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Solution of Fuzzy Non-linear Equations over Triangular Fuzzy Number using Modified Secant Algorithm

***Md. Yasin Ali*[1], *Kanak Ray Chowdhury*[2], *Abeda Sultana*[3] and *A F M Khodadad Khan*[4]**

[1]School of Science and Engineering
University of Information Technology & Sciences, Baridhara, Dhaka, Bangladesh
E-mail: ali.mdyasin56@gmail.com
[2]Department of Mathematics, Mohammadpur Model School and College
Mohammadpur, Dhaka
[3]Department of Mathematics, Jahangirnagar University, Savar, Bangladesh
[4]School of Engineering and Computer Science, Independent University
Bashundhara R/A, Dhaka, Bangladesh

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Abstract. The application of fuzzy non-linear equations has recently increased in many branches of pure and applied mathematics as well as engineering and social sciences. So solution of fuzzy non-linear equations has become an important tool in fuzzy Mathematics. There have been many numerical methods for the solution of fuzzy non-linear equations. In this study, we have solved fuzzy non-linear equations over Triangular Fuzzy Numbers (TFNs) using modified secant algorithm. Graphical representation of the solutions has also been drawn so that anyone can achieve the idea of converging to the root of a fuzzy non-linear equation.

Keywords: Fuzzy number, Triangular Fuzzy number, Fuzzy non-linear equation, Modified Secant method.

AMS Mathematics Subject Classification (2010): 90C70

1. Introduction

In science and engineering work, a frequently occurring problem is to find the roots of the non-linear equations of the form $F(x) = 0$. In classical mathematics there have been many methods for solving these kinds of equations. But in fuzzy mathematics all of these methods is not suitable for solving fuzzy non-linear equations, there need some modification for obtaining the roots of fuzzy non-linear equations using classical numerical methods. Subhash and Sathya [8] proposed a method using linear interpolation to solve a non-linear equation $f(x) = 0$, which is a modification of fuzzy Newton-Raphson method. Gautam and Shirin [5, 6] proposed methods to solve a fuzzy non-linear equation with the help of modified bisection algorithm and modified fixed point Algorithm. Abbasbandy and Asady [4] have considered Newton's method for solving fuzzy non-linear equations. In this study, we propose modified secant algorithm for the solution of fuzzy non-linear equations.

The paper is organized as follows. In section 2 we discuss about basic definitions and notations used in this article. In section 3 we, discuss modified Secant method for solving fuzzy non-linear equations. In section 4, the precise solution of the non-linear fuzzy equation is obtained with the help of the proposed algorithm and its graphical representation has also been shown as well.

2. Preliminaries

In this section, we discuss some definitions, which are needed to read rest of the paper.

Definition 2.1. (Fuzzy set) A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A the second element $\mu_A(x)$ belongs to the interval [0, 1], called Membership function.

Definition 2.2. (Fuzzy number**)** If a fuzzy set is convex and normalized, and its membership function is defined in R and piecewise continuous, it is called as fuzzy number. So fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy. Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number R. Since the boundary of this interval is ambiguous, the interval is also a fuzzy set. Generally a fuzzy interval is represented by two end points a_1 and a_3 and a peak point a_2 as [a_1, a_2, a_3] (Figure-1)

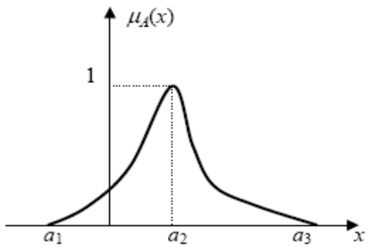

Figure 1: Fuzzy numbers

Definition 2.3. (Triangular fuzzy number) A fuzzy number $\tilde{A}$ on R is said to be a triangular fuzzy number (TFN) or linear real fuzzy number (LRFN) if its membership function $\mu : \mathrm{R} \to [0,1]$ has the following characteristics

$$\mu_{A(x)} = \begin{cases} 0 & , \ x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , \ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & , \ a_2 \le x \le a_3 \\ 0 & , \ x > a_3 \end{cases}$$

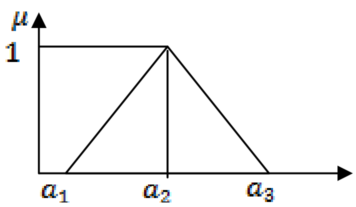

Figure 2: Triangular fuzzy numbers

The triangular fuzzy number is denoted by $\tilde{A} = \mu(a, b, c)$. We use $TF(R)$ denote the set of all triangular fuzzy numbers. We note that any real number b can be written as a triangular fuzzy number $r(b)$, where $r(b) = \mu(b,b,b)$ and therefore R. Now it is clear that $r(b)$ represents the real number b itself. Operation, fuzzy function and fuzzy non-linear equations in $TF(R)$ are also defined as follows.

Definition 2.4. (Operations on $TF(R)$) For given two triangular fuzzy numbers $\mu_1 = \mu(a_1, b_1, c_1)$ and $\mu_2 = \mu(a_2, b_2, c_2)$, we define addition, subtraction, multiplication, scalar multiplication and division by

1. $\mu_1 + \mu_2 = \mu(a_1 + a_2, b_1 + b_2, c_1 + c_2)$;
2. $\mu_1 - \mu_2 = \mu(a_1 - c_2, b_1 - b_2, c_1 - a_2)$;
3. $\mu_1.\mu_2 = \mu(\min\{a_1a_2, a_1c_2, a_2c_1, c_1c_2\}, b_1b_2, \max\{a_1a_2, a_1c_2, a_2c_1, c_1c_2\})$;
4. $\dfrac{\mu_1}{\mu_2} = \mu_1.\dfrac{1}{\mu_2}\, where \dfrac{1}{\mu_2} = \mu\left(\min\left\{\dfrac{1}{a_2}, \dfrac{1}{b_2}, \dfrac{1}{c_2}\right\}, median\left\{\dfrac{1}{a_2}, \dfrac{1}{b_2}, \dfrac{1}{c_2}\right\}, \max\left\{\dfrac{1}{a_2}, \dfrac{1}{b_2}, \dfrac{1}{c_2}\right\}\right)$;

   where a_2, b_2, c_2 are all non-zero real numbers.
5. $k\mu_1 = \mu(ka_1, kb_1, kc_1)$ for $k \geq 0$;
6. $k\mu_1 = \mu(kc_1, kb_1, ka_1)$ for $k \leq 0$;

Definition 2.5. (Function in $TF(R)$) If $f : R \to R$ is a real-valued function and $\mu(a,b,c)$ is a TFN, then triangular fuzzy-valued function $\tilde{f} : TF(R) \to TF(R)$ is defined as $\tilde{f}(\mu(a,b,c)) = \mu(\tilde{a}, \tilde{b}, \tilde{c})$, where $\tilde{a} = \min\{f(a), f(b), f(c)\}$, $\tilde{b} = median\{f(a), f(b), f(c)\}$, $\tilde{c} = \max\{f(a), f(b), f(c)\}$.

Definition 2.6. (n-th power of a fuzzy number) A fuzzy number $\mu(a,b,c)$ of power n is defined as $(\mu(a,b,c))^n = \mu(a^n, b^n, c^n)$ where $a, b, c \geq 0$.

Definition 2.7. (Fuzzy non-linear equation) Fuzzy non-linear equations can be found in many applications, all the way from light diffraction to planetary orbits for example. A non-linear equation over triangular fuzzy number is called a fuzzy non-linear equation. The equation of the form $\tilde{f}(\mu_x) = 0$ is called fuzzy non-linear equation, where $\tilde{f} : TF(R) \to TF(R)$ is a non-linear function. For example $\mu_x^3 + \mu_x^2 - 1 = 0$ is a fuzzy non-linear equation.

3. The modified secant method to solve fuzzy non-linear equation

Solving fuzzy non-linear equations over triangular fuzzy numbers of the form $\tilde{f}(\mu_x) = 0$ is possible with a modification of secant method over triangular fuzzy numbers. To solve fuzzy non-linear equation using this modified secant method over triangular fuzzy number, we have begun with two initial approximations for which $\tilde{f}(\mu_x^{(n)}) = 0$ have opposite sign. Let us assume that two initial approximations are

$\mu_x^{(0)} = \mu(a^{(0)}, b^{(0)}, c^{(0)})$ and $\mu_x^{(1)} = \mu(a^{(1)}, b^{(1)}, c^{(1)})$ for which $\tilde{f}(r(b^{(0)}))$ and $\tilde{f}(r(b^{(1)}))$ have opposite sign (say $\tilde{f}(r(b^{(0)})) < 0$ and $\tilde{f}(r(b^{(1)})) > 0$). Hence the method generates the sequences $\{\mu_x^{(n)}\}_{n=0}^{\infty}$ by

$$\mu_x^{(n)} = \mu_x^{(n-1)} - \frac{\tilde{f}(r(b^{(n-1)}))(r(b^{(n-1)}) - r(b^{(n-2)}))}{\tilde{f}(r(b^{(n-1)})) - \tilde{f}(r(b^{(n-2)}))}, \text{ for } n \geq 2 \qquad (1)$$

The stopping criteria of this method is $\left| r(b^{(n-1)}) - r(b^{(n-2)}) \right| < \varepsilon$, where ε is a preset small value $\varepsilon = 10^{-7}$

ALGORITHM:

To find a solution to $\tilde{f}(\mu_x) = 0$ given initial approximation $\mu_x^{(0)}$ and $\mu_x^{(1)}$:

INPUT: Initial approximations are

$\mu_x^{(0)} = \mu(a^{(0)}, b^{(0)}, c^{(0)})$ and $\mu_x^{(1)} = \mu(a^{(1)}, b^{(1)}, c^{(1)})$; tolerance TOL; maximum number of iterations n

OUTPUT: approximation solution

$\mu_x^{(n)} = \mu(a^{(n)}, b^{(n)}, c^{(n)})$ or message of failure

Step-1: Set $i = 2$

Step-2: While $i \leq n$ do steps 3-6.

Step-3: Set $\mu_x^{(n)} = \mu_x^{(n-1)} - \dfrac{\tilde{f}(r(b^{(n-1)}))(r(b^{(n-1)}) - r(b^{(n-2)}))}{\tilde{f}(r(b^{(n-1)})) - \tilde{f}(r(b^{(n-2)}))}$ (compute $\mu_x^{(n)}$)

Step-4: If $\left| r(b^{(n-1)}) - r(b^{(n-2)}) \right| < TOL$ then

OUTPUT ($\mu_x^{(n)}$); (The procedure was successful)

STOP

Step-5: Set $i = i + 1$

Step-6: If $\tilde{f}(r(b^{(0)})).\tilde{f}(r(b^{(n)})) < 0$ then set

$$\mu_x^{(n-1)} = \mu_x^{(n)}$$
$$\tilde{f}(r(b^{(n-1)})) = \tilde{f}(r(b^{(n)}))$$
$$r(b^{(n-1)}) = r(b^{(n)})$$

Or If $\tilde{f}(r(b^{(1)})).\tilde{f}(r(b^{(n)})) < 0$ then set

$$\mu_x^{(n-1)} = \mu_x^{(n-1)}$$
$$\tilde{f}(r(b^{(n-2)})) = \tilde{f}(r(b^{(n)}))$$
$$r(b^{(n-2})) = r(b^{(n)})$$

Step-7: OUTPUT (“The method failure after n iteration, n” n);

(The procedure is unsuccessful.)

STOP.

4. Numerical example

Solve a fuzzy non-linear equation $\mu(1,1,1)\mu_x^3 + \mu(1,1,1)\mu_x^2 - \mu(1,1,1) = 0$

Solution: Suppose that $\tilde{f}(\mu_x) = \mu(1,1,1)\mu_x^3 + \mu(1,1,1)\mu_x^2 - \mu(1,1,1)$ (2)

By the definition of 2.3 we have $r(1) = \mu(1,1,1) = 1$. Now the equation (2) can be written as $\tilde{f}(\mu_x) = \mu_x^3 + \mu_x^2 - 1$ (3)

Let the initial approximations are $\mu_x^{(0)} = \mu(0,0.5,1)$ and $\mu_x^{(1)} = \mu(0.5,1,1.5)$, the equation (3) generates the sequence $\mu_x^{(n)}$ of approximate solutions as follows:

Iteration-1: $\tilde{f}\left(\mu_x^{(0)}\right) = \mu^3(0,0.5,1) + \mu^2(0.5,1,1.5) - 1$

$$= \mu(0,0.125,1) + \mu(0,0.25,1) - 1 = \mu(-1,-0.625,1) < 0$$

$$\tilde{f}\left(\mu_x^{(1)}\right) = \mu^3(0.5,1,1.5) + \mu^2(0.5,1,1.5) - 1$$

$$= \mu(0.125,1,3.375) + \mu(0.25,1,2.25) - 1$$

$$= \mu(-0.625,1,4.625) > 0$$

Hence the root lies between $[\mu_x^{(0)}, \mu_x^{(1)}]$.

Then we get, $\mu_x^{(2)} = \mu_x^{(1)} - \dfrac{\tilde{f}(r(b^{(1)}))(r(b^{(1)}) - r(b^{(0)}))}{\tilde{f}(r(b^{(1)})) - \tilde{f}(r(b^{(0)}))}$

$$= \mu(0.192307693,0.692307693,1.192307693)$$

where $\tilde{f}(r(b^{(2)})) < 0$. Hence the root lies between $[\mu_x^{(2)}, \mu_x^{(1)}]$.

Proceeding in this way we can get the solutions shown in the following table:

Table 1: Approximation solutions of fuzzy non-linear equation over triangular fuzzy number using modified secant algorithm

Iteration no. n	Root's interval	Root $\mu_x^{(n)}$	Sign of $\tilde{f}(r(b^{(n)}))$
1	$[\mu_x^{(0)}, \mu_x^{(1)}]$	$= \mu(0.192307693,0.692307693,1.192307693)$	$(-)ve$
2	$[\mu_x^{(2)}, \mu_x^{(1)}]$	$= \mu(0.2411944883,0.741194488,1.241194488)$	$(-)ve$
3	$[\mu_x^{(3)}, \mu_x^{(1)}]$	$= \mu(0.251969256,0.751969256,1.251969256)$	$(-)ve$
4	$[\mu_x^{(4)}, \mu_x^{(1)}]$	$= \mu(0.254263303,0.754263303,1.25263303)$	$(-)ve$
5	$[\mu_x^{(5)}, \mu_x^{(1)}]$	$= \mu(0.255117059,0.755117059,1.255117059)$	$(+)ve$
6	$[\mu_x^{(5)}, \mu_x^{(6)}]$	$= \mu(0.254877517,0.754877517,1.254877517)$	$(-)ve$
7	$[\mu_x^{(7)}, \mu_x^{(6)}]$	$= \mu(0.255093106,0.755093106,1.255093106)$	$(+)ve$
8	$[\mu_x^{(7)}, \mu_x^{(8)}]$	$= \mu(0.254877666,0.754877666,1.2554877666)$	$(-)ve$
9	$[\mu_x^{(7)}, \mu_x^{(9)}]$	$= \mu(0.254877617,0.754877617,1.254877617)$	$(-)ve$

From the table it is clear that $\mu_x^{(10)} = \mu(0.2548776, 0.7548776, 1.2548776)$ is the desired root of the given non-linear equation. The graphical representations of the sequence of approximate roots are shown in fig. 3 and the optimum solution of fuzzy non-linear equation is shown in fig. 4.

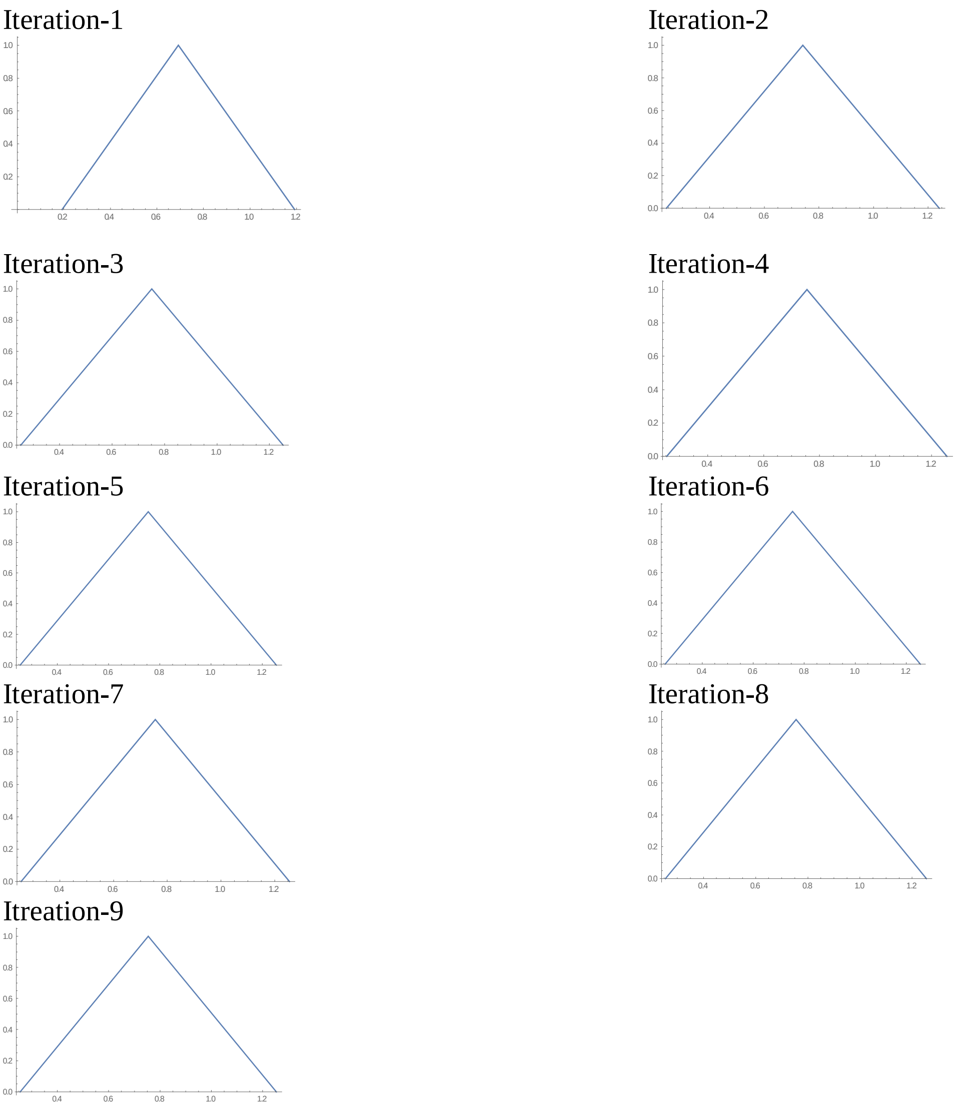

Figure 3: Graphical representation of approximate solutions of the given fuzzy non-linear equation

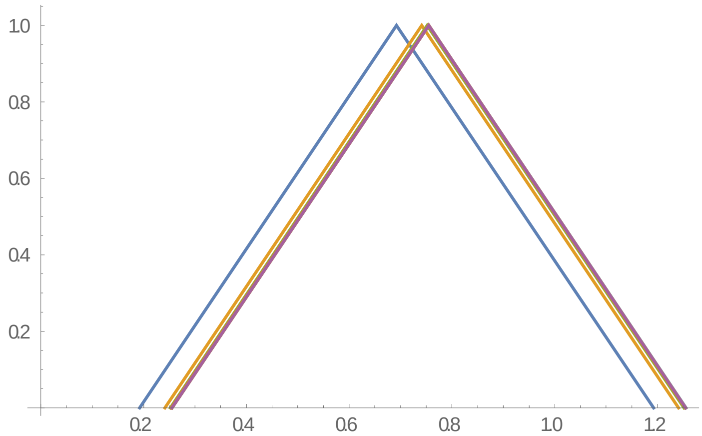

Figure 4: Graphical representation of optimum solution of a fuzzy non-linear equation given in the above example

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