Annals of Pure and Applied Mathematics
Vol. 11, No. 1, 2016, 133-144
ISSN: 2279-087X (P), 2279-0888(online)
Published on 29 February 2016
www.researchmathsci.org

Annals of Pure and Applied Mathematics

Fibonacci Cordial Labeling of Some Special Graphs

A.H.Rokad[1] **and** ***G.V.Ghodasara***[2]

[1]School of Science, RK University
Rajkot - 360020, Gujarat - India. E-mail: rokadamit@rocketmail.com

[2]H. & H. B. Kotak Institute of Science, Rajkot - 360001, Gujarat – India.
E-mail: gaurang_enjoy@yahoo.co.in

Received 1 February 2016; accepted 23 February 2016

Abstract. In this paper we introduce Fibonacci cordial labeling. An injective function f from vertex set V of a graph G to the set $\{F_0, F_1, F_2, \dots, F_n\}$, where F_j is the j^{th} Fibonacci number (j = 0, 1, . . . , n), is said to be Fibonacci cordial labeling if the induced function f^* from the edge set E of graph G to the set {0, 1} defined by $f^*(uv) = (f(u)+ f(v))(\mathrm{mod}\,2)$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. A graph which admits Fibonacci cordial labeling is called Fibonacci cordial graph. In this paper we discuss Fibonacci cordial labeling of different graphs.

Keywords: Fibonacci cordial labeling

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

The graphs considered here are finite, connected, undirected and simple. The vertex set and edge set of a graph G are denoted by V (G) and E(G) respectively. For various graph theoretic notations and terminology we follow Gross and Yellen [3]. Sridevi, Nagarajan, Nellaimurugan and Navanaeethakrishnan [4] proved that Path, Cycle are Fibonacci divisor cordial graph. A dynamic survey of graph labeling is published and updated every year by Gallian[2]. In this paper we introduce a new concept called Fibonacci cordial labeling. We have derived different graph families satisfying the conditions of Fibonacci cordial labeling. We have also discussed Fibonacci cordial labeling in context of different graph operations.

Definition 1.1. A function $f: V(G) \rightarrow \{0, 1\}$ is called a binary vertex labeling of a graph G and f (v) is called label of the vertex v of G under f.
For an edge e = uv, the induced edge labeling $f^*: E(G) \rightarrow \{0, 1\}$ is given by
$f^*(e) = |f(u) - f(v)|$.

Notations:
$v_f(0)$: number of vertices with label 0.
$v_f(1)$: number of vertices with label 1.
$e_f(0)$: number of edges with label 0.

$e_f(1)$: number of edges with label 1.

Definition 1.2. A binary vertex labeling of a graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Cahit [1] introduced the concept of cordial labeling.

Definition 1.3. Fibonacci numbers can be defined by the linear recurrence relation $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$, where $F_0 = 0$, $F_1 = 1$. This generates the infinite sequence of integers beginning 0,1,1,2,3,5,8,13,21,34,55,89,144,. . . .

Definition 1.4. An injective function f: V (G) → $\{F_0, F_1, F_2, \ldots, F_n\}$, where F_j is the j^{th} Fibonacci number (j = 0, 1, . . . , n), is said to be Fibonacci cordial labeling if the induced function $f^* : E(G) \rightarrow \{0, 1\}$ defined by $f^*(uv) = (f(u) + f(v))(\mathrm{mod}2)$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. A graph which admits Fibonacci cordial labeling is called Fibonacci cordial graph.

Definition 1.5. Bistar $B_{n,n}$, is the graph obtained from two copies of $K_{1,n}$ by joining the apex vertices by an edge.

Definition 1.6. The joint sum of two graphs G and H is the graph obtained by the joining a vertex of G with a vertex of H by an edge.

Definition 1.7. Ring sum $G_1 \oplus G_2$ of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G_1 \oplus G_2 = (V_1 \cup V_2, (E_1 \cup E_2) - (E_1 \cap E_2))$.

Definition 1.8. For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

Definition 1.9. Subdivision of a graph G is a graph resulting from the subdivision of edges in G. The subdivision of some edge e with endpoints {u, v} yields a graph containing one new vertex w, and with an edge set replacing e by two new edges, {u, w} and {w, v}.

2. Main results

Theorem 2.1. Petersen graph is Fibonacci cordial.

Proof. Let u_1, u_2, u_3, u_4, u_5 be the internal vertices and $u_6, u_7, u_8, u_9, u_{10}$ be the external vertices of Petersen graph such that each u_i is adjacent to u_{i+5}, $1 \leq i \leq 5$.

We define labeling function f: V (G) → $\{F_0, F_1, F_2, \ldots, F_{10}\}$ as follows.

$f(u_1) = F_0$, $f(u_2) = F_1$, $f(u_i) = F_i$, $3 \leq i \leq 10$.

Then we have $e_f(0) = 7$ and $e_f(1) = 8$. Therefore $|e_f(0) - e_f(1)| = 1$.

Hence, Petersen graph is Fibonacci cordial.

Example 2.1. Fibonacci cordial labeling of Petersen graph is shown in Fig. 1.

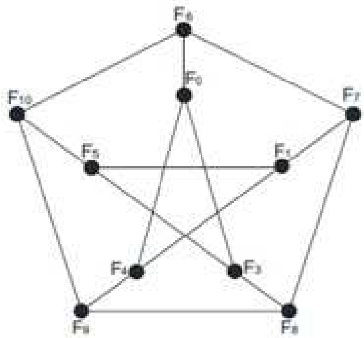


Figure 1: Fibonacci cordial labeling of Petersen graph

Theorem 2.2. Wheel W_n is Fibonacci cordial for $n \geq 3$, $n \in N$.
Proof. Let $u_1, u_2, \ldots, u_n$ be successive rim vertices and u_0 be the apex vertex of W_n.
We define labeling f: $V(W_n) \rightarrow \{F_0, F_1, F_2, \ldots, F_{n+1}\}$, we consider the following two cases.

<u>Case 1</u>: $n \equiv 0(\text{mod}3)$.
$f(u_0) = F_1$, $f(u_i) = F_{i+1}$, $1 \leq i \leq n$.

<u>Case 2</u>: n does not congruent to 0(mod3).
$f(u_0) = F_1$, $f(u_1) = F_0$, $f(u_i) = F_{i+1}$, $2 \leq i \leq n$.
Then in each case we have $e_f(0) = e_f(1) = n$.
Hence wheel W_n is Fibonacci cordial for $n \geq 3$, $n \in N$.

Example 2.2. Fibonacci cordial labeling of W_9 is shown in Fig. 2.

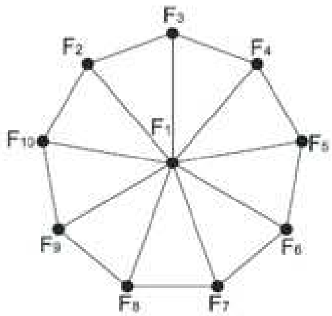


Figure 2: Fibonacci cordial labeling of W_9

Theorem 2.3. Shell S_n is Fibonacci cordial for $n \geq 3$, $n \in N$.
Proof. Let $u_1, u_2, \ldots, u_n$ be successive vertices of shell S_n, where u_1 is the apex vertex of shell S_n. We define labeling f: $V(S_n) \rightarrow \{F_0, F_1, F_2, \ldots, F_n\}$ as
$f(u_1) = F_1$, $f(u_2) = F_0$, $f(u_i) = F_i$, $3 \leq i \leq n$.
With this labeling the edge labels produced will satisfy the condition as shown in following table.
Let $n = 3a + b$, $a, b \in N$.

b	edge conditions
0,1	$e_f(0) + 1 = e_f(1)$
2	$e_f(0) = e_f(1) + 1$

Table 1: Table for Fibonacci cordial labeling of S_n.

From the above table $|e_f(0) - e_f(1)| \leq 1$.
Hence, shell S_n is Fibonacci cordial for $n \geq 3$, $n \in N$.

Example 2.3. Fibonacci cordial labeling of S_{11} is shown in Fig. 3.

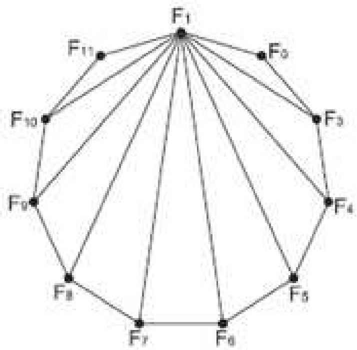


Figure 3: Fibonacci cordial labeling of S_{11}

Theorem 2.4. Bistar $B_{n,n}$ is a Fibonacci cordial, for all n.
Proof. Let u_0, v_0 be the apex vertices of $B_{n,n}$. Let $u_1, u_2, \ldots, u_n$ be the pendant vertices adjacent to the vertex u_0 and $v_1, v_2, \ldots, v_n$ be the pendant vertices adjacent to the vertex v_0.
We define labeling function $f : V(G) \to \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows.

<u>Case 1</u>: $n \equiv 2(\text{mod}3)$.
$f(u_0) = F_1$, $f(u_1) = F_2$, $f(u_i) = F_{i+2}$, $2 \leq i \leq n$.
$f(v_0) = F_0$, $f(v_1) = F_3$, $f(v_i) = F_{n+i+1}$, $2 \leq i \leq n$.
Then we have $e_f(0) = n + 1$ and $e_f(1) = n$.

<u>Case 2</u>: n does not congruent to 2(mod3).
$f(u_0) = F_1$, $f(u_i) = F_{i+1}$, $1 \leq i \leq n$.
$f(v_0) = F_0$, $f(v_i) = F_{n+i+1}$, $1 \leq i \leq n$.
Then we have $e_f(0) = n + 1$ and $e_f(1) = n$.
Hence, $B_{n,n}$ is a Fibonacci cordial graph, for all n.

Example 2.4. Fibonacci cordial labeling of $B_{7,7}$ is shown in Fig.4.

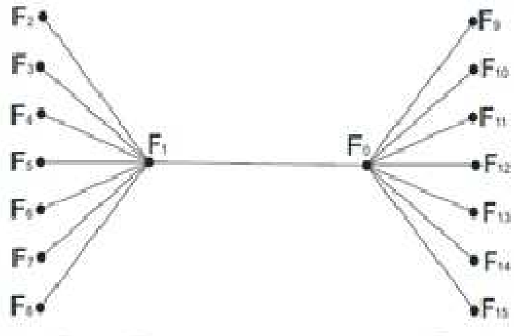


Figure 4: Fibonacci cordial labeling of $B_{7,7}$

Theorem 2.5. G =< $B_{n,n}$: w > is Fibonacci cordial graph.
Proof. Let u_0, v_0 be the apex vertices of $B_{n,n}$ and w be the vertex added as a result of subdivision of the edge joining u_0 and v_0. Let $u_1, u_2, \ldots, u_n$ be the pendant vertices adjacent to the vertex u_0 and $v_1, v_2, \ldots, v_n$ be the pendant vertices adjacent to the vertex v_0.
We define vertex labeling f: V (G) → $\{F_0, F_1, F_2, \ldots, F_{2n+3}\}$ as follows.
Case 1: $n \equiv 1(\text{mod}3)$.
$f(w) = F_2$, $f(u_0) = F_1$, $f(u_1) = F_{2n+3}$, $f(u_i) = F_{i+2}$, $2 \le i \le n$.
$f(v_0) = F_0$, $f(v_i) = F_{n+i+2}$, $1 \le i \le n$.
Then we have $e_f(0) = e_f(1) = n + 1$.

Case 2: n does not congruent to 1(mod3).
$f(w) = F_2$, $f(u_0) = F_1$, $f(u_i) = F_{i+2}$, $1 \le i \le n$.
$f(v_0) = F_0$, $f(v_i) = F_{n+i+2}$, $1 \le i \le n$.
Then we have $e_f(0) = e_f(1) = n + 1$.
Hence G =< $B_{n,n}$: w > is Fibonacci cordial graph.

Example 2.5. Fibonacci cordial labeling of subdivision of $B_{7,7}$ is shown in Fig.5.

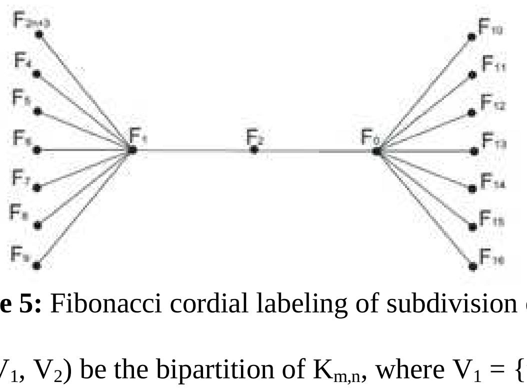


Figure 5: Fibonacci cordial labeling of subdivision of $B_{7,7}$

Definition 2.1. Let (V_1, V_2) be the bipartition of $K_{m,n}$, where $V_1 = \{u_1, u_2, \ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. The graph $K_{m,n} \odot u_i(K_1)$ is defined by attaching a pendant vertex to the vertex u_i for some i.

Theorem 2.6. The graph $K_{2,n} \odot u_2(K_1)$ is Fibonacci cordial graph.
Proof. Let $G = K_{2,n} \odot u_2(K_1)$. Let $V = V_1 \cup V_2$ be the bipartition of $K_{2,n}$ such that $V_1 = \{u_1, u_2\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$ and pendant vertex w is adjacent to vertex u_2 in G.
We define vertex labeling $f : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{n+3}\}$ as follows:
$f(u_1) = F_1$, $f(u_2) = F_3$. Assign the largest Fibonacci prime number to w and assign the remaining Fibonacci numbers to the vertices $v_1, v_2, \ldots, v_n$ in the Fibonacci sequence.
Then we have $e_f(0) = n$ and $e_f(1) = n + 1$.
Hence, $K_{2,n} \odot u_2(K_1)$ is a Fibonacci cordial graph.

Example 2.6. Fibonacci cordial labeling of the graph $K_{2,6} \odot u_2(K_1)$ is shown in Fig. 6.

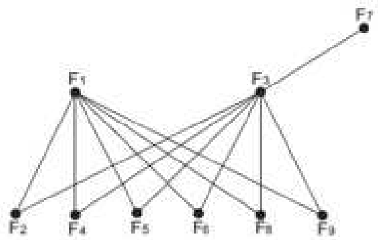


Figure 6: Fibonacci cordial labeling of $K_{2,6} \odot u_2(K_1)$

Theorem 2.7. The graph $C_n \oplus K_{1,n}$ is Fibonacci cordial, $n \geq 3$.
Proof. Let $V(G) = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \ldots, u_n\}$ be the vertex set of C_n, $V_2 = \{v, v_1, v_2, \ldots, v_n\}$ be the vertex set of $K_{1,n}$, $v = u_1$ and $v_1, v_2, \ldots, v_n$ are pendant vertices. Note that $|V(G)| = |E(G)| = 2n$.
We define $f: V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n}\}$ as follows.

Case I: $n \equiv 0 \pmod 3$
$f(u_1) = F_1$, $f(u_n) = F_0$, $f(u_i) = F_{n+i-1}$, $2 \leq i \leq n - 1$.
$f(v_1) = F_{2n-1}$, $f(v_j) = F_j$, $2 \leq j \leq n$.
Then we have $e_f(0) = e_f(1) = n$.

Case II: $n \equiv 1 \pmod 3$
$f(u_1) = F_1$, $f(u_i) = F_{n+i-1}$, $2 \leq i \leq n$.
$f(v_1) = F_0$, $f(v_j) = F_j$, $2 \leq j \leq n$.
Then we have $e_f(0) = e_f(1) = n$.

Case III: $n \equiv 2 \pmod 3$
$f(u_1) = F_1$, $f(u_n) = F_3$, $f(u_i) = F_{n+i+1}$, $2 \leq i \leq n - 1$.
$f(v_1) = F_2$, $f(v_j) = F_{j+2}$, $2 \leq j \leq n$.
Then in each case we have $e_f(0) = e_f(1) = n$.
Therefore $|e_f(0) - e_f(1)| \leq 1$.
Hence $C_n \oplus K_{1,n}$ is a Fibonacci cordial.

Example 2.7. Fibonacci cordial labeling of the graph $C_9 \oplus K_{1,9}$ is shown in Fig.7.

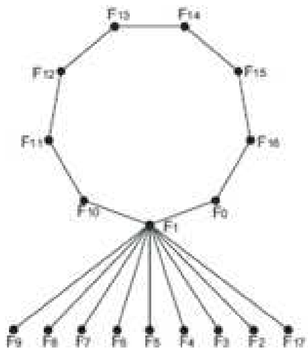


Figure 7: Fibonacci cordial labeling of the graph $C_9 \oplus K_{1,9}$

Theorem 2.8. $B^2_{n,n}$ is Fibonacci cordial.

Proof. Consider Bn,n with vertex set $\{u, v, ui, vi/1 \leq i \leq n\}$, where ui, vi are pendant vertices and u, v are apex. Let G be the graph $B^2_{n,n}$. Here $|V(G)| = 2n + 2$ and $|E(G)| = 4n + 1$.

We define vertex labeling $f : V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows.

$f(u) = F_1$,

$f(u_i) = F_{i+2}$, $1 \leq i \leq n$.

$f(v) = F_0$,

$f(v_i) = F_{n+i+2}$, $1 \leq i \leq n$.

Then we have $e_f(0) = 2n$ and $e_f(1) = 2n + 1$.

Hence, $B^2_{n,n}$ is Fibonacci cordial.

Example 2.8. The Fibonacci cordial labeling of $B^2_{5,5}$ is as shown in Fig. 8.

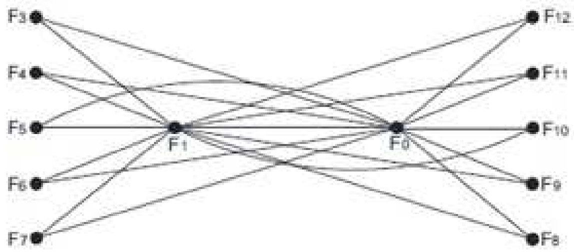


Figure 8: Fibonacci cordial labeling of $B^2_{5,5}$

Theorem 2.9. The graph obtained by vertex switching of cycle C_n is Fibonacci cordial.

Proof. Let $u_1, u_2, \ldots, u_n$ be the successive vertices of cycle C_n. Let $(C_n)u_1$ denote the vertex switching of C_n with respect to the vertex u_1.

We define labeling $f: V((C_n)u_1) \rightarrow \{F_0, F_1, F_2, \ldots, F_n\}$ as

$f(u_1) = F_1$, $f(u_2) = F_0$, $f(u_i) = F_i$, $3 \leq i \leq n$.

With this labeling the edge labels produced will satisfy the condition as shown in following table.

Let $n = 3a + b$, $a, b \in N$.

b	edge conditions
1	$e_f(0) + 1 = e_f(1)$
0,2	$e_f(0) = e_f(1) + 1$

Table 2: Table for Fibonacci cordial labeling of vertex switching of cycle C_n

From the above table $|e_f(0) - e_f(1)| \leq 1$.
Hence vertex switching of cycle C_n is Fibonacci cordial.

Example 2.9. Fibonacci cordial labeling for the graph obtained by vertex switching of cycle C_{10} is shown in Fig.9.

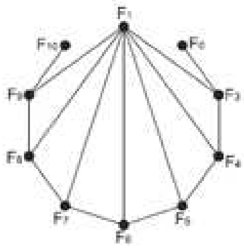


Figure 9: Fibonacci cordial labeling for the graph obtained by vertex switching of cycle C_{10}

Theorem 2.10. The graph obtained by vertex switching of any vertex of cycle C_n with one chord is Fibonacci cordial, for all $n \leq 4$.
Proof. Let G be the cycle with one chord. Let $u_1, u_2, \ldots, u_n$ be consecutive vertices of cycle C_n and $e = u_2u_n$ be the chord of cycle C_n. The vertices u_1, u_2, u_n form a triangle with chord e. If u_i and u_j are of same degree then the graph obtained by switching of vertex u_i and the graph obtained by switching of vertex u_j are isomorphic to each other. Hence we require to discuss two cases: (i) vertex switching of a vertex of C_n of degree 2 and (ii) vertex switching of a vertex of C_n of degree 3. Let $(G)u_1$ denote the vertex switching of G with respect to the vertex u_1.
To define labeling function $f: V((G)u_1) \rightarrow \{F_0, F_1, F_2, \ldots, F_n\}$ we consider the following cases.

Case 1: $\deg(u_1) = 2$
Subcase I: $n \equiv 0(\text{mod}3)$
$f(u_1) = F_1$, $f(u_2) = F_2$, $f(u_3) = F_0$, $f(u_i) = F_{i-1}$, $4 \leq i \leq n$.
Then we have $e_f(0) = e_f(1) = n - 2$.

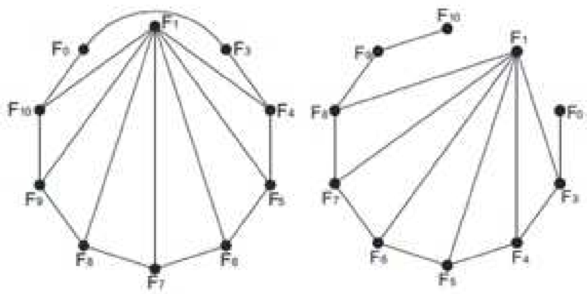
Subcase II: $n \equiv 1(\text{mod}3)$
$f(u_1) = F_1$, $f(u_n) = F_0$, $f(u_i) = F_{i+1}$, $2 \leq i \leq n - 1$.
Then we have $e_f(0) = e_f(1) = n - 2$.

Subcase III: $n \equiv 2(\text{mod}3)$
$f(u_i) = F_i$, $1 \leq i \leq n$.
Then we have $e_f(0) = e_f(1) = n - 2$.

Case 2: $\deg(u_1) = 3$
$f(u_1) = F_1$, $f(u_2) = F_0$, $f(u_i) = F_i$, $3 \leq i \leq n$.
Then we have $e_f(0) = e_f(1) = n - 3$.
Hence the graph obtained by vertex switching of cycle with one chord is Fibonacci cordial.

Example 10.
(a) Fibonacci cordial labeling of the graph obtained by switching of a vertex of degree 2 in cycle C_{10} with one chord is shown in Fig. 10(a).

(b) Fibonacci cordial labeling of the graph obtained by switching of a vertex of degree 3 in cycle C_{10} with one chord is shown in Fig. 10(b).



(a) (b)

Figure 10:

Theorem 2.11. The graph obtained by vertex switching of cycle C_n with twin chords $C_{n,3}$ is Fibonacci cordial, where chords form two triangles and one cycle C_{n-2}.

Proof. Let G be the cycle C_n with twin chords. Let $u_1, u_2, \ldots, u_n$ be the successive vertices of cycle C_n. Let $e_1 = u_nu_2$ and $e_2 = u_nu_3$ be the chords of cycle C_n. If u_i and u_j are of same degree then the graph obtained by switching of vertex u_i and the graph obtained by switching of vertex u_j are isomorphic to each other. Hence we require to discuss two cases: (i) vertex switching of a vertex of C_n of degree 2, (ii) vertex switching of a vertex of C_n of degree 3 and (iii) vertex switching of a vertex of C_n of degree 4. Let $(G)u_1$ denote the vertex switching of G with respect to the vertex u_1.
To define labeling $f: V((G)u_1) \rightarrow \{F_0, F_1, F_2, \ldots, F_n\}$ we consider the following cases.

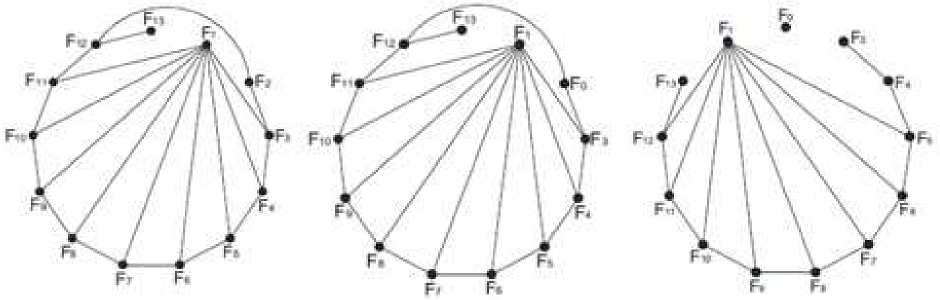
Case 1: $\deg(u_1) = 2$
$f(u_i) = F_i$, $1 \leq i \leq n$.

Case 2: $\deg(u_1) = 3$, $\deg(u_1) = 4$
$f(u_1) = F_1$, $f(u_2) = F_0$, $f(u_i) = F_i$, $3 \leq i \leq n$.
In each case $|e_f(0) - e_f(1)| \leq 1$.
Hence vertex switching of cycle C_n with twin chords is a Fibonacci cordial graph.

Example 2.11.
(a) Fibonacci cordial labeling of the graph obtained by switching of a vertex of degree 2 in cycle C_{13} with twin chords is shown in Fig. 11(a).
(b) Fibonacci cordial labeling of the graph obtained by switching of a vertex of degree 3

in cycle C_{13} with twin chords is shown in Fig. 11(b).

(c) Fibonacci cordial labeling of the graph obtained by switching of a vertex of degree 4 in cycle C_{13} with twin chords is shown in Fig. 11(c).



(a) (b) (c)

Figure 11:

Theorem 2.12. The graph obtained by joint sum of two copies of fan F_n is Fibonacci cordial.

Proof. Let G be the joint sum of two copies of fan F_n. Let $\{u_1, u_2, \ldots, u_n\}$ and $\{v_1, v_2, \ldots, v_n\}$ be the vertices of first and second copy of F_n respectively. Let u be the apex vertex of first copy of F_n and v be the apex vertex of second copy of F_n.
We define $f: V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows.

Case 1: $n \equiv 0, 1 (\text{mod}3)$.
$f(u_0) = F_1$, $f(u_1) = F_0$, $f(u_i) = F_{i+1}$, $2 \leq i \leq n$.
$f(v_0) = F_2$, $f(v_i) = F_{n+i+1}$, $1 \leq i \leq n$.

Case 2: $n \equiv 2(\text{mod}3)$.
$f(u_0) = F_1$, $f(u_i) = F_{i+2}$, $1 \leq i \leq n$.
$f(v_0) = F_2$, $f(v_n) = F_0$, $f(v_i) = F_{n+i+2}$, $1 \leq i \leq n - 1$.
In all cases, we have $|e_f(0) - e_f(1)| \leq 1$.
Hence the joint sum of two copies of F_n is Fibonacci cordial graph.

Example 2.12. The Fibonacci cordial labeling of joint sum of two copies of F_5 is shown in Fig. 12.

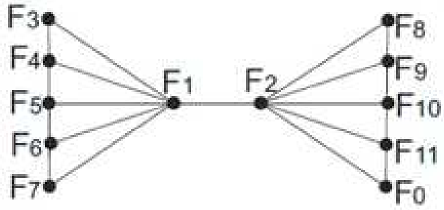


Figure 12: Fibonacci cordial labeling o joint sum of two copies of F_5

Theorem 2.13. The graph obtained by joint sum of two copies of wheel W_n is Fibonacci cordial.

Proof. Let G be the joint sum of two copies of W_n. Let $\{u_0, u_1, u_2, \ldots, u_n\}$ and $\{v_0, v_1, v_2, \ldots, v_n\}$ be the vertices of first and second copy of W_n respectively, where u_0 and v_0 be the apex vertices of first and second copy of W_n respectively.
Here we define labeling function $f: V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{2n+2}\}$ as follows.

Case 1: $n \equiv 0 (\mathrm{mod} 3)$
$f(u_0) = F_1$, $f(u_i) = F_{i+2}$, $1 \leq i \leq n$.
$f(v_0) = F_2$, $f(v_i) = F_{n+i+2}$, $1 \leq i \leq n$.
Then we have $e_f(0) = 2n + 1$ and $e_f(1) = 2n$.

Case 2: $n \equiv 1 (\mathrm{mod} 3)$
$f(u_0) = F_1$, $f(u_1) = F_0$, $f(u_i) = F_{i+1}$, $2 \leq i \leq n$.
$f(v_0) = F_2$, $f(v_i) = F_{n+i+1}$, $1 \leq i \leq n$.
Then we have $e_f(0) = 2n + 1$ and $e_f(1) = 2n$.

Case 3: $n \equiv 2 (\mathrm{mod} 3)$
$f(u_0) = F_1$, $f(u_1) = F_0$, $f(u_i) = F_{i+1}$, $2 \leq i \leq n$.
$f(v_0) = F_2$, $f(v_1) = F_{2n+2}$, $f(v_i) = F_{n+i}$, $2 \leq i \leq n$.
Then in each case we have $e_f(0) = 2n$ and $e_f(1) = 2n + 1$.
Therefore $|e_f(0) - e_f(1)| = 1$ in each case.
Hence joint sum of two copies of W_n is Fibonacci cordial.

Example 2.13. The Fibonacci cordial labeling of joint sum of two copies of W_9 is shown in Fig. 13.

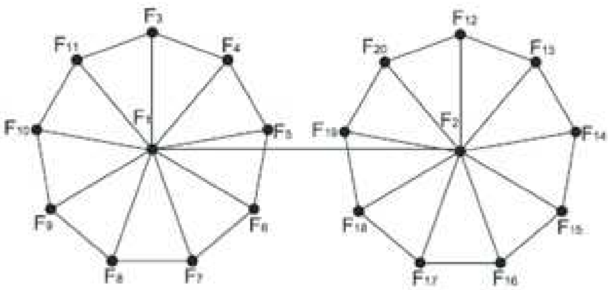


Figure 13: Fibonacci cordial labeling of joint sum of two copies of W_9

Theorem 2.14. The graph obtained by joint sum of two copies of petersen graph is Fibonacci cordial.
Proof. Let G be the joint sum of two copies of petersen graph. Let $\{u_1, u_2, \ldots, u_{10}\}$ and $\{v_1, v_2, \ldots, v_{10}\}$ be the vertices of first and second copy of petersen graph respectively.
We define labeling function $f: V(G) \rightarrow \{F_0, F_1, F_2, \ldots, F_{20}\}$ as follows.
$f(u_1) = F_0$, $f(u_2) = F_1$, $f(u_i) = F_i$, $3 \leq i \leq 10$.
$f(v_i) = F_{10+i}$, $1 \leq i \leq 10$.
Then we have $e_f(0) = 15$, $e_f(1) = 16$
Therefore, $|e_f(0) - e_f(1)| \leq 1$.
Hence the graph obtained by joint sum of two copies of petersen graph is Fibonacci cordial.

Example 2.14. Fibonacci cordial labeling of joint sum of two copies of petersen graph is as shown in Fig. 14.

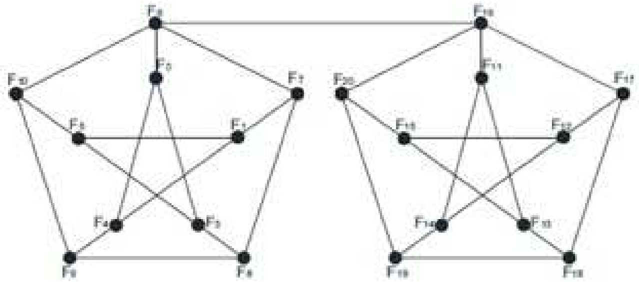


Figure 14: Fibonacci cordial labeling of joint sum of two copies of Petersen graph

3. Conclusion

We introduce here new concept of Fibonacci cordial labeling. This will add new dimension to the research work in the area connecting two branches - graph labeling and number theory. Here we have investigated nine new graph families which admit Fibonacci cordial labeling. Further we have discussed Fibonacci cordial labeling in context of vertex switching and joint sum of different graph families and derived five more results.

REFERENCES

1. Cahit, Cordial graphs: A weaker version of graceful and Harmonic graphs, *Ars Combinatoria*, 23(1987) 201 − 207.
2. J.A.Gallian, A dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, 16(2013) DS 61 − 308.
3. J.Gross and J.Yellen, *Graph Theory and its Applications*, CRC Press, 1999.
4. R.Sridevi, K.Nagarajun, A.Nellaimurugan and S.Navanaeethakrishnan, Fibonacci divisor cordial graphs, *International Journal of Mathematics and Soft Computing*, 3 (3) (2013) 33 − 39.