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Time to Recruitment in a Two Graded Manpower System with Different Epochs for Decisions and Exits and Geometric Process for Inter-Decision Times

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Abstract. In this paper, the problem of time to recruitment is studied for a two graded manpower system using a univariate policy of recruitment in which exits take place due to policy decisions. Assuming that the policy decisions and exits occur at different epochs, three mathematical models are constructed based on shock model approach and the variance of time to recruitment is obtained when the inter-policy decision times form a geometric process and the inter-exit times form an ordinary renewal process. The analytical results are numerically illustrated and the effect of nodal parameters on the performance measure is studied.

Keywords: Two graded manpower system; Decision and exit epochs; geometric process; Ordinary renewal process; Univariate CUM policy of recruitment; Mean and variance of time to recruitment.

AMS Mathematics Subject Classification (2010): 60H30

1. Introduction

Attrition is a common phenomenon in many organizations when policy decisions such as revising sales target, emoluments etc. are announced and this leads to the depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. As the depletion of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. One univariate recruitment policy which is often used in the literature is based on shock model approach for replacement of system in reliability theory. In this policy, known as univariate CUM policy of recruitment, the cumulative loss of manpower is permitted till it reaches a level, called the breakdown threshold and when this cumulative loss exceeds the threshold, recruitment is carried out. In [1,2] the authors have discussed several manpower planning models using Markovian and renewal theoretic approach. In [3] this problem is studied for the first time using this policy. In [7], the authors have studied the problem of time to recruitment for a single grade manpower system and obtained the variance of time to recruitment when the loss of manpower forms a sequence of independent and identically distributed exponential random variables, the inter-decision times form a geometric process and the mandatory threshold for the loss of manpower is an exponential random variables using univariate CUM policy of recruitment. This work has been extended by the authors in [8] using bivariate CUM policy of recruitment. In [9], the authors have extended the work of [7] using indicatory technique method. The concept of non-instantaneous loss of manpower in decision epochs has been introduced for the first time in [10] for a single grade manpower system and the performance measures are obtained for the same. In [11, 12, 13, 14, 19], the authors have extended the research work in [10] when the inter decision times form a geometric process by using Laplace transform technique and indicatory function technique. In [15, 16, 17, 18], the authors have extended the work in [10] for a two graded manpower system and obtained the performance measures according as the inter decision times are independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables using the above cited techniques. In this paper, the research work in [15] is studied when the inter-decision times form a geometric process.

2. Model description and analysis

Consider an organization with two grades (grade-1and grade-2) taking policy decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the epochs for decisions and exits are different and the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) in the organization at the ith exit point with probability distribution $M(.)$, density function $m(.)$ and mean $\frac{1}{\lambda}$ (λ >0). Let S_i be the cumulative loss of manpower up to i-th exit. Let U_i be the continuous random variable representing the time between (i-1)th and ith policy decisions. It is assumed that U_i's form a geometric process of independent random variables with rate 'a' (a>0). Let $F(.)$ and $f(.)$ be the distribution function and density function of U_1 respectively with mean $\frac{1}{\theta}$ (θ>0). Let $f^*(.)$ be the Laplace transform of $f(.)$. Let $F_n(.)$ and $f_n(.)$ be the distribution and density function of the random variable $U_1+U_2+...+U_n$ respectively and $f_n^*(.)$ be the Laplace transform of $f_n(.)$. Let R_i be the time between (i-1)th and ith exits. It is assumed that R_i's are independent and identically distributed random variables with distribution function $G(.)$ and density function $g(.)$. Let D_{i+1} be the waiting time up to (i+1)th exit. Let $E(R)$ and $V(R)$ be the mean and variance of the inter-exit times respectively. Let Y_1, Y_2

be continuous random variables representing the thresholds for the cumulative loss of man hours in grades 1 and 2 respectively. Let Y be the breakdown threshold for the cumulative loss of manhours in the organization with distribution function $H(.)$ and density function $h(.)$. Let q (q≠0) be the probability that every policy decision produces an attrition. Let $I(A)$ be the indicatory function of the event A. Let T be a continuous random variable denoting the time for recruitment with mean $E(T)$ and variance $V(T)$.

The univariate CUM policy of recruitment employed in this paper is stated as follows: **Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold Y**.

We now obtain the variance of time to recruitment. By the probabilistic arguments, the time to recruitment can be written as

$$T = \sum_{i=0}^{\infty} D_{i+1}\, I(S_i \le Y < S_{i+1})$$

and

$$E(T) = E(R)\sum_{i=0}^{\infty} (i+1)\, P(S_i \le Y < S_{i+1}) \tag{1}$$

Similarly $T^2 = \sum_{i=0}^{\infty} {D_{i+1}}^2\, I(S_i \le Y < S_{i+1})$

and

$$E(T^2) = V(R)\sum_{i=0}^{\infty} (i+1)\, P(S_i \le Y < S_{i+1}) + [E(R)]^2 \sum_{i=0}^{\infty} (i+1)^2\, P(S_i \le Y < S_{i+1}) \tag{2}$$

By the law of total probability,

$$\begin{aligned} P(S_i \le Y < S_{i+1}) &= P(0 \le Y - S_i < S_{i+1} - S_i) \\ &= \int_0^{\infty}\int_0^{\infty} P(X_{i+1} > y - x)\, dM_i(x)\, dH(y) \\ &= \int_0^{\infty}\left[\int_0^{y} P(X_{i+1} > y - x)\, dM_i(x)\right] dH(y) \end{aligned}$$

i.e., $$P(S_i \le Y < S_{i+1}) = \int_0^{\infty}\left[\int_0^{y} \overline{M}(y-x)\, dM_i(x)\right] dH(y) \tag{3}$$

We now obtain the explicit expression for E(T) and E(T^2) for different forms of Y by assuming specific distribution to Y_1 and Y_2.

<u>Case (i)</u>: $Y = Max(Y_1, Y_2)$

Suppose Y_1 and Y_2 follows exponential distribution with parameters α_1, α_2 respectively.

In this case

$$H(y) = 1 - e^{-\alpha_1 y} - e^{-\alpha_2 y} + e^{-(\alpha_1+\alpha_2)y}$$

$$P(S_i \le Y < S_{i+1}) = \overline{A_1} A_1^{\,i} + \overline{A_2} A_2^{\,i} - \overline{A_3} A_3^{\,i} \tag{4}$$

$$where\ A_1 = \frac{\lambda}{\lambda+\alpha_1}; A_2 = \frac{\lambda}{\lambda+\alpha_2}; A_3 = \frac{\lambda}{\lambda+\alpha_1+\alpha_2} \ \&\ \overline{A_i} = 1 - A_i \ \ for\ all\ i \tag{5}$$

It can be proved that

$$G(x) = \sum_{n=1}^{\infty} q(1-q)^{n-1} F_n(x) \tag{6}$$

Since $U_i\,'s$ follow a geometric process with rate **"a",** from [7]

$$L[f_{U_1+U_2+\ldots+U_n}(t)] = f_n^*(s) = \prod_{i=1}^{n} f^*\left(\frac{s}{a^{i-1}}\right) \tag{7}$$

From (6) and (7), we can get

$$E(R) = \frac{a}{\theta\left(a-1+q\right)}$$

and

$$V(R) = \left(\frac{a}{\theta\left(a-1+q\right)}\right)^2 \left[\frac{a^2+(2a-1)(q-1)}{a^2-1+q}\right] \tag{8}$$

Using (4), (5) and (8) in (1) and (2) and after simplification, we get

$$E(T) = \left(\frac{a}{\theta\left(a-1+q\right)}\right)\left[\frac{1}{\overline{A_1}} + \frac{1}{\overline{A_2}} - \frac{1}{\overline{A_3}}\right] \tag{9}$$

and

$$E(T^2) = \left(\frac{a}{\theta\left(a-1+q\right)}\right)^2 \left[\frac{a^2+(2a-1)(q-1)}{a^2-1+q}\right]\left[\frac{1}{\overline{A_1}} + \frac{1}{\overline{A_2}} - \frac{1}{\overline{A_3}}\right] + \left(\frac{a^2}{\theta^2\left(a-1+q\right)^2}\right)\left[\frac{(2\lambda+\alpha_1)}{\alpha_1\overline{A_1}} + \frac{(2\lambda+\alpha_2)}{\alpha_2\overline{A_2}} - \frac{(2\lambda+\alpha_1+\alpha_2)}{(\alpha_1+\alpha_2)\overline{A_3}}\right]. \tag{10}$$

It is known that

$$V(T) = E(T^2) - \left[E(T)\right]^2 . \tag{11}$$

Equation (11) together with equations (9) and (10) will give V(T) for case (i).

Case (ii) : $Y = Min(Y_1, Y_2)$

Suppose Y_1 and Y_2 are as in case (i).

In this case, $H(y) = 1 - e^{-(\alpha_1+\alpha_2)y}$

$$P(S_i \le Y < S_{i+1}) = \overline{A_3}(A_3)^i \tag{12}$$

where A_3 is given in (5).

Using (12) in (1) and (2) and proceeding as in case (i), we get

$$E(T)=\left(\frac{a}{\theta\left(a-1+q\right)}\right)\left[\frac{1}{\overline{\overline{A_3}}}\right] \tag{13}$$

and

$$E(T^2)=\left(\frac{a}{\theta\left(a-1+q\right)}\right)^2\left[\frac{a^2+\left(2a-1\right)\left(q-1\right)}{a^2-1+q}\right]\left[\frac{1}{\overline{\overline{A_3}}}\right]+\left(\frac{a^2}{\theta^2\left(a-1+q\right)^2}\right)\left[\frac{(2\lambda+\alpha_1+\alpha_2)}{(\alpha_1+\alpha_2)\overline{\overline{A_3}}}\right]. \tag{14}$$

(13) and (14) together with (11) will give V(T) for case (ii).

<u>Case (iii)</u>: $Y=Y_1+Y_2$

Suppose Y_1 and Y_2 are as in case (i).

In this case, it can be shown that

$$H(y)=1+\frac{\alpha_2}{\alpha_1-\alpha_2}e^{-\alpha_1 y}-\frac{\alpha_1}{\alpha_1-\alpha_2}e^{-\alpha_2 y}$$

$$P(S_i\le Y<S_{i+1})=\frac{\alpha_1}{\alpha_1-\alpha_2}\overline{A_2}A_2^{\,i}-\frac{\alpha_2}{\alpha_1-\alpha_2}\overline{A_1}A_1^{\,i} \tag{15}$$

$$E(T)=\left(\frac{a}{\theta\left(a-1+q\right)}\right)\left[\frac{\alpha_1}{\left(\alpha_1-\alpha_2\right)\overline{\overline{A_2}}}-\frac{\alpha_2}{(\alpha_1-\alpha_2)\overline{\overline{A_1}}}\right] \tag{16}$$

and

$$\begin{aligned}E(T^2)=&\left(\frac{a}{\theta\left(a-1+q\right)}\right)^2\left[\frac{a^2+\left(2a-1\right)\left(q-1\right)}{a^2-1+q}\right]\left[\frac{\alpha_1}{\left(\alpha_1-\alpha_2\right)\overline{\overline{A_2}}}-\frac{\alpha_2}{\left(\alpha_1-\alpha_2\right)\overline{\overline{A_1}}}\right]\\&+\left(\frac{a^2}{\theta^2\left(a-1+q\right)^2}\right)\left[\frac{\alpha_1(2\lambda+\alpha_2)}{(\alpha_1-\alpha_2)\alpha_2\overline{\overline{A_2}}}-\frac{\alpha_2(2\lambda+\alpha_1)}{(\alpha_1-\alpha_2)\alpha_1\overline{\overline{A_1}}}\right]\end{aligned} \tag{17}$$

where A_1 and A_2 are given in (5).

Equation (11) together with equations (16) and (17) will give V(T) for case (iii).

Note:

a) When a=1, our result agrees with [15].

b) In the cases of the thresholds having extended exponential distribution [4] or thresholds having Setting the Clock Back to Zero property [5], the distribution H(y) will have just additional exponential terms. Hence V(T) can be calculated as in the case of exponential thresholds.

3. Numerical illustration

The mean and variance of time to recruitment is numerically illustrated by varying one parameter and keeping other parameter fixed. The influence of nodal parameters λ, θ, q and **a** on the performance measures namely mean and variance of the time to recruitment for all the models is shown in the following table. In all the computations, it is assumed that $\alpha_1 = 0.2$ and $\alpha_2 = 0.4$

Table 1: Effect of λ, θ, q and a on performance measures E(T) and V(T)

λ	θ	q	a	Model – I		Model – II		Model – III	
				E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.1	0.5	0.8	0.5	5.2778	96.9907	3.8889	66.9753	0.3111	20.6933
0.2	0.5	0.8	0.5	7.2222	143.5185	4.4444	79.0123	0.4889	36.0573
0.3	0.5	0.8	0.5	9.1667	195.1389	5.0000	91.6667	0.6667	53.9259
0.1	0.25	0.8	0.5	10.5556	387.9630	7.7778	267.9012	0.6222	151.9091
0.1	0.5	0.8	0.5	5.2778	96.9907	3.8889	66.9753	0.3111	20.6933
0.1	0.75	0.8	0.5	3.5185	43.1070	2.5926	29.7668	0.2074	6.6365
0.1	0.5	0.8	0.5	5.2778	96.9907	3.8889	66.9753	0.3111	20.6933
0.1	0.5	1.6	0.5	1.4394	1.0564	1.0606	0.4443	0.0848	0.2742
0.1	0.5	2.4	0.5	0.8333	0.2915	0.6140	0.1028	0.0491	0.0871
0.1	0.5	0.8	0.5	5.2778	96.9907	3.8889	66.9753	0.3111	20.6933
0.1	0.5	0.8	1	3.9583	14.9740	2.9167	8.5069	0.2333	3.3761
0.1	0.5	0.8	1.5	3.6538	11.9362	2.6923	6.6424	0.2154	2.6692

4. Findings

From the above table, we observe the following results which agrees with reality **:**

1. When **λ** increases and keeping other parameter fixed, the mean and variance of the time to recruitment increase in all the models.

2. When θ increases and keeping other parameter fixed, the mean and variance of the time to recruitment decrease in all the three models.

3. As **q** increases and keeping other parameter fixed, the mean and variance of the time to recruitment decrease in all the models.

4. As **a** increases, the mean and variance of time to recruitment decrease in all the cases.

5. Conclusion

The model discussed in this paper are found to be more realistic and new for a two grade manpower system in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points and (iii) geometric process of inter-decision times. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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