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On the Zagreb Indices of Semi total Point Graphs of Some Graphs

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Abstract. The first Zagreb index $M_1(G)$ of a graph G is the sum of the square of the degrees of the vertices of a graph G and the second Zagreb index $M_2(G)$ of a graph G is the sum of the products of the degrees of the pair of adjacent vertices of G. In this paper, the degree based topological indices $M_1(G)$ and $M_2(G)$ of r^{th} -semi total point graphs of some graphs are obtained.

Keywords: Degree of a vertex, Topological index, Semi total point graph.

2000 Mathematics Subject Classification (2010): 05C12, 05C75

1. Introduction

Throughout this paper, we consider only finite connected graphs without loops and multiple edges. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex $v \in V(G)$ is the number of edges incident to v and is denoted by $d_G(v)$.

In theoretical chemistry, the physico-chemical properties of chemical compounds are often modeled by means of a molecular graph based structure descriptors, which are also referred to as topological indices. In 1972, Gutman and Trinajsti´c [2], introduced the degree based topological indices $M_1(G)$ and $M_2(G)$, called Zagreb indices.

The first and second Zagreb indices are respectively defined as,

$$M_1(G) = \sum_{v \in V(G)} d_G(v)^2 \tag{1.1}$$

and

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v) \tag{1.2}$$

For more details and recent results on topological indices, we encourage the reader to consult the papers [3-19] and the references cited therein.

2. Semi total point graphs

In [1, 3], the semi-total point graph and the r^{th}- semi total point graphs are defined as follows,

Definition 2.1. [1] Let G be a simple graph of order n. Then $R(G)$ be the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end points, of the edge corresponding to it. It is called the *semi total point graph.*

Definition 2.2. [2] Let G be a simple graph of order *n* possessing *m* edges. The r^{th} - *semi total point graph of G,* denoted by $R^r(G)$, is the graph obtained by adding *r* vertices to each edge of *G* and joining them to the end points of the respective edge.

Obviously this is equivalent to adding *r* triangles to each edge of *G*. The construction of $R(G)$ and $R^r(G)$ is illustrated by Figure 1.

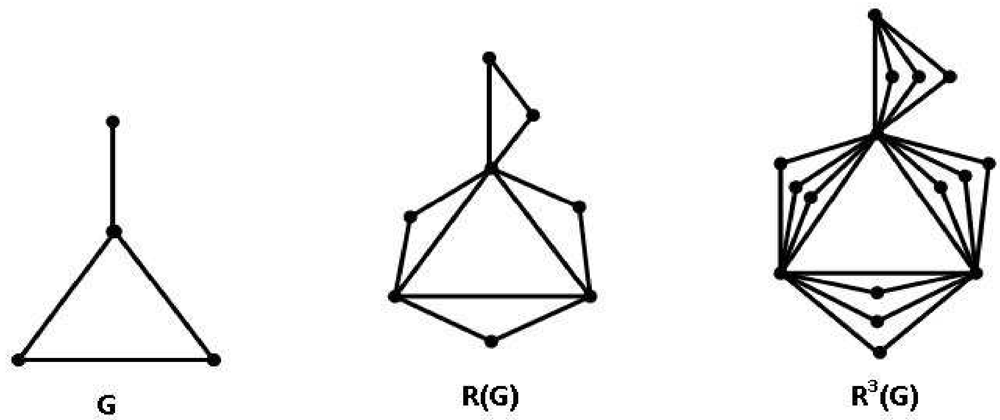

Figure 1: A graph G and its semi total point graphs

It is clear that, the semi total point graph of Definition 2.1 is the special case of $R^r(G)$ for *r = 1*. In [2], S. R. Jog et al. obtained the characteristic polynomial, second stage spectrum and discussed some characteristic features of the r^{th}- semi total point graphs of some graphs.

In this paper, we have obtained the explicit formulae for first and second Zagreb indices of the r^{th}- semi total point graphs of the graphs Path P_n, Cycle C_n, Star graph S_n, Complete graph K_n on n vertices and a Complete bipartite graph $K_{a,b}$. Also, in continuation to this the exact formulae for the first and second Zagreb indices of the k - regular graph, linegraph *L(G)* of a regular graph G and the corona product graph $G \circ \overline{K_t}$ are presented.

In this paper, for the sake of convenience, here onwards we use the notation $R^r(G)$ as just $R(G)$.

3. Main results

In this section, we obtain the exact formulae for first and second Zagreb indices of r^{th} -semi total point graphs of some graphs.

Let P_n be a path on n vertices. Then P_n has $n - 1$ edges.

Theorem 3.1. *The first Zagreb index of $R(P_n)$ is*

$$M_1(R(P_n)) = (4n - 6)(r + 1)^2 + 4r(n - 1) \tag{3.1}$$

Proof. In $R(P_n)$ there are n + (n − 1)r vertices, of which n − 2 vertices, each of degree 2r+2, 2 vertices, each of degree r+1 and (n−1)r vertices, each of degree 2.
Hence by Eqn.(1.1), we have,

$$\begin{aligned} M_1(R(P_n)) &= (n - 2)(2r + 2)^2 + 2(r + 1)^2 + (n - 1)r(2)^2 \\ &= 4(n - 2)(r + 1)^2 + 2(r + 1)2 + 4r(n - 1) \\ &= (4n - 6)(r + 1)^2 + 4r(n - 1) \end{aligned}$$

Theorem 3.2. *For n > 2, the second Zagreb index of $R(P_n)$ is*

$$M_2(R(P_n)) = 4(r + 1)[n(3r + 1) - 5r - 2] \tag{3.2}$$

Proof. Let $v_1e_1v_2e_2v_3e_3 \dots v_{n-1}e_{n-1}v_n$ be the path P_n on n vertices. In $R(P_n)$, there are (n − 1) + (n − 1)2r = (n − 1)(2r + 1) edges.

Among these (n − 1)(2r + 1) edges, there are 2 edges, namely, e_1 and e_{n-1} of P_n have the end vertices with degrees r + 1 and 2r + 2. The remaining n − 3 edges of P_n, namely, $e_2, e_3, \dots, e_{n-2}$, each have both the end vertices with degree 2r + 2. There are r edges, incident to each of the vertices v_1 and v_n (other than the edges of P_n), have the end vertices with degrees r + 1 and 2. Lastly, each of the remaining 2r + (n − 3)(2r) = 2r(n − 2) edges of $R(P_n)$, have the end vertices with degrees 2r + 2 and 2. Hence by Eqn.(1.2), we have,

$$\begin{aligned} M_2(R(P_n)) &= 2(r + 1)(2r + 2) + (n - 3)(2r + 2)(2r + 2) + 2r(r + 1)(2) \\ &\quad + 2r(n - 2)(2r + 2)(2) \\ &= 4(r + 1)^2 + 4(n - 3)(r + 1)^2 + 4r(r + 1) + 8r(r + 1)(n - 2) \\ &= 4(r + 1)[n(3r + 1) - 5r - 2] \end{aligned}$$

Let C_n be a cycle graph on n vertices. Then C_n has n edges.

Theorem 3.3. *The first Zagreb index of $R(C_n)$ is*

$$M_1(R(C_n)) = 4n(r^2 + 3r + 1) \tag{3.3}$$

Proof. In $R(C_n)$ there are n + nr vertices, of which, each of the n vertices have degree 2r + 2 and each of the remaining nr vertices have degree 2.

Hence by Eqn.(1.1),

$$\begin{aligned} M_1(R(C_n)) &= n(2r + 2)^2 + nr(2)^2 \\ &= 4n(r + 1)^2 + 4nr \\ &= 4n(r^2 + 3r + 1) \end{aligned}$$

Theorem 3.4. *The second Zagreb index of $R(C_n)$ is*

$$M_2(R(C_n)) = 4n(r + 1)(3r + 1) \tag{3.4}$$

Proof. In $R(C_n)$ there are n + n(2r) = n(2r + 1) edges, of which each of the n edges have end vertices of degree 2r + 2 and each of the remaining 2nr edges have the end vertices of degree 2r + 2 and 2. Hence by Eqn.(1.2), we have,

$$M_2(R(C_n)) = n(2r + 2)(2r + 2) + 2nr(2r + 2)(2)$$
$$= 4n(r + 1)(3r + 1)$$

Let S_n is a star graph on *n* vertices. Then, S_n has n − 1 edges.

Theorem 3.5. *The first Zagreb index of $R(S_n)$ is*

$$M_1(R(S_n)) = (n - 1)[4r + n(r + 1)^2] \tag{3.5}$$

Proof. In $R(S_n)$ there are (n−1)r+(n−1)+1 = (n−1)(r+1)+1 vertices, of which 1 vertex is of degree (n−1)+(n−1)r = (n−1)(r+1), n−1 vertices, each of degree r + 1 and each of the remaining (n − 1)r vertices of degree 2. Hence by Eqn.(1.1),

$$M_1(R(S_n)) = (n - 1)^2(r + 1)^2 + (n - 1)(r + 1)^2 + (n - 1)r(2)^2$$
$$= (n - 1)[4r + n(r + 1)^2]$$

Theorem 3.6. *The second Zagreb index of $R(S_n)$ is*

$$M_2(R(S_n)) = (n - 1)(r + 1)(3nr + n - r - 1) \tag{3.6}$$

Proof. In $R(S_n)$ there are (n − 1)(2r + 1) edges, of which there are n − 1 edges with end vertices of degrees r +1 and (n−1)(r +1), (n−1)r edges with end vertices of degrees 2 and (n−1)(r +1), (n−1)r edges with end vertices of degrees 2 and
(r + 1). Hence by Eqn.(1.2),

$$M_2(R(S_n)) = (n - 1)(r + 1)(n - 1)(r + 1) + (n - 1)r(2)(n - 1)(r + 1)$$
$$+ (n - 1)r(2)(r + 1)$$
$$= (n - 1)(r + 1)(3nr + n - r - 1)$$

Let K_n be a complete graph on *n* vertices. Then, K_n has $\binom{n}{2}$ edges.

Theorem 3.7. *The first Zagreb index of $R(K_n)$ is*

$$M_1(R(K_n)) = n(n - 1)^2(r + 1)^2 + 2nr(n - 1) \tag{3.7}$$

Proof. In $R(K_n)$ there are $n+\binom{n}{2}r = \left[(n-1)r+2\right]$ vertices, of which n vertices of degree (n − 1) + (n − 1)r = (n − 1)(r + 1), $\binom{n}{2}r$ vertices, each of degree 2.

Hence by Eqn.(1.1), we have,

$$M_1(R(K_n)) = n(n - 1)^2(r + 1)^2 + \binom{n}{2} r(2)^2$$
$$= n(n - 1)^2(r + 1)^2 + 2nr(n - 1)$$

Theorem 3.8. *For $n \geq 3$, the second Zagreb index of $R(K_n)$ is*

$$M_2(R(K_n)) = \frac{1}{2} n(n - 1)^2(r + 1)[(n - 1)(r + 1) + 4r] \tag{3.8}$$

Proof. In $R(K_n)$ there are $\binom{n}{2}+\binom{n}{2}(2r)=\frac{n(n-1)(2r+1)}{2}$ edges, of which there are $\binom{n}{2}$ edges, each with degree of both the end vertices equal to (n −1) + (n − 1)r = (n − 1)(r

+ 1), $\binom{n}{2}(2r)$ edges, each with degree of end vertices (n − 1)(r + 1) and 2. Hence by Eqn.(1.2), we have,

$$M_2(R(K_n)) = \binom{n}{2}(n-1)^2(r+1)^2 + \binom{n}{2}(2r)(n-1)(r+2)(2)$$
$$= \frac{n(n-1)}{2}(n-1)^2(r+1)^2 + \frac{n(n-1)}{2}(2r)(n-1)(r+1)(2)$$
$$= \frac{1}{2}n(n-1)^2(r+1)[(n-1)(r+1) = 4r]$$

Let $K_{a,b}$ be a complete bipartite graph. $K_{a,b}$ has a + b vertices and *ab* edges.

Theorem 3.9. *The first Zagreb index of $R(K_{a,b})$ is*

$$M_1(R(K_{a,b})) = ab(a + b)(r + 1)^2 + 4abr \qquad (3.9)$$

Proof. In $R(K_{a,b})$ there are a+b+abr vertices, of which there are a vertices of degree b+br, b vertices of degree a+ar and each of the remaining abr vertices of degree 2. Hence by Eqn.(1.1), we have,

$$M_1(R(K_{a,b})) = a(b + br)^2 + b(a + ar)^2 + abr(2)^2$$
$$= ab^2(r + 1)^2 + a^2b(r + 1)^2 + 4abr$$
$$= ab(a + b)(r + 1)^2 + 4abr$$

Theorem 3.10. *The second Zagreb index of $R(K_{a,b})$ is*

$$M_2(R(K_{a,b})) = a^2b^2(r + 1)^2 + 2abr(r + 1)(a + b) \qquad (3.10)$$

Proof. In $R(K_{a,b})$ there are ab + ab(2r) = ab(2r + 1) edges, of which there are ab edges with end vertices with degrees b + br and a + ar, abr edges with end vertices of each edge are of degrees b + r and 2 and abr edges with end vertices of each edge are of degrees a+r and 2. Hence by Eqn.(1.2), we have,

$$M_2(R(K_{a,b})) = ab(b + br)(a + ar) + abr(b + br)(2) + abr(a + ar)(2)$$
$$= a^2b^2(r + 1)^2 + 2ab^2r(r + 1) + 2a^2br(r + 1)$$
$$= a^2b^2(r + 1)^2 + 2abr(r + 1)(a + b)$$

For the special case a = b, we have the following Corollary:

Corollary 3.11. Let $K_{a,a}$ be the complete bipartite graph on 2a vertices. Then

$$M_1(R(K_{a,a})) = 2a^2[a(r + 1)^2 + 2r]$$
$$M_2(R(K_{a,a})) = a^3(r + 1)[a(r + 1) + 4r] \qquad (3.11)$$

Let G be a k- regular graph of order n. Then G has $\frac{nk}{2}$ edges. It is easy to see that, R(G) have $n+\frac{nk}{2}r = \frac{n}{2}(2+kr)$ vertices and $\frac{nk}{2}+\frac{nk}{2}(2r) = \frac{nk}{2}(2r+1)$ edges.

In the following Theorems 3.12 and 3.13, we present the direct formulae for the first and second Zagreb indices of R(G).

Theorem 3.12. *The first Zagreb index of R(G) is*

$$M_1(R(G)) = nk^2(r + 1)^2 + 2nrk \qquad (3.12)$$

Proof. Among $\frac{n}{2}(2+kr)$ vertices of R(G) there are, n vertices, each of degree kr + k = k(r + 1) and $\frac{nk}{2}r$ vertices, each with degree 2. Hence by Eqn.(1.1),

$$M_1(R(G)) = nk^2(r + 1)^2 + \frac{nk}{2}r(2)^2$$
$$= nk^2(r + 1)^2 + 2nrk$$

Theorem 3.13. *The second Zagreb index of R(G) is*

$$M_2(R(G)) = \frac{nk^2}{2}(r+1)\left[k(r+1)+4r\right] \tag{3.13}$$

Proof. Among $\frac{nk}{2}$(2r + 1) edges of R(G) there are $\frac{nk}{2}$ edges with end vertices, each of degree kr +k and $\frac{nk}{2}$(2r) = nkr edges, each with end vertices of degree kr + k and 2. Hence by Eqn.(1.2), we have,

$$M_2(R(G)) = \frac{nk}{2}(kr+k)(kr+k) + nkr(2)(kr+k)$$
$$= \frac{nk^3}{2}(r+1)^2 + 2nk^2r(r+1)$$
$$= \frac{nk^2}{2}(r+1)\left[k(r+1)+4r\right]$$

Let G be a graph of order *n*. Then the line graph of G will be denoted by L(G). For basic properties of line graphs, we refer to [20].

It is known about the line graphs that, the line graph of a regular graph is a regular graph [12]. In particular, the line graph of a regular graph of order *n* and of degree *k*, is a regular graph with order n_1 and regularity k_1, where,

$$n_1 = \frac{1}{2}nk; \qquad k_1 = 2k - 2 = 2(k - 1) \tag{3.14}$$

Further, it is easy to notice that the semi total point graph R(L(G)) of a line graph L(G) have, $\frac{nk}{2}((k-1)r+1)$ vertices and $\frac{nk(k-1)}{2}(2r+1)$ edges.

Therefore, we have the following corollaries.

Corollary 3.14. *Let G be a k-regular graph of order n, then the first Zagreb index of R(L(G)) is*

$$M_1(R(L(G))) = 2nk(k - 1)[(k - 1)(r + 1)^2 + r] \tag{3.15}$$

From (3.14) and (3.12), the proof of the Corollary 3.14 is straight forward.

Corollary 3.15. *Let G be a k-regular graph of order n, then the second Zagreb index of R(L(G)) is*

$$M_2(R(L(G))) = 2nk(k - 1)^2(r + 1)[(k - 1)(r + 1) + 2r] \tag{3.16}$$

From (3.14) and (3.13), the proof of the Corollary 3.15 is straight forward.

Corona product of two graphs: Let G and H be two graphs. The corona product $G \circ H$ is obtained by taking one copy of G and |V (G)| copies of H, and by joining each vertex of the i^{th} copy of H to the i^{th} vertex of G.

Let G be a k-regular graph of order n and $\overline{K_t}$ be the complement of complete graph K_t of order t. Then $G \circ \overline{K_t}$ has nt(r + 1) + $\frac{n}{2}(kr+2)$ vertices and $\frac{nk}{2}+\frac{nk}{2}(2r)+nt+nt(2r)=\frac{n}{2}(k+2t)(1+2r)$ edges.

Now we proceed to find M_1 and M_2 for $R\left(G \circ \overline{K_t}\right)$

Theorem 3.16. *The first Zagreb index of* $R\left(G \circ \overline{K_t}\right)$ *is*

$$M_1\left(R\left(G \circ \overline{K_t}\right)\right) = n(r + 1)^2[(k + t)^2 + t] + 2nr(k + 2t) \qquad (3.17)$$

Proof. Among nt(r + 1) + $\frac{n}{2}(kr+2)$ vertices of $R\left(G \circ \overline{K_t}\right)$, there are n vertices, each of degree k + t + (k + t)r = (k + t)(r + 1), *nt* vertices, each of degree r + 1, $\frac{nk}{2}r$ and *ntr* vertices, each of degree 2. Hence by Eqn.(1.1), we have,

$$\begin{aligned} M_1\left(R\left(G \circ \overline{K_t}\right)\right) &= n(k + t)^2(r + 1)^2 + nt(r + 1)^2 + \frac{nk}{2} r(2)^2 + ntr(2)^2 \\ &= n(r + 1)^2[(k + t)^2 + t] + 2nr(k + 2t) \end{aligned}$$

Theorem 3.17. *The second Zagreb index of* $R\left(G \circ \overline{K_t}\right)$ *is*

$$\begin{aligned} M_2\left(R\left(G \circ \overline{K_t}\right)\right) &= \frac{nk}{2} (k + t)^2(r + 1)^2 + 2nkr(k + t)(r + 1) + nt(k + t)(r + 1)^2 \\ &\quad + 2ntr(r + 1)(k + t + 1) \end{aligned} \qquad (3.18)$$

Proof. In $R\left(G \circ \overline{K_t}\right)$, there are $\frac{nk}{2}$ + $\frac{nk}{2}$ *(2r)* + *nt* +*nt(2r)* = $\frac{n}{2}$ (k+2t)(1+2r) edges. Among these edges, there are $\frac{nk}{2}$ edges, with the degree of each of the end vertices (k+kr)+tr+t = (k+t)(r+1), *nkr* edges with degree of the end vertices (k+t)(r+1) and 2, *nt* edges with degree of the end vertices *(k+t)(r+1)* and *(r + 1)*, *ntr* edges with degree of the end vertices (k + t)(r + 1) and 2 and the remaining *ntr* edges with degree of the end vertices (r + 1) and 2. Hence by Eqn.(1.2), we have,

$$\begin{aligned} M_2\left(R\left(G \circ \overline{K_t}\right)\right) &= \frac{nk}{2} (k + t)^2(r + 1)^2 + nkr(k + t)(r + 1)(2) \\ &\quad + nt(k + t)(r + 1)(r + 1) + ntr(k + t)(r + 1)(2) \\ &\quad + ntr(r + 1)(2) \end{aligned}$$

$$= \frac{nk}{2}(k+t)^2 nkr(k+t)(r+1) + nt(k+t)(r+1)^2$$
$$+ 2ntr(r+1)(k+t+1)$$

Remarks:

(1) For a = 1 and b = n − 1, formula (3.9) reduces to (3.5) and (3.10) reduces to (3.6).

(2) For k = 2, formula (3.12) reduces to (3.3) and (3.13) reduces to (3.4).

(3) For k = n − 1, formula (3.12) reduces to (3.7) and (3.13) reduces to (3.8).

(4) For n = 2a and k = a, formulae (3.12) and (3.13) coincide with the formulae in Corollary 3.11 and are as given below:

$$M_1(R(G)) = M_1(R(K_{a,a})) = 2a^2[a(r+1)^2 + 2r]$$
$$M_2(R(G)) = M_2(R(K_{a,a})) = a^3(r+1)[a(r+1) + 4r] \qquad (3.19)$$

(5) For n = 3, formulae (3.1) and (3.2) coincide with formulae (3.5) and (3.6) coincide. And are as given below:

$$M_1(R(P_3)) = M_1(R(S_3)) = 6(r+1)^2 + 8r$$
$$M_2(R(P_3)) = M_2(R(S_3)) = 4(r+1)(4r+1) \qquad (3.20)$$

(6) As $L(C_n) \cong C_n$, therefore for $K = 2$, formula (3.15) reduces to (3.3) and (3.16) reduces to (3.4).

(7) For $t = 0$, formula (3.17) reduces to (3.12) and (3.18) reduces to (3.13).

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