Common Fixed Point Theorems In Intuitionistic Fuzzy Symmetric Spaces For Occasionally Weakly Compatible Maps Satisfying Contractive Condition Of Integral Type

Aarti Sugandhi\(^1\), Sandeep Kumar Tiwari\(^2\) and Aklesh Pariya\(^3\)

\(^1\)School of Studies in Mathematics, Vikram University Ujjain (M.P), India  
\(^2\)Lakshmi Narain College of Technology and Science, Indore (M.P), India  
\(^3\)Email: skt tiwari75@yahoo.co.in, akleshpariya3@yahoo.co.in  
\(^1\)Corresponding author: email: aartivhs@gmail.com

Received 16 February 2016; accepted 15 March 2016

Abstract. The aim of this paper is to prove common fixed point theorem for occasionally weakly compatible mappings satisfying general contractive condition of integral type in intuitionistic fuzzy symmetric space.

Keywords: Occasionally weakly compatible, contractive condition of integral type, symmetric spaces.

AMS Mathematics Subject Classification (2010): 47H10, 54H25.

1. Introduction


2. Basic definitions and preliminaries

We recall some definitions and known results in intuitionistic fuzzy metric spaces

**Definition 2.1.** [12] A binary operation *:\([0,1]x[0,1]\rightarrow[0,1]\) is called a t-norm * satisfies the following conditions:

i. * is continuous,
ii. * is commutative and associative,
iii. \(a * 1 = a\) for all \(a \in [0, 1]\),
iv. \(a * b \leq c * d\) whenever \(a \leq c\) and \(b \leq d\) for all \(a, b, c, d \in [0, 1]\).
Example 2.1. \(a * b = ab\) and \(a * b = \min(a, b)\).

**Definition 2.2.** [12] A binary operation \(\hat{\diamond}: [0,1] \times [0,1] \to [0,1]\) is said to be continuous \(t\)-conorm if it satisfied the following conditions:

i. \(\hat{\diamond}\) is associative and commutative,

ii. \(a \hat{\diamond} 0 = a\) for all \(a \in [0,1]\),

iii. \(\hat{\diamond}\) is continuous,

iv. \(a \hat{\diamond} b \leq c \hat{\diamond} d\) whenever \(a \leq c\) and \(b \leq d\) for each \(a, b, c, d \in [0,1]\).

Example 2.2. \(\hat{\diamond} = \min(a+b, 1)\) and \(\hat{\diamond} = \max(a, b)\).

Recall that a symmetric on \(X\) is a nonnegative real valued function \(d\) on \(X \times X\) such that

\[(I)\] \(\quad d(x, y) = 0\) if and only if \(x = y\), and

\[(II)\] \(\quad d(x, y) = d(y, x)\)

**Definition 2.3.** [8] A subset \(S\) of a symmetric space \((X, d)\) is said to be \(d\)-closed if for a sequence \(\{x_n\}\) in \(S\) and a point \(x \in X\), \(\lim_{n \to \infty} d(x_n, x) = 0\) implies \(x \in S\).

For a symmetric space \((X, d)\), \(d\)-closedness implies \(\exists d\)-closedness, and if \(d\) is a symmetric, the converse is also true.

Yaoyao [14] gave intuitionistic fuzzy version of the definition of symmetric spaces.

**Definition 2.5.** [14] A 3-tuple \((X, M, N)\) is called intuitionistic fuzzy symmetric space if \(X\) is an arbitrary set and \(M, N\) are fuzzy sets on \(X \times X\) satisfying the following conditions:

For all \(x, y, z \in X\) and \(t, s > 0\)

\[\text{IFSym-1}\] \(M(x, y, t) + N(x, y, t) \leq 1\),

\[\text{IFSym-2}\] \(M(x, y, 0) > 0\),

\[\text{IFSym-3}\] \(M(x, y, t) = 1\) if and only if \(x = y\),

\[\text{IFSym-4}\] \(M(x, y, t) = M(y, x, t)\),

\[\text{IFSym-5}\] \(M(x, y, t): (0, \infty) \to (0, 1]\) is continuous,

\[\text{IFSym-6}\] \(N(x, y, 0) < 1\),

\[\text{IFSym-7}\] \(N(x, y, t) = 0\) if and only if \(x = y\),

\[\text{IFSym-8}\] \(N(x, y, t) = N(y, x, t)\),

\[\text{IFSym-9}\] \(N(x, y, t): (0, \infty) \to (0, 1]\) is continuous.

Then \((M, N)\) is called an intuitionistic fuzzy symmetric on \(X\). The function \(M(x, y, t)\) and \(N(x, y, t)\) denote the degree of nearness and degree of non nearness between \(x\) and \(y\) with respect to \(t\), respectively.

Example 2.3. [14] Let \(d\) be a symmetric on \(X\) defined by for all \(x, y \in X\),

\[\quad d(x, y) = e^{|x-y|} - 1\]

Let \(M(x, y, t) = \frac{t}{t+d(x,y)}\) and \(N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}\) for all \(x, y \in X\) and \(t > 0\).
Common Fixed Point Theorems In Intutionistic Fuzzy Symmetric …..

Then \((X, M, N)\) is an intuitionistic fuzzy symmetric space induced by the symmetric \(d\). It is obvious that \(N(x, y, t) = 1 - M(x, y, t)\).

Now consider an intuitionistic fuzzy symmetric space with the following two conditions:

**IFW.1.** [14] Given \(x, y \in X\),
\[
\lim_{n \to \infty} M(x_n, x, t) = 1, \quad \lim_{n \to \infty} N(x_n, x, t) = 0
\]
and
\[
\lim_{n \to \infty} M(x_n, y, t) = 1, \quad \lim_{n \to \infty} N(x_n, y, t) = 0
\]

imply \(x = y\).

**IFW.2.** [14] Given \(x_n, y_n \in X\) and \(x \in X\),
\[
\lim_{n \to \infty} M(x_n, x, t) = 1, \quad \lim_{n \to \infty} N(x_n, x, t) = 0
\]
and
\[
\lim_{n \to \infty} M(y_n, x_n, t) = 1, \quad \lim_{n \to \infty} N(y_n, x_n, t) = 0
\]
imply \(\lim_{n \to \infty} M(y_n, x, t) = 1, \quad \lim_{n \to \infty} N(y_n, x, t) = 0\).

**Definition 2.6.** [14] Let \(f\) and \(g\) be self mappings of an intuitionistic fuzzy symmetric space \((X, M, N)\). \(f\) and \(g\) are called compatible if
\[
\lim_{n \to \infty} M(fx_n, y, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(fx_n, y, t) = 0
\]
whenever \(\{x_n\}\) is a sequence in \(X\) such that
\[
\lim_{n \to \infty} M(fx_n, x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(fx_n, x_n, t) = 0
\]
and
\[
\lim_{n \to \infty} M(gx_n, y, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(gx_n, y, t) = 0 \quad \text{for some} \ y \in X.
\]

**Definition 2.7.** [14] Let \(f\) and \(g\) be self mappings of an Intuitionistic Fuzzy symmetric space \((X, M, N)\). \(f\) and \(g\) are said to be weakly compatible if they commute at their coincidence points i.e. \(fu = gu \) for some \(u \in X\). then \(fgu = gfu\).

Now we define occasionally weakly compatible in an intuitionistic fuzzy symmetric space as:

**Definition 2.8.** Self mappings \(f\) and \(g\) of an intuitionistic fuzzy symmetric space \((X, M, N)\) is said to be occasionally weakly compatible (owc) if there exists a point \(x \in X\) which is a coincidence point of \(f\) and \(g\) at which \(f\) and \(g\) commute.

**Definition 2.9.** [14] Let \(f\) and \(g\) be self mappings of an Intuitionistic Fuzzy symmetric space \((X, M, N)\), we say that \(f\) and \(g\) satisfy the property (IFE.A.) if there exists a sequence \(\{x_n\}\) such that
\[
\lim_{n \to \infty} M(fx_n, y, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(fx_n, y, t) = 0
\]
and
\[
\lim_{n \to \infty} M(gx_n, y, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(gx_n, y, t) = 0 \quad \text{for some} \ y \in X.
\]

**Remark 2.2.** It is clear from the above Definition 2.9 that two self mappings \(f\) and \(g\) of an intuitionistic fuzzy symmetric space \((X, M, N)\) will be non – compatible if there exists at least one sequence \(\{x_n\}\) such that
\[
\lim_{n \to \infty} M(fx_n, y, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(fx_n, y, t) = 0
\]
Aarti Sugandhi, Sandeep Kumar Tiwari and Aklesh Pariya

and

\[ \lim_{n \to \infty} M(gx_n, y, t) = 1 \text{ and } \lim_{n \to \infty} N(gx_n, y, t) = 0 \]

for some \( y \in X \), but

\[ \lim_{n \to \infty} M(\int g(x_n, g\int x_n), t) \neq 1 \text{ and } \lim_{n \to \infty} N(\int g(x_n, g\int x_n), t) \neq 0 \]

or do not exists.

Clearly, two non–compatible self mappings of an intuitionistic fuzzy symmetric space \((X, M, N)\) satisfy the property \(\text{IFE.A}\).

**Definition 2.10.** [14] Let \((X, M, N)\) be an intuitionistic fuzzy symmetric space, we say that \((X, M, N)\) satisfies the property \(\text{IFE.A.}\) if given sequences \(\{x_n\}, \{y_n\}\) such that

\[ \lim_{n \to \infty} M(x_n, x, t) = 1, \quad \lim_{n \to \infty} N(y_n, x, t) = 0 \]

and

\[ \lim_{n \to \infty} M(y_n, x, t) = 1, \quad \lim_{n \to \infty} N(y_n, x, t) = 0 \]

imply that

\[ \lim_{n \to \infty} M(y_n, x_n, t) = 1, \quad \lim_{n \to \infty} N(y_n, x_n, t) = 0 \]

**Lemma 2.1.** [10] Let \(A\) and \(B\) be self maps on \(X\) and let \(A\) and \(B\) have a unique point of coincidence, \(w = Ax = Bx\), then \(w\) is unique fixed point of \(A\) and \(B\).

**Definition 2.11.** Let \(\phi, \psi : R^+ \to R^+\) are continuous, non–increasing, non–decreasing functions respectively satisfying the conditions

\[ \phi(0) = 1, \quad \phi(t) > t, \quad \psi(0) = 0, \quad \psi(t) < t \quad \text{for every} \quad t > 0. \]

3. Main result

**Theorem 3.1.** Let \((X, M, N, *, \diamond)\) be a intuitionistic fuzzy symmetric space that satisfy \((IFW1), (IFW2), (IFE.A)\) and let \(A, B, S,\) and \(T\) be self mapping of \(X\) such that

(I) \(A(X) \subseteq T(X)\) and \(B(X) \subseteq S(X)\).

(II) For all \(x,y \in X\), let \(\phi, \psi : R^+ \to R^+\) are continuous, non–increasing, non–decreasing functions respectively satisfying the conditions \(\phi(0) = 1, \quad \phi(t) > t, \quad \psi(0) = 0, \quad \psi(t) < t \quad \text{for every} \quad t > 0\) such that

\[ \int_0^{\int M(Ax, By, t)} \phi(t)dt \geq \phi \left( \int_0^{\int M(x, y, t)} \phi(t)dt \right) \]

and

\[ \int_0^{\int N(Ax, By, t)} \psi(t)dt \leq \psi \left( \int_0^{\int N(x, y, t)} \psi(t)dt \right) \]

where \(\phi, R^+ \to R^+\) is a lebesgue integrable mapping which is summable, non-negative and such that \(\int_0^\infty \phi(t)dt > 0\) for each \(\varepsilon > 0\) and

\[ m(x, y, t) = \min \{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), \frac{1}{2}(M(Sx, By, t) + M(Ax, Ty, t)) \} \]

\[ n(x, y, t) = \max \{N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), \frac{1}{2}(N(Sx, By, t) + N(Ax, Ty, t)) \} \]

(III) Suppose that \((B, T)\) satisfied property \((\text{IFE.A.})\) respectively, \((A, S)\) satisfies property \((\text{IFE.A.})\) and
Common Fixed Point Theorems In Intuitionistic Fuzzy Symmetric ….

(IV) the pairs (A, S) and (B, T) are occasionally weakly compatible.

(V) S(X) is a d- closed subset of X( resp., T(X) is a d- closed subset of X).

Then A, B, S and T have a unique common fixed point in X.

Proof: Since the pair (B, T) satisfies property (E.A.), so there exists a sequence \( \{x_n\} \) in X, and a point \( z \in X \) such that \( \lim_{n \to \infty} M(Tx_n, z, t) = \lim_{n \to \infty} M(Bx_n, z, t) = 1 \) and \( \lim_{n \to \infty} N(Tx_n, z, t) = \lim_{n \to \infty} N(Bx_n, z, t) = 0 \).

From (I), \( B(X) \subset S(X) \), there exists a sequence \( \{y_n\} \) in X such that \( Bx_n = Sy_n \) and hence \( \lim_{n \to \infty} M(Sy_n, z, t) = 1 \) and \( \lim_{n \to \infty} N(Sy_n, z, t) = 0 \).

By property (IFH), \( \lim_{n \to \infty} M(Bx_n, Tx_n, t) = \lim_{n \to \infty} M(Sy_n, Tx_n, t) = 1 \) and \( \lim_{n \to \infty} N(Bx_n, Tx_n, t) = \lim_{n \to \infty} N(Sy_n, Tx_n, t) = 0 \).

From (V), \( S(X) \) is a d- closed subset of X there exists a point \( u \in X \) such that \( Su = z \).

Now we will prove that \( Au = Su \). Suppose not then

\[
\int_0^{M(Au,x,t)} \varphi(t) \, dt = \int_0^r \varphi(t) \, dt, \quad \text{where } r = \lim_{n \to \infty} m(Au, Bx_n, t)
\]

\[
\geq \varphi \left( \int_0^{m(u,x_n,t)} \varphi(t) \, dt \right)
\]

and

\[
\int_0^{N(Au,x,t)} \varphi(t) \, dt = \int_0^s \varphi(t) \, dt, \quad \text{where } s = \lim_{n \to \infty} n(Au, Bx_n, t)
\]

\[
\leq \psi \left( \int_0^{n(u,x_n,t)} \varphi(t) \, dt \right)
\]

where

\[
\lim_{n \to \infty} m(u,x_n,t) = \lim_{n \to \infty} \min \{ M(Su,Tx_n,t), M(Au,Su,t), M(Bx_n,Tx_n,t), \frac{1}{2}(M(Su,Bx_n,t) + M(Au,Tx_n,t)) \}
\]

and

\[
\lim_{n \to \infty} n(u,x_n,t) = \lim_{n \to \infty} \max \{ N(Su,Tx_n,t), N(Au,Su,t), N(Bx_n,Tx_n,t), \frac{1}{2}(N(Su,Bx_n,t) + N(Au,Tx_n,t)) \}
\]

On using the property (IFH), we get

\[
\lim_{n \to \infty} m(u,x_n,t) = \lim_{n \to \infty} \min \{ M(Au,z,t), 1, \frac{1}{2} (1 + M(Au,z,t)) \}
\]

and

\[
\lim_{n \to \infty} n(u,x_n,t) = \lim_{n \to \infty} \max \{ 0, N(Au,z,t), 0, \frac{1}{2} (0 + N(Au,z,t)) \}
\]

we have

\[
\int_0^{M(Au,x,t)} \varphi(t) \, dt \geq \varphi \left( \int_0^{M(Au,x,t)} \varphi(t) \, dt \right)
\]

\[
\int_0^{N(Au,x,t)} \varphi(t) \, dt \leq \psi \left( \int_0^{N(Au,x,t)} \varphi(t) \, dt \right)
\]

which is contradiction. Hence \( Au = Su = z \).

Again by (I) \( A(X) \subset T(X) \), there exists a point \( w \in X \) such that \( Au = Tw \). Now we will show that \( Tw = Bw \). Suppose not, then by (II) we have
Aarti Sugandhi, Sandeep Kumar Tiwari and Aklesh Pariya

\[ \int_0^{M(Au,Bw,t)} \varphi(t)dt \geq \Phi \left( \int_0^{m(u,w,t)} \varphi(t)dt \right) \]
\[ \geq \Phi \left( \int_0^{\min \{M(Su,Tw,t),M(Au,Su,t),M(Bw,Tw,t),\frac{1}{2}(M(Su,Bw,t)+M(Au,Tw,t))\}} \varphi(t)dt \right) \]
\[ \geq \Phi \left( \int_0^{\max \{0,0,M(Au,Bw,t)\}} \frac{1}{2}(M(Au,Bw,t)+0) \varphi(t)dt \right) \]
\[ \leq \Psi \left( \int_0^{\min \{N(Su,Tw,t),N(Au,Su,t),N(Bw,Tw,t),\frac{1}{2}(N(Su,Bw,t)+N(Au,Tw,t))\}} \varphi(t)dt \right) \]
\[ \leq \Psi \left( \int_0^{\max \{0,0,N(Au,Bw,t)\}} \frac{1}{2}(N(Au,Bw,t)+0) \varphi(t)dt \right) \]
\[ \leq \Psi \left( \int_0^{N(Au,Bw,t)} \varphi(t)dt \right) \]
\[ < \int_0^{N(Au,Bw,t)} \varphi(t)dt \]

which is a contradiction. Hence Tw = Bw.

Thus Au = Su = Tw = Bw = z.

Now by (IV), (A, S) and (B, T) are occasionally weakly compatible, we have

AAu = ASu = SSu and BTw = TBw = TTw = BBw.

Now we will show that Au = w. Suppose Au ≠ w then by (II)

\[ \int_0^{M(Au,Au,u,t)} \varphi(t)dt = \Phi \left( \int_0^{m(Au,Au,u,t)} \varphi(t)dt \right) \]
\[ \geq \Phi \left( \int_0^{\min \{M(SAu,Tw,t),M(AAu,SAu,t),M(Bw,Tw,t),\frac{1}{2}(M(SAu,Bw,t)+M(AAu,Tw,t))\}} \varphi(t)dt \right) \]
\[ \geq \Phi \left( \int_0^{\max \{0,0,M(AAu,Bw,t)\}} \frac{1}{2}(M(AAu,Bw,t)+0) \varphi(t)dt \right) \]
\[ \leq \Psi \left( \int_0^{\min \{N(AAu,Bw,t),N(AAu,Su,t),N(Bw,Tw,t),\frac{1}{2}(N(Su,Bw,t)+N(Au,Tw,t))\}} \varphi(t)dt \right) \]
\[ \leq \Psi \left( \int_0^{\max \{0,0,N(AAu,Bw,t)\}} \frac{1}{2}(N(AAu,Bw,t)+0) \varphi(t)dt \right) \]
\[ \leq \Psi \left( \int_0^{N(AAu,Bw,t)} \varphi(t)dt \right) \]
\[ < \int_0^{N(AAu,Bw,t)} \varphi(t)dt \]
Common Fixed Point Theorems In Intuitionistic Fuzzy Symmetric ….

\[ > \int_0^M(Au,Bw) \varphi(t) dt = \int_0^M(Au,Au) \varphi(t) dt \]
i.e. \( \int_0^M(Au,Au) \varphi(t) dt > \int_0^M(Au,Au) \varphi(t) dt \)
and \( \int_0^N(Au,Au) \varphi(t) dt = \int_0^N(Au,Bw) \varphi(t) dt \)
\[ \leq \psi \left( \int_0^n(Au,Au) \varphi(t) dt \right) \]
\[ \leq \psi \left( \int_0^n \max \{ N(Au,Bw) \} \varphi(t) dt \right) \]
\[ \leq \psi \left( \int_0^n \max \{ N(Au,Bw),0\} \varphi(t) dt \right) \]
\[ \leq \psi \left( \int_0^n \max \{ N(Au,Bw)+N(Au,Bw),0\} \varphi(t) dt \right) \]
i.e. \( \int_0^n \max \{ N(Au,Bw),0\} \varphi(t) dt < \int_0^n \max \{ N(Au,Bw),0\} \varphi(t) dt \)
which is a contradiction. Hence \( Au = Su = w \). Similarly if \( Bw \neq u \),
we have a contradiction. Thus \( w = Au = Su = Bw = Tw = u \), so \( w = u \) is a
common fixed point of \( A, B, S \) and \( T \).

For the uniqueness, let \( v \) be another common fixed point of \( A, B, S \) and \( T \).
If \( w \neq v \), then from (II) we have
\[ \int_0^M(v,w) \varphi(t) dt = \int_0^M(Av,Bw) \varphi(t) dt \]
\[ > \int_0^M(v,w) \varphi(t) dt \]
\[ \geq \phi \left( \int_0^m \min \{ M(Sv,Tw),M(Sv,Av),M(Bw,Tw),M(Bw,Av) \} \varphi(t) dt \right) \]
\[ \geq \phi \left( \int_0^m \min \{ M(v,w),1,1,M(v,w) \} \varphi(t) dt \right) \]
\[ \geq \phi \left( \int_0^m \varphi(t) dt \right) \]
\[ > \int_0^m \varphi(t) dt \]
and
\[ \int_0^N(v,w) \varphi(t) dt = \int_0^N(Av,Bw) \varphi(t) dt \]
\[ \leq \psi \left( \int_0^n \max \{ N(Sv,Tw),N(Sv,Av),N(Bw,Tw),N(Bw,Av) \} \varphi(t) dt \right) \]
\[ \leq \psi \left( \int_0^n \max \{ N(v,w),0\} \varphi(t) dt \right) \]

17
Aarti Sugandhi, Sandeep Kumar Tiwari and Aklesh Pariya

\[ \psi \left( \int_0^{N(v,w)} \varphi(t) \, dt \right) \leq \psi \left( \int_0^{N(v,w)} \varphi(t) \, dt \right) \]

which is a contradiction. Hence \( w = v \).

This complete the proof.

**Corollary 3.1.** Let \( (X, M, N, \ast, \Diamond) \) be a Intuitionistic fuzzy symmetric space that satisfy (IFW1), (IFW2), (IFH), and let \( A, B, S, \) and \( T \) be self mapping of \( X \) satisfy the conditions (I), (II), (III) and (V) and the pairs \((A, S), (B, T)\) are weakly compatible then \( A, B, S \) and \( T \) have a unique common fixed point in \( X \).

**Proof:** Since weakly compatible mappings are occasionally weakly compatible mappings result follows from theorem 3.1.

**Corollary 3.2.** Let \( (X, M, N, \ast, \Diamond) \) be a Intuitionistic fuzzy symmetric space that satisfy (IFW1), (IFW2), (IFH), and let \( A, B, S, \) and \( T \) be self mapping of \( X \) such that

(I) \( A(X) \subseteq T(X) \) and \( B(X) \subseteq S(X) \),

(II) for all \( x, y \in X \), let \( \phi, \psi : \mathbb{R}^+ \to \mathbb{R}^+ \) are continuous, non – increasing, non – decreasing functions respectively satisfying the conditions \( \phi(0) = 1, \phi(t) > 0, \psi(0) = 0, \psi(t) < t \) for every \( t > 0 \) such that

\[
M(Ax, By, t) \geq \phi(m(x, y, t))
\]

and

\[
N(Ax, By, t) \leq \psi(n(x, y, t))
\]

where \( m(x, y, t) = \min \{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\} \frac{1}{2} (M(Sx, By, t) + MAx, Ty, t) \)

and \( n(x, y, t) = \max \{N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t)\} \frac{1}{2} (N(Sx, By, t) + NAx, Ty, t) \)

(III) Suppose that \( (B, T) \) satisfied property (IFE.A.) respectively, \( (A, S) \) satisfies property (IFE.A.) and (IV) the pairs \((A, S)\) and \((B, T)\) are occasionally weakly compatible.

(V) \( S(X) \) is a \( d \)- closed subset of \( X \) (resp., \( T(X) \) is a \( d \)- closed subset of \( X \)).

Then \( A, B, S \) and \( T \) have a unique common fixed point in \( X \).

**Proof:** If we put \( \varphi(t) = 1 \) in theorem 3.1, the result follows.

**REFERENCES**

Common Fixed Point Theorems In Intuitionistic Fuzzy Symmetric ….. 