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Hamiltonian-*t*-laceability Property of Edge Fault Hierarchical Hypercube Network

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Abstract. Fault tolerance is an important property of network performance. A graph G^* is $K-$ edge fault tolerant with respect to a graph G, if every graph obtained by removing any $K-$ edges from G^* contains G. A connected graph G is Hamiltonian-t-laceable if there exists a Hamiltonian path between every pair of vertices u and v in G with the property $d(u,v)=t$ where t is a positive integer such that $1 \le t \le diam(G)$ .The diameter of a graph G denoted by $diam(G)$ is the maximum eccentricity of any vertex in the graph. That is $diam(G)$ is the greatest distance between any pair of vertices in G. In this paper we explore the Hamiltonian-t-laceability property of the edge fault tolerant n -dimensional Hierarchical hypercube network for $n \ge 7$ where t is even and $2 \le t \le diam(G)$.

Keywords: Hamiltonian-t-laceable graph, Hierarchical hypercube graphs, Edge fault tolerance.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

The Hierarchical hypercube (HHC) network was proposed as an alternative to the hypercube network with thousands of processors are feasible to implement. Structural fault tolerance is defined as the ability to reconfigure around faults, so that the reconfigured system is isomorphic to the original one. If the interconnection structures changes due to the failure of processors or links, it is necessary to reconfigure around faults in order to preserve the basic interconnection structure. The fundamental to the reconfiguration process is the one to one mapping of faulty processors on to fault free ones. If F is a faulty edge in G, then $G-F$ denotes the graph obtained by deleting the fault from G. A graph G^* is 1- edge fault tolerant with respect to a graph G, if the graph obtained by removing one edge from G^* contains G. A connected graph G is Hamiltonian-t-laceable if there exists a Hamiltonian path between every pair of vertices u and v in G with the property $d(u,v)=t$ where t is a positive integer such that $1 \le t \le diam(G)$. The diameter of a graph G denoted by $diam(G)$ is the maximum eccentricity of any vertex in the graph. That is $diam(G)$ is the greatest distance between any pair of vertices in G. Hamiltonian laceability of n-dimensional Hierarchical hypercube network for $n \ge 6$ was explored by R.A. Daisy singh and R.Murali in [5]. In this paper, the authors have shown that the graph of 6-HHC is Hamiltonian-t-laceable for even t with one fault edge. In this paper we show that the edge fault tolerant n-dimensional Hierarchical hypercube graph for $n \ge 7$ is Hamiltonian-t-laceable for even t such that $2 \le t \le diam(G)$.

2. Hierarchical hypercube network

An n-dimensional Hierarchical hypercube network can be obtained by replacing each vertex say P of Q_{2m} with Q_m where each vertex of Q_m is uniquely connected to an adjacent vertex of P. Each vertex of an $n-HHC$ network can be identified with a two tuple (S,P) where $S=S_{n-m-1}S_{n-m-2}..........S_0$ is a binary sequence of length $(n-m)$, telling which Q_m the vertex is located in and $P=P_{m-1}P_{m-2}..........P_0$ is a binary sequence of length m, giving the address of the vertex in the located Q_m.

The vertex adjacency of a $n-HHC$ network is defined as follows. (S,P) is adjacent to internal edge $(S,P^{(1)})$ for all $0 \le l \le m-1$ and (S,P) is adjacent to external edge $(S^{(dec(p))},P)$ where $dec(P)$ is the decimal value of P. Internal edges are those which are contained in Q_m and external edges are those which connects two Q_m's. HHC networks have been described by Malluhi and Bayoumi [1] and they are symmetrical and have the special property that, addresses of any two adjacent nodes differ in one bit position. HHCs are able to connect a large number of nodes while retaining a small diameter and low degree. An HHC connects 2^n nodes where $n=2^m+m$ and the diameter is 2^{m+1} and the degree is $m+1$.

3. Results

Theorem 3.1. The 1-edge fault tolerant graph of 7-HHC network is Hamiltonian-t-laceable for even t such that $2 \le t \le diam(G)$.

Proof: Let G be a 7-HHC graph and it can be partitioned into two smaller HHC graph G_1 and G_2 where G_1 is a copy of 6-HHC graph which is bipartite and is properly colored with two different colors and G_2 is also a 6-HHC graph with one fault edge which makes it non bipartite. Let the distance between the two vertices which are of same color (different color) be even (odd) . We have three cases to discuss.

Case i: $S, E \in G_1$

Let S and E be the white vertices in G_1. We choose two black vertices A and B in G_1. Construct a path from S to A and a path from E to B such that the two paths are disjoint and covers all the vertices of G_1. Also by considering two vertices $\phi(A)$ and $\phi(B)$ in G_2, we construct the Hamiltonian path from $\phi(A)$ to $\phi(B)$. Then the path $P_1 : \{s \to A; (A, \phi(A)); \phi(A) \to \phi(B); (\phi(B), B); B \to E\}$ is a Hamiltonian path which includes all the vertices of G.

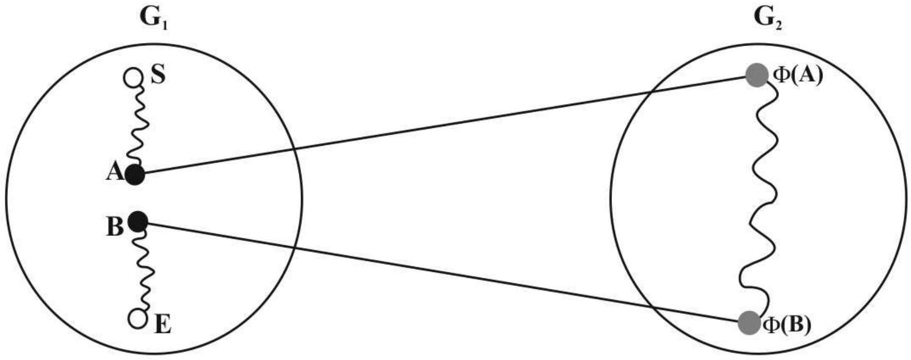

Figure 1:

Case ii: $S, E \in G_2$

Let S and E be the vertices of G_2 whose distance is even. Since G_1 is bipartite, it is properly colored with two colors. Then there exists vertices A_i and B_i in G_2 such that $A_i \neq \phi(A_i)$ and $B_i \neq \phi(B_i)$ where $\phi(A_i)$ is the black vertex and $\phi(B_i)$ is the white vertex of G_1. Then the path

$P_2 : \{S \to A_i; (A_i, \phi(A_i)); \phi(A_i) \to \phi(B_i); (\phi(B_i), B_i); B_i \to E\}$ is Hamiltonian.

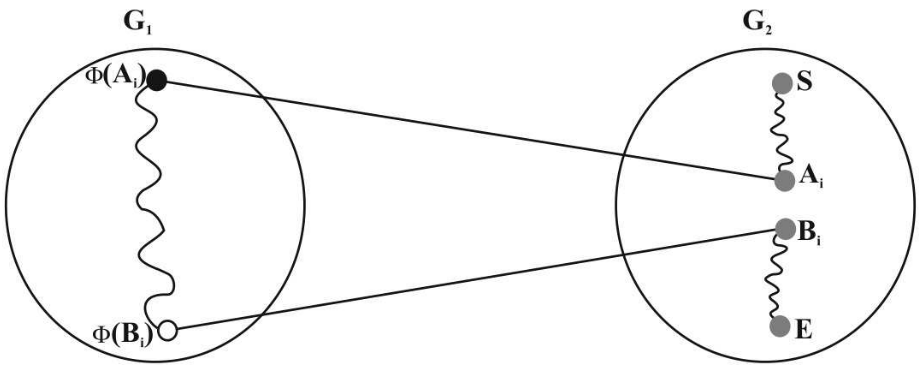

Figure 2:

The figure below depicts the actual Hamiltonian path of the graph,1-edge fault tolerant 7-HHC from $S\ to\, E \in G_2$.

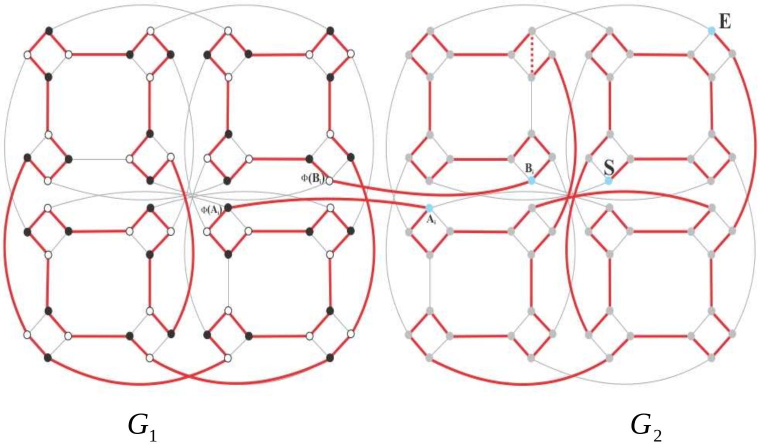

Figure 2(a):

Case iii: $S \in G_1$ and $E \in G_2$.

Let S be the vertex of G_1 and E be the vertex of G_2. Choose a vertex A of G_1 such that S and A are of different colors. Also choose $\phi(A)$ in G_2 such that $\phi(A) \neq E$. Then the path $P_3 : \{S \to A; (A, \phi(A)); \phi(A) \to E\}$ forms a Hamiltonian path.

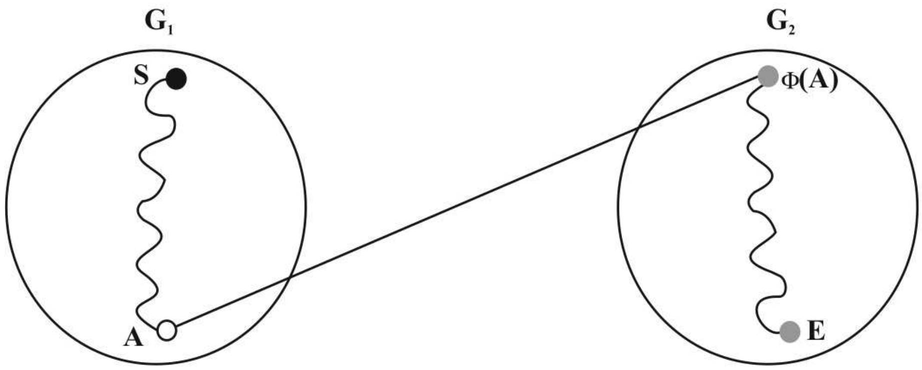

Figure 3:

From the above cases we can conclude that the 1- edge fault tolerant graph of 7-HHC network is Hamiltonian -t - laceable for even t such that $2 \leq t \leq diam\,(G)$.

Note: we construct a new graph G_k consisting of two subgraphs $g_k^{'}$ and $g_k^{''}$ where $g_k^{'}$ has $K-1$ copies of 6-HHC and $g_k^{''}$ has only one copy of 6-HHC with one fault edge added to it which makes the graph G_k, 1- edge fault tolerant.

Theorem 3.2. The 1-edge fault tolerant graph G_k is Hamiltonian -t- laceable for even t such that $2 \le t \le diam\,(G)$.

Proof: Let G_k be the graph consisting of K copies of 6-HHC of which one copy is added with one false edge. The graph G_k is partitionable into two smaller HHC graphs $g_k^{'}$ and $g_k^{''}$ of which $g_k^{'}$ has $K-1$ copies of 6-HHC and $g_k^{''}$ has only one copy of 6-HHC with one false edge. The graph $g_k^{'}$ is bipartite where as the graph $g_k^{''}$ is non bipartite. We consider three different cases.

Case i: Let $S, E \in g_k^{'}$

Subcase i: Vertices of same color S, E belongs to one copy of 6-HHC in $g_k^{'}$.

Since the graph $g_k^{'}$ is bipartite, it is properly colored with two colors. Let S, E be the two white vertices that belong to one copy of 6-HHC in $g_k^{'}$. Also we choose two black vertices A and B in the same copy. We construct a path from S to A and a path from E to B such that the paths are disjoint and covers all the vertices of the copy. Considering the vertex set $(A_i, B_i);(\phi(A_i), \phi(B_i))$ where $i = 1,2,......n-1$, in the remaining copies of $g_k^{'}$.Here (A_i, B_i) are all white vertices and $(\phi(A_i), \phi(B_i))$ are the black vertices. Then the paths $A_i \to \phi(A_i)$ and $B_i \to \phi(B_i)$ are disjoint, covers the remaining vertices of $g_k^{'}$. Let $\phi(A_n) \ne \phi(B_n)$ in $g_k^{''}$ such that there exists a Hamiltonian path from $\phi(A_n) \to \phi(B_n)$ in $g_k^{''}$, includes all the vertices. Then we join the end vertices $\phi(A_i)\,\&\,\phi(A_n)$ and $\phi(B_i)\,\&\,\phi(B_n)$ where $i = n-1$. The complete Hamiltonian path is

$$P_1 : \{S \to A; (A, A_i); A_i \to \phi(A_i); \phi(A_i) \to \phi(A_n); \phi(A_n) \to \phi(B_n); \phi(B_n) \to \phi(B_i); \phi(B_i) \to B_i; (B_i, B); B \to E\}.$$

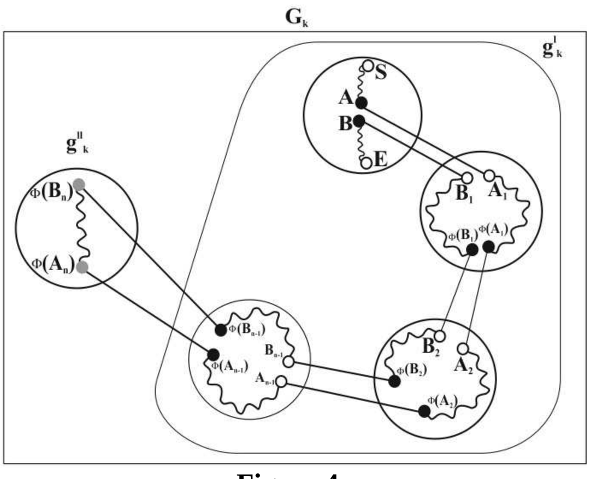

Figure 4:

Subcase ii: Vertices of same color S, E belongs to different copies of 6-HHC in $g_k^{'}$.
Let S & E be the two vertices which belongs to two different copies of 6-HHC in $g_k^{'}$

Here $P_2 : \{S \to A; (A, \phi(A)); \phi(A \to \phi(B)); (\phi(B), B); B \to S^{'}; S^{'} \to E\}$ is the Hamiltonian path of G_k.

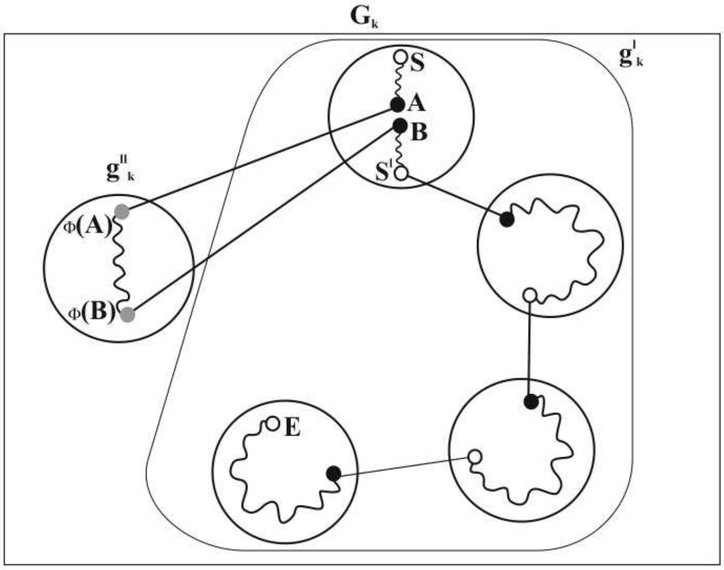

Figure 5:

Case ii: Let $S, E \in g_k^{''}$.

Since $g_k^{'}$ is properly colored with two colors there exists a vertex A such that the color of A and $\phi(A)$ are different and also there exists a vertex B such that the color of B and $\phi(B)$ are different. Then there exists two disjoint paths S to $S^{'}$ and E to $E^{'}$ in $g_k^{''}$, which includes all the vertices. Then the Hamiltonian path of G_k is
$P_3 : \{S \to S^{'}; (S^{'}, A); A \to \phi(A); \phi(A) \to \phi(B); \phi(B) \to B; (B, E^{'}); E^{'} \to E\}$.

Case iii: Let $S \in g_k^{'}$ and $E \in g_k^{''}$ whose distance is even.

We choose a vertex A of $g_k^{'}$ such that S and A are of different colors and $E \neq \phi(A)$ in $g_k^{''}$ and construct a Hamiltonian paths $P^{'}$ from S to A and P'' from $\phi(A)$ to E in each of $g_k^{'}$ and $g_k^{''}$ respectively. Then the complete Hamiltonian path is obtained by joining the paths $P^{'}$ and $P^{''}$. Thus $P_4 : \{S \to A; (A, \phi(A)); \phi(A) \to E\}$ is the Hamiltonian path.

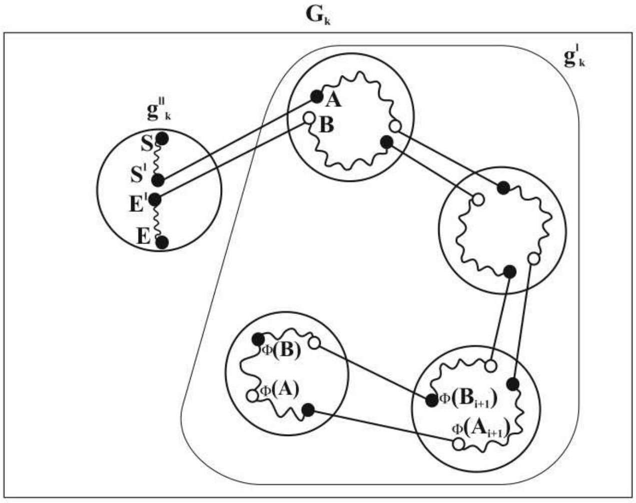

Figure 6:

From the above cases we can conclude that the graph G_k is Hamiltonian-t - laceable.

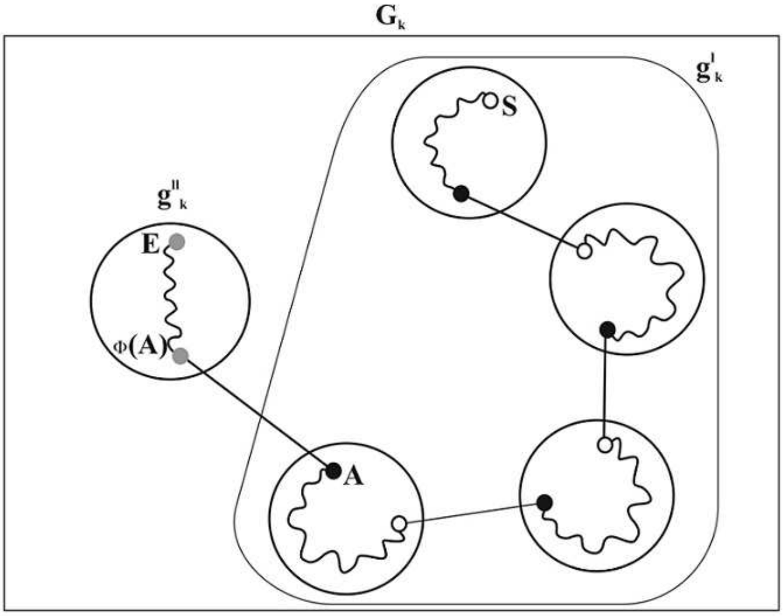

Figure 7:

Theorem 3.3. The 1-edge fault tolerant n -dimensional HHC graph for $n \geq 7$ is Hamiltonian-t - laceable such that $2 \leq t \leq diam\,(G)$.

Proof: The proof is by induction on M , where M is the number of copies of 6-HHC. The result is true for $M \leq K$. We prove the result is true for $M = K+1$. We Construct the graph G_{k+1} which is partitionable to two HHC graphs $g^{'}_{k+1}$ and $g^{''}_{k+1}$. The graph $g^{'}_{k+1}$ has Q copies of 6-HHC where Q is a finite integer and $g^{''}_{k+1}$ has only one copy of 6-HHC with one fault edge added to it. The graph $g^{'}_{k+1}$ is bipartite and is properly colored with two colors where as the graph g''_{k+1} is non bipartite. we shall prove that the graph G_{k+1} is Hamiltonian-t- laceable. Here we consider three cases.

Case i: Let $S, E \in g'_{k+1}$.

Subcase i: Vertices of same color S, E belongs to one copy of 6-HHC in g'_{k+1}.

Subcase ii: Vertices of same color S, E belongs different copies of 6-HHC in g'_{k+1}.

Case ii: Let $S, E \in g''_{k+1}$.

Case iii: Let $S \in g'_{k+1}$ and $E \in g''_{k+1}$.

In all the above cases there exists Hamiltonian paths between the vertices $S \,\&\, E$ of G_{k+1} (as in theorem 3.2) which includes all the vertices of the graph. Thus by induction if the theorem is true for $M = K$, then it is true for $M = K + 1$. Hence the theorem.

4. Conclusion

In this present study the concept of Hamiltonian-t- laceability of 1-edge fault tolerant n-dimensional HHC network is discussed. In particular we have proved that the 1-edge fault tolerant n- dimensional HHC network for $n \geq 7$ is Hamiltonian-t-laceable for even t such that $2 \leq t \leq diam\,(G)$.

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