+C_8e^{-\tilde{H}t}-C_9e^{-\tilde{I}t}}$$

ii. Probability that recruitment takes place in (t, t+dt) given that there is no recruitment in [0,t]= $P(t<T<t+dt/T>t)$ =

$$\frac{C_1e^{-\tilde{A}t}(1-e^{-\tilde{A}dt})+C_2e^{-\tilde{B}t}(1-e^{-\tilde{B}dt})+C_3e^{-\tilde{C}t}(1-e^{-\tilde{C}dt})-C_4e^{-\tilde{D}t}(1-e^{-\tilde{D}dt})+C_5e^{-\tilde{E}t}(1-e^{-\tilde{E}dt})-C_6e^{-\tilde{F}t}(1-e^{-\tilde{F}dt})-C_7e^{-\tilde{G}t}(1-e^{-\tilde{G}dt})+C_8e^{-\tilde{H}t}(1-e^{-\tilde{H}dt})-C_9e^{-\tilde{I}t}(1-e^{-\tilde{I}dt})}{C_1e^{-\tilde{A}t}+C_2e^{-\tilde{B}t}+C_3e^{-\tilde{C}t}-C_4e^{-\tilde{D}t}+C_5e^{-\tilde{E}t}-C_6e^{-\tilde{F}t}-C_7e^{-\tilde{G}t}+C_8e^{-\tilde{H}t}-C_9e^{-\tilde{I}t}}$$

iii. Average residual time for recruitment given that there is no recruitment upto time t.

$$= E(T-t/T>t) = \frac{C_1e^{-\tilde{A}t}/\tilde{A}+C_2e^{-\tilde{B}t}/\tilde{B}+C_3e^{-\tilde{C}t}/\tilde{C}-C_4e^{-\tilde{D}t}/\tilde{D}+C_5e^{-\tilde{E}t}/\tilde{E}-C_6e^{-\tilde{F}t}/\tilde{F}-C_7e^{-\tilde{G}t}/\tilde{G}+C_8e^{-\tilde{H}t}/\tilde{H}-C_9e^{-\tilde{I}t}/\tilde{I}}{C_1e^{-\tilde{A}t}+C_2e^{-\tilde{B}t}+C_3e^{-\tilde{C}t}-C_4e^{-\tilde{D}t}+C_5e^{-\tilde{E}t}-C_6e^{-\tilde{F}t}-C_7e^{-\tilde{G}t}+C_8e^{-\tilde{H}t}-C_9e^{-\tilde{I}t}}$$

where
$$\tilde{A} = \mu_{1A}(1-\bar{g}_A(\theta_{A1}+\theta_{B1})) + \mu_{1B}(1-\bar{g}_B(\theta_{A1}+\theta_{B1})) + \mu_{2A}(1-\bar{h}_A(\theta_{A1}+\theta_{B1})) + \mu_{2B}(1-\bar{h}_B(\theta_{A1}+\theta_{B1}))$$
$$\tilde{B} = \mu_{1A}(1-\bar{g}_A(\theta_{A2}+\theta_{B2})) + \mu_{1B}(1-\bar{g}_B(\theta_{A2}+\theta_{B2})) + \mu_{2A}(1-\bar{h}_A(\theta_{A2}+\theta_{B2})) + \mu_{2B}(1-\bar{h}_B(\theta_{A2}+\theta_{B2}))$$
$$\tilde{C} = \mu_{1A}(1-\bar{g}_A(\theta_{A2}+\theta_{B2})) + \mu_{1B}(1-\bar{g}_B(\theta_{A2}+\theta_{B2})) + \mu_{2A}(1-\bar{h}_A(\theta_{A2}+\theta_{B2})) + \mu_{2B}(1-\bar{h}_B(\theta_{A2}+\theta_{B2}))$$
$$\tilde{D} = \mu_{1A}(1-\bar{g}_A(\theta_{A1}+\theta_{B2})) + \mu_{1B}(1-\bar{g}_B(\theta_{A1}+\theta_{B2})) + \mu_{2A}(1-\bar{h}_A(\theta_{A1}+\theta_{B2})) + \mu_{2B}(1-\bar{h}_B(\theta_{A1}+\theta_{B2}))$$
$$\tilde{E} = \mu_{1A}(1-\bar{g}_A(\theta_{A1}+\theta_{B2})) + \mu_{1B}(1-\bar{g}_B(\theta_{A1}+\theta_{B2})) + \mu_{2A}(1-\bar{h}_A(\theta_{A1}+\theta_{B2})) + \mu_{2B}(1-\bar{h}_B(\theta_{A1}+\theta_{B2}))$$
$$\tilde{F} = \mu_{1A}(1-\bar{g}_A(\theta_{A2}+\theta_{B1})) + \mu_{1B}(1-\bar{g}_B(\theta_{A2}+\theta_{B1})) + \mu_{2A}(1-\bar{h}_A(\theta_{A2}+\theta_{B1})) + \mu_{2B}(1-\bar{h}_B(\theta_{A2}+\theta_{B1}))$$
$$\tilde{G} = \mu_{1A}(1-\bar{g}_A(\theta_{A2}+\theta_{B2})) + \mu_{1B}(1-\bar{g}_B(\theta_{A2}+\theta_{B2})) + \mu_{2A}(1-\bar{h}_A(\theta_{A2}+\theta_{B2})) + \mu_{2B}(1-\bar{h}_B(\theta_{A2}+\theta_{B2}))$$
$$\tilde{H} = \mu_{1A}(1-\bar{g}_A(\theta_{A2}+\theta_{B1})) + \mu_{1B}(1-\bar{g}_B(\theta_{A2}+\theta_{B1})) + \mu_{2A}(1-\bar{h}_A(\theta_{A2}+\theta_{B1})) + \mu_{2B}(1-\bar{h}_B(\theta_{A2}+\theta_{B1}))$$
$$\tilde{I} = \mu_{1A}(1-\bar{g}_A(\theta_{A2}+\theta_{B2})) + \mu_{1B}(1-\bar{g}_B(\theta_{A2}+\theta_{B2})) + \mu_{2A}(1-\bar{h}_A(\theta_{A2}+\theta_{B2})) + \mu_{2B}(1-\bar{h}_B(\theta_{A2}+\theta_{B2})) \quad (11)$$

iv. Average number of policy and transfer decisions required to make recruitment at T $=(\mu_{1A}+\mu_{1B}+\mu_{2A}+\mu_{2B})E(T)$

v. Average total loss of manpower due to $N_P(T)$ and $N_{Trans.}(T)$ decisions $= \{\mu_{1A}E(X_{Ai}) + \mu_{2A}E(Y_{Aj}) - \mu_{1B}E(Z_{Bi}) + \mu_{1B}E(Y_{Bj})\}E(T)$

Case (ii) $\mathbf{Z = max}(Z_{A1}+Z_{A2}+Z_{A3},\ Z_{B1}+Z_{B2}+Z_{B3})$.

Since $P(Z \le z) = \{P(Z_{A1} + Z_{A2} + Z_{A3} \le z)P(Z_{B1} + Z_{B2} + Z_{B3} \le z)\}$, from the hypothesis and on simplification it can be shown that

$$k(z) = -C_1(\theta_{A1}+\theta_{B1})e^{-(\theta_{A1}+\theta_{B1})z} - C_2(\theta_{A2}+\theta_{B2})e^{-(\theta_{A2}+\theta_{B2})z} - C_3(\theta_{A3}+\theta_{B3})e^{-(\theta_{A3}+\theta_{B3})z} + C_4(\theta_{A1}+\theta_{B2})e^{-(\theta_{A1}+\theta_{B2})z} - C_5(\theta_{A1}+\theta_{B3})e^{-(\theta_{A1}+\theta_{B3})z} + C_6(\theta_{A2}+\theta_{B1})e^{-(\theta_{A2}+\theta_{B1})z} + C_7(\theta_{A2}+\theta_{B3})e^{-(\theta_{A2}+\theta_{B3})z} - C_8(\theta_{A3}+\theta_{B1})e^{-(\theta_{A3}+\theta_{B1})z} + C_9(\theta_{A3}+\theta_{B2})e^{-(\theta_{A3}+\theta_{B2})z} + C_{10}\theta_{A1}e^{-\theta_{A1}z} - C_{11}\theta_{A2}e^{-\theta_{A2}z} + C_{12}\theta_{A3}e^{-\theta_{A3}z} + C_{13}\theta_{B1}e^{-\theta_{B1}z} - C_{14}\theta_{B2}e^{-\theta_{B2}z}$$

where $C_i, i = 1,2,\dots,9$, are given by (4).

$$C_{10} = \frac{\theta_{A2}\theta_{A3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A1})}, C_{11} = \frac{\theta_{A1}\theta_{A3}}{(\theta_{A2}-\theta_{A1})(\theta_{A3}-\theta_{A2})}, C_{12} = \frac{\theta_{A1}\theta_{A2}}{(\theta_{A3}-\theta_{A2})(\theta_{A3}-\theta_{A1})},$$

$$C_{13} = \frac{\theta_{B2}\theta_{B3}}{(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B1})}, C_{14} = \frac{\theta_{B1}\theta_{B3}}{(\theta_{B2}-\theta_{B1})(\theta_{B3}-\theta_{B2})} \text{ and } C_{15} = \frac{\theta_{B1}\theta_{B2}}{(\theta_{B3}-\theta_{B2})(\theta_{B3}-\theta_{B1})}$$

Proceeding as in case (i), we get

$$E(T) = C_{10}E_{\theta_{A1}} - C_{11}E_{\theta_{A2}} + C_{12}E_{\theta_{A3}} + C_{13}E_{\theta_{B1}} - C_{14}E_{\theta_{B2}} + C_{15}E_{\theta_{B3}} - C_1E_{\theta_{A1},\theta_{B1}} - C_2E_{\theta_{A2},\theta_{B2}} - C_3E_{\theta_{A3},\theta_{B3}} + C_4E_{\theta_{A1},\theta_{B2}} - C_5E_{\theta_{A1},\theta_{B3}} + C_6E_{\theta_{A2},\theta_{B1}} + C_7E_{\theta_{A2},\theta_{B3}} - C_8E_{\theta_{A3},\theta_{B1}} + C_9E_{\theta_{A3},\theta_{B2}} \quad (12)$$

and

$$E(T^2) = C_{10}E^2_{\theta_{A1}} - C_{11}E^2_{\theta_{A2}} + C_{12}E^2_{\theta_{A3}} + C_{13}E^2_{\theta_{B1}} - C_{14}E^2_{\theta_{B2}} + C_{15}E^2_{\theta_{B3}} - C_1E^2_{\theta_{A1},\theta_{B1}} - C_2E^2_{\theta_{A2},\theta_{B2}} - C_3E^2_{\theta_{A3},\theta_{B3}} + C_4E^2_{\theta_{A1},\theta_{B2}} - C_5E^2_{\theta_{A1},\theta_{B3}} + C_6E^2_{\theta_{A2},\theta_{B1}} + C_7E^2_{\theta_{A2},\theta_{B3}} - C_8E^2_{\theta_{A3},\theta_{B1}} + C_9E^2_{\theta_{A3},\theta_{B2}} \quad (13)$$

While (12) gives the mean time to recruitment, from (12) and (13), the variance of the time to recruitment can be computed for case (ii).

We now obtain the following additional results for this case related to time to recruitment:

vi. Hazard rate at time t

$$= \frac{C_{10}\bar{J}e^{-\bar{J}t} - C_{11}\bar{K}e^{-\bar{K}t} + C_{12}\bar{L}e^{-\bar{L}t} + C_{13}\bar{M}e^{-\bar{M}t} - C_{14}\bar{N}e^{-\bar{N}t} + C_{15}\bar{O}e^{-\bar{O}t} - C_1\bar{A}e^{-\bar{A}t} - C_2\bar{B}e^{-\bar{B}t} - C_3\bar{C}e^{-\bar{C}t} + C_4\bar{D}e^{-\bar{D}t} - C_5\bar{E}e^{-\bar{E}t} + C_6\bar{F}e^{-\bar{F}t} + C_7\bar{G}e^{-\bar{G}t} - C_8\bar{H}e^{-\bar{H}t} + C_9\bar{I}e^{-\bar{I}t}}{C_{10}e^{-\bar{J}t} - C_{11}e^{-\bar{K}t} + C_{12}e^{-\bar{L}t} + C_{13}e^{-\bar{M}t} - C_{14}e^{-\bar{N}t} + C_{15}e^{-\bar{O}t} - C_1e^{-\bar{A}t} - C_2e^{-\bar{B}t} - C_3e^{-\bar{C}t} + C_4e^{-\bar{D}t} - C_5e^{-\bar{E}t} + C_6e^{-\bar{F}t} + C_7e^{-\bar{G}t} - C_8e^{-\bar{H}t} + C_9e^{-\bar{I}t}}$$

vii. Probability that recruitment takes place in (t, t+dt) given that there is no recruitment in [0,t]$=P(t < T < t + dt/T > t)=$

$$\frac{\begin{matrix} C_{10}e^{-\bar{J}t}(1-e^{-\bar{J}t}) - C_{11}e^{-\bar{K}t}(1-e^{-\bar{K}t}) + C_{12}e^{-\bar{L}t}(1-e^{-\bar{L}t}) + C_{13}e^{-\bar{M}t}(1-e^{-\bar{M}t}) - C_{14}e^{-\bar{N}t}(1-e^{-\bar{N}t}) + C_{15}e^{-\bar{O}t}(1-e^{-\bar{O}t}) - C_1e^{-\bar{A}t}(1-e^{-\bar{A}dt}) - C_2e^{-\bar{B}t}(1-e^{-\bar{B}dt}) \\ - C_3e^{-\bar{C}t}(1-e^{-\bar{C}dt}) + C_4e^{-\bar{D}t}(1-e^{-\bar{D}dt}) - C_5e^{-\bar{E}t}(1-e^{-\bar{E}dt}) + C_6e^{-\bar{F}t}(1-e^{-\bar{F}dt}) + C_7e^{-\bar{G}t}(1-e^{-\bar{G}dt}) - C_8e^{-\bar{H}t}(1-e^{-\bar{H}dt}) + C_9e^{-\bar{I}t}(1-e^{-\bar{I}dt}) \end{matrix}}{C_{10}e^{-\bar{J}t} - C_{11}e^{-\bar{K}t} + C_{12}e^{-\bar{L}t} + C_{13}e^{-\bar{M}t} - C_{14}e^{-\bar{N}t} + C_{15}e^{-\bar{O}t} - C_1e^{-\bar{A}t} - C_2e^{-\bar{B}t} - C_3e^{-\bar{C}t} + C_4e^{-\bar{D}t} - C_5e^{-\bar{E}t} + C_6e^{-\bar{F}t} + C_7e^{-\bar{G}t} - C_8e^{-\bar{H}t} + C_9e^{-\bar{I}t}}$$

viii. Average residual time for recruitment given that there is no recruitment upto time t= $E\left(T - \frac{t}{T}/T > t\right)$

$$= \frac{C_{10}e^{-\bar{J}t}/\bar{J} - C_{11}e^{-\bar{K}t}/\bar{K} + C_{12}e^{-\bar{L}t}/\bar{L} + C_{13}e^{-\bar{M}t}/\bar{M} - C_{14}e^{-\bar{N}t}/\bar{N} + C_{15}e^{-\bar{O}t}/\bar{O} - C_1e^{-\bar{A}t}/\bar{A} - C_2e^{-\bar{B}t}/\bar{B} - C_3e^{-\bar{C}t}/\bar{C} + C_4e^{-\bar{D}t}/\bar{D} - C_5e^{-\bar{E}t}/\bar{E} + C_6e^{-\bar{F}t}/\bar{F} + C_7e^{-\bar{G}t}/\bar{G} - C_8e^{-\bar{H}t}/\bar{H} + C_9e^{-\bar{I}t}/\bar{I}}{C_{10}e^{-\bar{J}t} - C_{11}e^{-\bar{K}t} + C_{12}e^{-\bar{L}t} + C_{13}e^{-\bar{M}t} - C_{14}e^{-\bar{N}t} + C_{15}e^{-\bar{O}t} - C_1e^{-\bar{A}t} - C_2e^{-\bar{B}t} - C_3e^{-\bar{C}t} + C_4e^{-\bar{D}t} - C_5e^{-\bar{E}t} + C_6e^{-\bar{F}t} + C_7e^{-\bar{G}t} - C_8e^{-\bar{H}t} + C_9e^{-\bar{I}t}}$$

where $\bar{A}, \bar{B}, \bar{C}, \bar{D}, \bar{E}, \bar{F}, \bar{G}, \bar{H}, \bar{I}$ are given by (11) and

$$\bar{J} = \mu_{1A}(1-\bar{g}_A(\theta_{A1})) + \mu_{1B}(1-\bar{g}_B(\theta_{A1})) + \mu_{2A}(1-\bar{h}_A(\theta_{A1})) + \mu_{2B}(1-\bar{h}_B(\theta_{A1}))$$
$$\bar{K} = \mu_{1A}(1-\bar{g}_A(\theta_{A2})) + \mu_{1B}(1-\bar{g}_B(\theta_{A2})) + \mu_{2A}(1-\bar{h}_A(\theta_{A2})) + \mu_{2B}(1-\bar{h}_B(\theta_{A2}))$$
$$\bar{L} = \mu_{1A}(1-\bar{g}_A(\theta_{A3})) + \mu_{1B}(1-\bar{g}_B(\theta_{A3})) + \mu_{2A}(1-\bar{h}_A(\theta_{A3})) + \mu_{2B}(1-\bar{h}_B(\theta_{A3}))$$
$$\bar{M} = \mu_{1A}(1-\bar{g}_A(\theta_{B1})) + \mu_{1B}(1-\bar{g}_B(\theta_{B1})) + \mu_{2A}(1-\bar{h}_A(\theta_{B1})) + \mu_{2B}(1-\bar{h}_B(\theta_{B1}))$$
$$\bar{N} = \mu_{1A}(1-\bar{g}_A(\theta_{B2})) + \mu_{1B}(1-\bar{g}_B(\theta_{B2})) + \mu_{2A}(1-\bar{h}_A(\theta_{B2})) + \mu_{2B}(1-\bar{h}_B(\theta_{B2}))$$

$$\tilde{O} = \mu_{1A}(1 - g_A(\theta_{B2})) + \mu_{1B}(1 - g_B(\theta_{B2})) + \mu_{2A}\left(1 - \bar{h}_A(\theta_{B2})\right) + \mu_{2B}\left(1 - \bar{h}_B(\theta_{B2})\right)$$

ix. Average number of policy and transfer decisions required to make recruitment at T=$(\mu_{1A} + \mu_{1B} + \mu_{2A} + \mu_{2B})E(T)$

x. Average total loss of manpower due to $N_p(T)$ and $N_{Trans.}(T)$ decisions $= \{\mu_{1A}E(X_{Ai}) + \mu_{2A}E(Y_{Aj}) + \mu_{1B}E(X_{Bi}) + \mu_{2B}E(Y_{Bj})\}E(T)$

Case (iii) $\boldsymbol{Z = Z_{A1} + Z_{A2} + Z_{A3} + Z_{B1} + Z_{B2} + Z_{B3}}$.

In this case, it can be shown that

$$k(z) = C_{16}\theta_{A1}e^{-\theta_{A1}z} - C_{17}\theta_{A2}e^{-\theta_{A2}z} + C_{18}\theta_{A3}e^{-\theta_{A3}z} + C_{19}\theta_{B1}e^{-\theta_{B1}z} - C_{20}\theta_{B2}e^{-\theta_{B2}z} + C_{21}\theta_{B3}e^{-\theta_{B3}z}$$

Where

$$C_{16} = \frac{\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A1})(\theta_{B1} - \theta_{A1})(\theta_{B2} - \theta_{A1})(\theta_{B3} - \theta_{A1})}, C_{17} = \frac{\theta_{A1}\theta_{A3}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A2} - \theta_{A1})(\theta_{A3} - \theta_{A2})(\theta_{B1} - \theta_{A2})(\theta_{B2} - \theta_{A2})(\theta_{B3} - \theta_{A2})},$$

$$C_{18} = \frac{\theta_{A1}\theta_{A2}\theta_{B1}\theta_{B2}\theta_{B3}}{(\theta_{A3} - \theta_{A2})(\theta_{A3} - \theta_{A1})(\theta_{B1} - \theta_{A3})(\theta_{B2} - \theta_{A3})(\theta_{B3} - \theta_{A3})}, C_{19} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B2}\theta_{B3}}{(\theta_{B1} - \theta_{A1})(\theta_{B1} - \theta_{A2})(\theta_{B1} - \theta_{A3})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})},$$

$$C_{20} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B3}}{(\theta_{B2} - \theta_{A1})(\theta_{B2} - \theta_{A2})(\theta_{B2} - \theta_{A3})(\theta_{B2} - \theta_{B1})(\theta_{B3} - \theta_{B1})}, C_{21} = \frac{\theta_{A1}\theta_{A2}\theta_{A3}\theta_{B1}\theta_{B2}}{(\theta_{B3} - \theta_{A1})(\theta_{B3} - \theta_{A2})(\theta_{B3} - \theta_{A3})(\theta_{B3} - \theta_{B1})(\theta_{B3} - \theta_{B2})}$$

Proceeding as in case (i), we get

$$E(T) = C_{16}E_{\theta_{A1}} - C_{17}E_{\theta_{A2}} + C_{18}E_{\theta_{A3}} + C_{19}E_{\theta_{B1}} - C_{20}E_{\theta_{B2}} + C_{21}E_{\theta_{B3}} \quad (14)$$

and

$$E(T^2) = C_{16}E^2_{\theta_{A1}} - C_{17}E^2_{\theta_{A2}} + C_{18}E^2_{\theta_{A3}} + C_{19}E^2_{\theta_{B1}} - C_{20}E^2_{\theta_{B2}} + C_{21}E^2_{\theta_{B3}} \quad (15)$$

In (9), (10), (12), (13), (14) and (15) $E_{\alpha,\beta}$, E_α, $E^2{}_{\alpha,\beta}$ and $E^2{}_\alpha$ are given below.

$$E_{\alpha,\beta} = \frac{1}{\mu_{1A}[1-g_A(\alpha+\beta)]+\mu_{2A}[1-\bar{h}_A(\alpha+\beta)]+\mu_{1B}[1-g_B(\alpha+\beta)]+\mu_{2B}[1-\bar{h}_B(\alpha+\beta)]}$$

$$E_\alpha = \frac{1}{\mu_{1A}[1-g_A(\alpha)]+\mu_{2A}[1-\bar{h}_A(\alpha)]+\mu_{1B}[1-g_B(\alpha)]+\mu_{2B}[1-\bar{h}_B(\alpha)]}$$

$$E^2{}_{\alpha,\beta} = \frac{1}{[\mu_{1A}[1-g_A(\alpha+\beta)]+\mu_{2A}[1-\bar{h}_A(\alpha+\beta)]+\mu_{1B}[1-g_B(\alpha+\beta)]+\mu_{2B}[1-\bar{h}_B(\alpha+\beta)]]^2}$$

and $E^2{}_\alpha = \dfrac{1}{[\mu_{1A}[1-g_A(\alpha)]+\mu_{2A}[1-\bar{h}_A(\alpha)]+\mu_{1B}[1-g_B(\alpha)]+\mu_{2B}[1-\bar{h}_B(\alpha)]]^2}$

While (14) gives the mean time to recruitment, from (14) and (15), the variance of the time to recruitment can be computed for case (iii).

We now obtain the following additional results related to time to recruitment.

xi. Hazard rate at time t= $\dfrac{C_{10}\tilde{J}e^{-Jt} - C_{11}\tilde{K}e^{-Kt} + C_{12}\tilde{L}e^{-Lt} + C_{13}\tilde{M}e^{-Mt} - C_{14}\tilde{N}e^{-Nt} + C_{15}\tilde{O}e^{-Ot}}{C_{10}e^{-Jt} - C_{11}e^{-Kt} + C_{12}e^{-Lt} + C_{13}e^{-Mt} - C_{14}e^{-Nt} + C_{15}e^{-Ot}}$

xii. Probability that recruitment takes place in (t, t+dt) given that there is no recruitment in [0,t]=$P(t < T < t + dt/T > t)$

$$= \frac{C_{16}e^{-\tilde{J}t}(1-e^{-\tilde{J}t}) - C_{17}e^{-\tilde{K}t}(1-e^{-\tilde{K}t}) + C_{18}e^{-\tilde{L}t}(1-e^{-\tilde{L}t}) + C_{19}e^{-\tilde{M}t}(1-e^{-\tilde{M}t}) - C_{20}e^{-\tilde{N}t}(1-e^{-\tilde{N}t}) + C_{21}e^{-\tilde{O}t}(1-e^{-\tilde{O}t})}{C_{16}e^{-\tilde{J}t} - C_{11}e^{-\tilde{K}t} + C_{12}e^{-\tilde{L}t} + C_{13}e^{-\tilde{M}t} - C_{14}e^{-\tilde{N}t} + C_{15}e^{-\tilde{O}t}}$$

xiii. Average residual time for recruitment given that there is no recruitment upto time t

$$= E\left(T - \tfrac{t}{T}/T > t\right) = \frac{C_{16}e^{-\tilde{J}t}/\tilde{J} - C_{17}e^{-\tilde{K}t}/\tilde{K} + C_{18}e^{-\tilde{L}t}/\tilde{L} + C_{19}e^{-\tilde{M}t}/\tilde{M} - C_{20}e^{-\tilde{N}t}/\tilde{N} + C_{21}e^{-\tilde{O}t}/\tilde{O}}{C_{16}e^{-\tilde{J}t} - C_{17}e^{-\tilde{K}t} + C_{18}e^{-\tilde{L}t} + C_{19}e^{-\tilde{M}t} - C_{20}e^{-\tilde{N}t} + C_{21}e^{-\tilde{O}t}}$$

where $\tilde{J}, \tilde{K}, \tilde{L}, \tilde{M}, \tilde{N}, \tilde{O}$ are given by (24).

xiv. Average number of policy and transfer decisions required to make recruitment at T $=(\mu_{1A} + \mu_{1B} + \mu_{2A} + \mu_{2B})E(T)$

xv. Average total loss of manpower due to $N_p(T)$ and $N_{Trans.}(T)$ decisions $= \{\mu_{1A}E(X_{Ai}) + \mu_{2A}E(Y_{Aj}) + \mu_{1B}E(X_{Bi}) + \mu_{2B}E(Y_{Bj})\}E(T)$

Special Case:

Suppose X_{Ai}, X_{Bi}, Y_{Ai} and Y_{Bi} follow exponential distribution with parameters $\alpha_{1A}, \alpha_{1B}, \alpha_{2A}$ and α_{2B} respectively.

In this case $\bar{g}_A(\theta) = \frac{\alpha_{1A}}{\alpha_{1A}+\theta}$, $\bar{g}_B(\theta) = \frac{\alpha_{1B}}{\alpha_{1B}+\theta}$, $\bar{h}_A(\theta) = \frac{\alpha_{2A}}{\alpha_{2A}+\theta}$, $\bar{h}_B(\theta) = \frac{\alpha_{2B}}{\alpha_{2B}+\theta}$ (16)

Using (16) in (9), (10), (12), (13), (14) and (15), we get explicit form of the results for the performance measures related to time to recruitment.

4. Conclusion

The model discussed in this paper is found to be more realistic and new in the context of considering three components for the breakdown threshold. In the context, the model developed in this paper can be utilized to plan for adequate provision of manpower in the organization. The goodness of fit for the distribution assumed in this paper can be tested by collecting relevant data. The results of this paper will be very useful in planning recruitments in future for those marketing organizations with depletion of manpower due to attrition.

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