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Mixed Convective Nanofluid Flow over an Inclined Stretching Plate with MHD and Effects of Suction and Internal Heat Generation

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Abstract. Investigation of two-dimensional steady laminar flow with mixed-convection due to a inclined stretching vertical sheet with MHD and partial slip over nanofluidsis done numerically in this paper. Two types of nanofluids namely silver and titanium oxide with water as base fluid are considered. Using a similarity approach, the governing partial differential equations are transformed into ordinary differential equations and they are solved numerically using MATLAB. Numerical investigations are carried out for different values of physical parameters and the effect of all these parameters over the flow field and temperature are discussed. The skin friction coefficient decreases while the Nusselt number increases as the slip parameter increases. The numerical values of skin friction coefficient and rate of heat transfer for various values of physical parameters are also obtained.

Keywords: Nanofluids; stagnation-point flow; inclined stretching sheet; partial slip

AMS Mathematics Subject Classification (2010): 76Mxx

1. Introduction

The study of boundary layer flow and heat transfer over a stretching surface has attracted the attention of many researchers due to its huge industrial and engineering applications. In the field of industry, metallurgical processes such as drawing of continuous filaments through quiescent fluids, annealing and tinning of copper wires, manufacturing of plastic and rubber sheets, crystal growing, and continuous cooling and fiber spinning, in addition to wide-ranging applications in many engineering processes, such as extrusion of polymer, wire drawing, manufacturing foods and paper, in textile and glass fiber production etc. During the manufacturing of these sheets, the melt issues from a slit and it is stretched to achieve the desired thickness. The final product depends on two characteristics first is the rate of cooling in the process and the other is stretching rate.

Commonly used fluids, such as water, mineral oil, etc., have very low thermal conductivity. Taking into account, there is a necessity to emerge new kind of fluids that will be more efficacious in heat transfer. Nanofluids were introduced in order to get past the above requirement. Fluids with nano-meter sized particles suspended into them was called nanofluids. The use of nanoparticles in the base fluid enhancesheat transfer. Convection heat transfer of nanofluids in enclosure was investigated by some researchers by considering different models of nanofluid properties. It aims at manipulating the structure of the matter at the molecular level with the goal for innovation in virtually every industry and public endeavor including biological sciences, physical sciences, electronics cooling, transportation, the environment and national security. Choi and Eastman [1] were probably the first to employ a mixture of nanoparticles and base fluid that such fluids were designated as "nano-fluids".

In nano fluids due to the increase of surface area to the volume, some physical properties such as thermal, electrical, mechanical, optical and magnetic property of the materials can be changed significantly. The most important point is that nano structured materials exhibit different and unique properties as compared to the bulk materials with the same compositions. Experimental studies have displayed that with 1%–5% volume of solid metallic or metallic oxide particles, the effective thermal conductivity of the resulting mixture can be increased by 20% compared to that of the base fluid [2]. A variety of research papers on nano-fluids and their different applications can be found in [3].

Sakiadis [4] was the first person to discuss the laminar boundary layer flow of a viscous and incompressible fluid caused by a continuous moving rigid surface. The flow over a linearly stretching sheet for the steady two - dimensional problem was analysed by Crane [5]. These types of flows usually occur in the drawing of plastic films and artificial fibres. The hydromagnetic mixed convective flows over a stretching surface were investigated by [6-9]. Mucoglu and Chen [10] analysed the study on mixed convection along an inclined flat plate, with the angle of inclination from the vertical and the plate is kept at a uniform temperature. Later, many investigations were proposed on hydromagnetic flow over inclined stretching surface considering various physical situations and few of them are [11-15].

The gravity-driven convection heat transfer is a vital phenomenon in the cooling mechanism of many engineering systems like the electronics industry, solar collectors and cooling systems for nuclear reactors because of its minimum cost, low noise, smaller size and reliability. There has been increasing interest in studying the problem of MHD with convection boundary layer flow and heat transfer characteristics over a vertical plate[16]. The MHD boundary layer flow over a vertical stretching/shrinking sheet in a nano-fluid was investigated by Makinde [17] and [18] and Das [19], and recently Das [20] studied the MHD Nanofluid flow past an impulsively started porous flat plate in a rotating frame. Nadeem [21], [22], [23], [24] and [25] has investigated MHD flow of different types of nano fluids over a convective surface. Ellahi [26] considered Non Newtonian nanofluid flow. Sheikholeslami [27] and [28] discussed effects of thermal radiation on magneto hydrodynamics nanofluid flow.

In the present study, our main objective is to investigate the effects of suction and internal heat generation over the mixed convective hydromagnetic flow over an inclined stretching plate with two different types of nanofluids namely, copper-water and alumina-water nanofluids. Through an appropriate similarity transformation, the governing partial differential equations are reduced into ordinary differential equations, which are then solved numerically using MATLAB. The effects of the various non-dimensional parameters namely, Magnetic parameter, volume fraction, angle of inclination, Suction parameter, heat generation parameter, mixed convection parameter and Prandtl number over the flow field and temperature distribution are discussed with the aid of graphs. The numerical values of skin friction coefficient and Nusselt number are also analysed with the help of tables.

2. Formulation of problem

Consider a steady, two dimensional stagnation point flow of a viscous and incompressible fluid towards a linear stretching vertical sheet inclined to angle α with vertical. X axis is choosen as direction of stretching sheet and Y axis normal to it .The velocity of the stretching sheet is $U_w = ax\,; a > 0$ is constant acceleration parameter. A transverse magnetic field of strength B_0 is applied parallel to the y-axis. Suction and internal heat generation are also considered .The pressure gradient and external forces are neglected. Under the Boussinque and the Prandtl boundary layer approximations with MHD factor, the basic equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + \frac{g(\rho\beta)_{nf}(T-T_\infty)}{\rho_{nf}}\cos\alpha - \frac{\sigma B_0^2 u}{\rho_{nf}} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0(T-T_\infty)}{(\rho C_p)_{nf}} \tag{3}$$

where u and v are velocity components in x and y directions, respectively, T is temperature , ϑ_{nf} is kinematic viscosity of nanofluids ,α_{nf} is thermal diffusivity, k_{nf} thermal conductivity , $(C p)_{nf}$ is specific heat and ρ_{nf} density of nanofluids.

Apart from these equations, the boundary condition for velocity and temperature are

$$y = 0\,,\ \ u = ax + l\left(\frac{\partial u}{\partial y}\right)_{y=0}\ ,\ \ v = -v_w\ ,\ \ T = T_w + \mathrm{b\,x}$$

$$y \to \infty\,, u \to 0\,, T \to T_\infty \tag{4}$$

The viscosity, heat capacity and thermal conductivity of the nanofluids depends upon volume fraction ϕ of nanoparticles used . The effective density of nanofluid is given by

$$\rho_{nf=(1-\emptyset)\rho_f} + \phi\ \rho_s \tag{5}$$

And heat capacitance of nanofluid is given by

$$(\rho C_p)_{nf\,=\,(1-\emptyset)(\rho C_p)_f} + \ \emptyset(\rho C_p)_s \tag{6}$$

$$(\rho\beta)_{nf}\ = (1 -\ \phi)(\rho\beta)_f\ +\ \phi(\rho\beta)_s \tag{7}$$

As given by Santra et al. (Santra et al. 2009) , where ρ_f and ρ_s are density, $(C p)_f$ and $(C p)_s$ are specific heat capacitance of base fluid and solid particle respectively.

The dynamic viscosity of nanofluids as given by (Brinkman 1952) is as follows :

$$\mu = \frac{\mu_f}{(1-\emptyset)^{2.5}} \tag{8}$$

$$\frac{k_{nf}}{K_f} = \frac{(k_s+2k_f)-2\emptyset\,(k_f-k_s)}{(k_s+2k_f)+\emptyset\,(k_f-k_s)} \tag{9}$$

$\alpha_{nf} = \frac{k_{nf}}{(C_p)_{nf}}$ (10)

	ρ(kg/m^3)	β(k^{-1})	**κ**(W/m.k)	C_p(J/kg.k)
Water	997.1	21×10^{-5}	0.613	4179
Silver	10,500	1.89×10^{-5}	429	235
Titanium oxide	4250	0.9×10^{-5}	8.9538	686.2

Table 1: Physical properties of base fluid water, silver, titanium oxide

To solve the equations following dimensionless variables are introduced:

$\psi = (a\,\vartheta_{nf})^{1/2} x f(\eta)$ (11)

$\eta = (\frac{a}{\vartheta_{nf}})^{1/2} y$ (12)

$u = ax\, f'(\eta)$ (13)

$v = -\left(a\vartheta_{nf}\right)^{\frac{1}{2}} f\,(\eta\,)$ (14)

where **ψ**(x, y) is stream function and $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, η is similarity variable.

and, $\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$ (15)

Continuity equation is satisfied and equations (2) and (3) along with boundary conditions (4) are transformed and are written as :

$$f''' + (1-\emptyset)^{2.5}\left\{\left(ff''-f'^2\right)\left(1-\emptyset+\emptyset\frac{\rho_s}{\rho_f}\right) - M^2 f' + \lambda\theta\left(1-\emptyset+\emptyset\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\cos\alpha\right\} = 0 \quad (16)$$

$$\frac{1}{Pr_{nf}}\frac{k_{nf}}{k_f}\theta'' + \left(1-\emptyset+\emptyset\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)(f\theta' - f'\theta) + \beta_1\theta = 0 \quad (17)$$

Along with boundary conditions :

At $\eta = 0\,,\ f = A, f' = 1 + kf'', \theta = 1$

As $\eta \to \infty\,,\ , f' \to 0\,, \theta \to 0$ (18)

where $k = l(\frac{a}{\vartheta})^{1/2}$; A= $\frac{v_w}{(a\vartheta_f)^{1/2}}$

‘ denotes differentiation w.r.t . η

Terms used in equation are :

$(Rex)_f = \frac{U_w x}{\vartheta_f}$; $c = \frac{g\beta(T_w - T_\infty)x^3}{\vartheta_f^2}$; $\lambda = \frac{g\beta(T_w - T_\infty)}{a^2 x}$

$Pr = \frac{\vartheta_f}{\alpha_f}$; $M^2 = \frac{\sigma B_0^2}{a\rho_f}$; $A = \frac{v_w}{(a\vartheta_f)^{1/2}}$; $\beta_1 = \frac{Q_0}{a(\rho C_p)_f}$

where $(Rex)_f$ is local Reynold’s number ; $(Grx)_f$ is Grashouf number ; λ is mixed convection parameter ; Pr is Prandtl number ; M^2 is magnetic parameter ; A is suction parameter ; β_1 is heat source parameter.

2.1. Skin friction coefficient

The skin friction friction coefficient cf is given by

$C_f = \frac{\tau_w}{\rho_f U_w^2}$ where $\tau_w = \mu_{nf}(\frac{\partial u}{\partial y})_{y=0}$

Using (13) we get

$$C_f Re_x^{1/2} = \frac{1}{(1-\emptyset)^{2.5}} f''(0)$$

Nusselt number

Nusselt number is defined by :

$Nu = \frac{xq_w}{k_f(T_w - T_f)}$ where $q_w = -k_{nf}(\frac{\partial T}{\partial y})_{y=0}$

Using (15) we get $NuRe_x^{-1/2} = -\frac{k_{nf}}{K_f}\theta'(0)$

Solution. The set of nonlinear equation (16) and (17) along with conditions (18) are solved with help of MATLAB software . Equations (16) and (17) are converted to five first order equations .Then they are solved numerically in the symbolic computation software MATLAB for various values of the governing parameters such as Magnetic interaction parameter, angle of inclination, volume fraction, slip parameter, suction parameter and heat generation parameter with fixed values of Prandtl number and mixed convection parameter. The asymptotic boundary conditions given by equation (9) were replaced by using a value of 15 for the similarity variable η_{max} as follows

$\eta_{max}=15$, $f'(15)=0$, $\theta(15)=0$

The choice of $\eta_{ma}=15$ ensured that all numerical solutions approached the asymptotic values correctly. The absolute error tolerance for this method is 10-^{6}. The numerical values for skin friction coefficient and the Nusselt number are also obtained and are tabulated for different values of M^2, ϕ, A and β_1. The value of η is found on each iteration loop by $\eta_{n+1}= \eta_n+\Delta\eta$. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution with $\eta_{max} = 15$ and by considering the six decimal place as a criterion for convergence.

3. Results and discussions

We consider two types of water based nanofluids containing Silver (Ag) and Titanium oxide (TiO2).The mixed convection problem associated with two-dimensional laminar flow of these nanofluids over an inclined stretching sheet in the presence of various physical parameters magnetic field, slip parameter, suction and internal heat generation is considered and numerical results are obtained.

Numerical solutions of the problem are obtained for various values of physical parameters involved in the study such as M^2 ,α, ϕ ,k ,β_1 ,A, λ and Pr. The Prandtl number is kept constant at Pr = 6.2 and the mixed convection parameter is fixed at $\lambda=1.5$ for different values of physical parameters such as M^2=0,1,2,4;α=, 300, 450,600; ϕ=0.01,0.03,0.05,0.1; A =0.1, 0.3, 0.5,0.7, k= 0.0,0.1,0.2,0.3 and β_1=0.0,0.5,0.7,1.0. Numerical computations of results are demonstrated through graphs over the flow field and temperature. Further, Skin friction coefficient and the non-dimensional rate of heat transfer are found out and are presented by means of tables.

The effect of magnetic interaction parameter over the dimensionless velocity for both the Silver-water and Titanium oxide-water nanofluids is shown in Fig.1. The presence of transverse magnetic field sets in Lorentz force effect, which results in the retarding effect on the velocity field. Increasing values of magnetic field, the retarding force increases and consequently the velocity gets decelerated. Thus, the presence of the magnetic field reduces the momentum boundary layer thickness due to the Lorentz force effect for both the types of nanofluids.

The influence of Magnetic field on temperature distribution for both silver-water and Titanium oxide -water nanofluids is depicted in Fig 2. It is observed that for increasing values of M^2, the temperature increases which gives the effect of Magnetic field is to enhance the temperature. It is observed that the thermal boundary layer thickness increases due to increase in Magnetic field for both the types of nanofluids

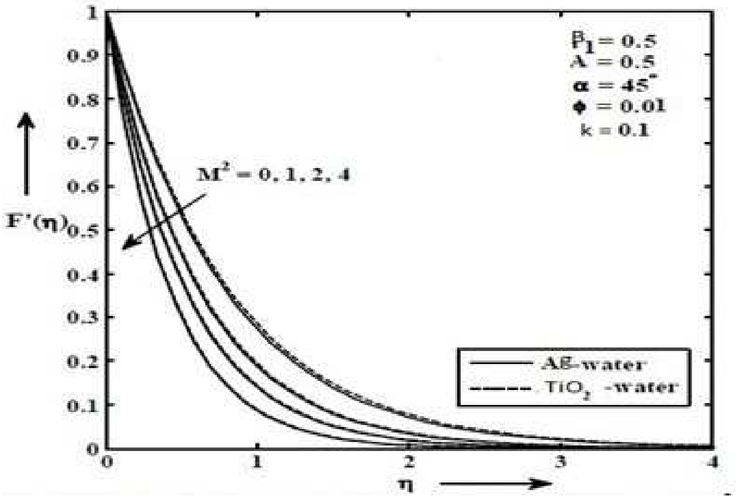

Figure 1: Dimensionless velocity profiles for different M^2

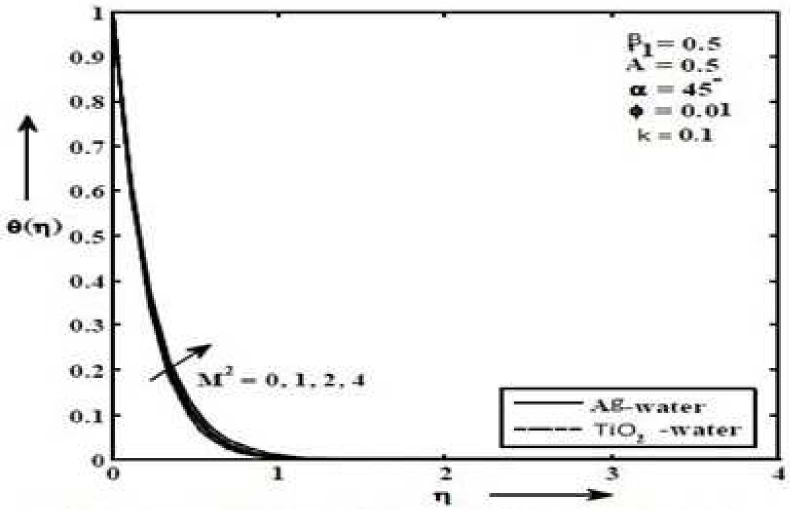

Figure 2: Temperature distribution for different M^2

For both the silver-water and Titanium oxide -water nanofluids, the dimensionless velocity for different values of α is depicted in Fig. 3. It is observed that the increase in the value of α , the velocity of the fluid gets decelerated. It is also found that the thickness of the momentum boundary layer decreases for increase in α. The effect of inclination angle α on temperature distribution for specified parameters for both the silver-water and Titanium oxide-waternanofluids is demonstrated by Fig 4. When α increases, temperature also increases. However, the change is not significant.

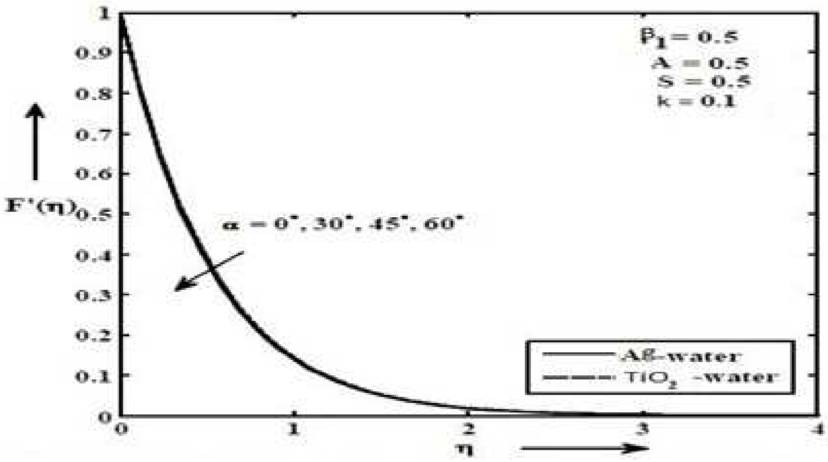

Figure 3: Dimensionless velocity profiles for different α

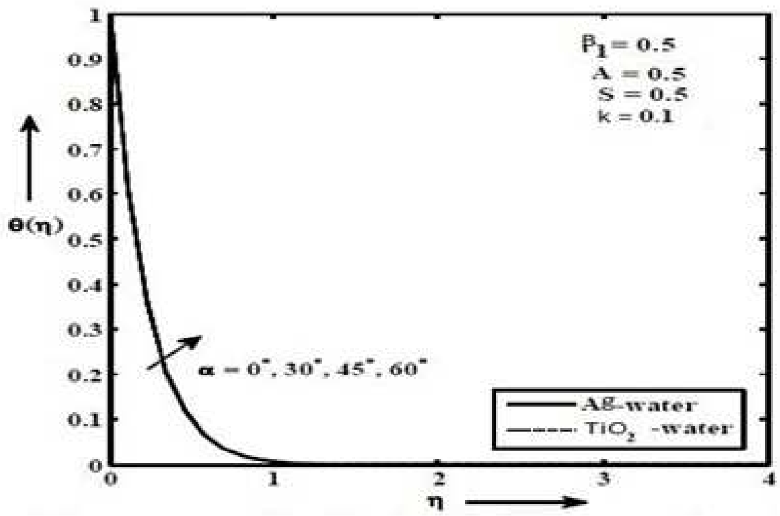

Figure 4: Temperature distribution for different values of α

The influence of heat generation parameter on dimensionless velocity for both silver-water and Titanium oxide-water nanofluids is demonstrated through Fig.5. When the heat is generated the buoyant force increases which causes the rate of flow to increase which gives rise to the increase in the velocity profile for both the silver –water and Titanium oxide water nanofluids. As a consequence, the hydrodynamic boundary layer thickness of the nanofluid increases with increasing heat generation parameter. Fig.6 shows the heat generation parameter on the temperature distribution for both the silver-water and Titanium oxide -water nanofluids. With the presence of heat generation ($\beta_1 > 0$), it is apparent that there is an increase in the thermal state of the fluid. Thus, the thermal boundary layer thickness increases due to increase in heat generation parameter for both the types of nanofluids.

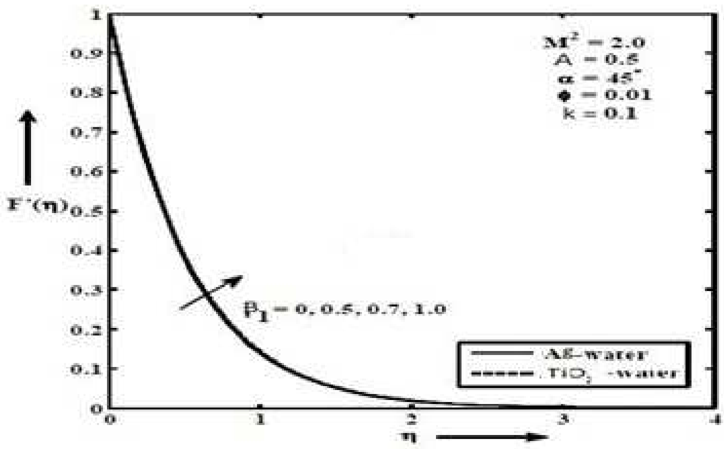

Figure 5: Dimensionless velocity profiles for different values of β_1

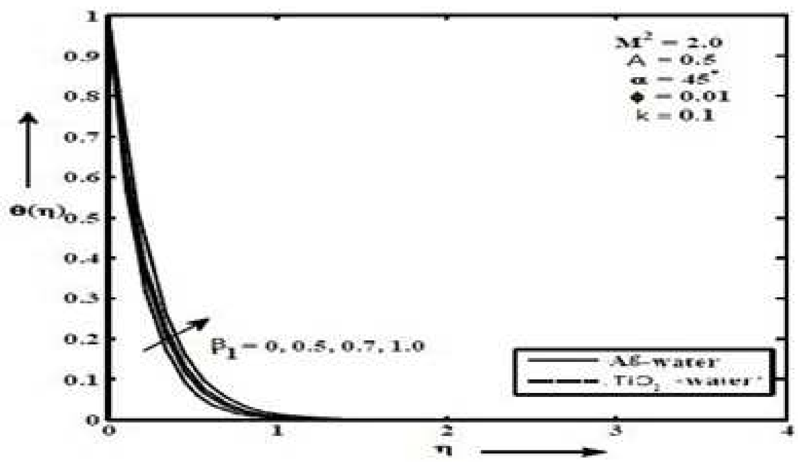

Figure 6: Temperature distribution for different values of β_1

Fig.7 depicts the effect of Suction over dimensionless velocity for both the silver-water and Titanium-oxidewaternanofluids. The velocity is found todecrease which leads a rise in A. This is because the suction pulls the fluid toward the wall, and the buoyant force acts as the pulling force. It is also found that the momentum boundary layer thickness reduces for both the silver-water and Titanium-oxidewater-water nanofluids. Fig.8 displays the temperature distribution for both the silver-water and Titanium-oxidewater-water nanofluids for different values of Suction. The increase of suction parameter accelerates the transverse fluid motion it has tendency to decrease the temperature. Moreover, the thermal boundary layer thickness and the surface temperature are also decreasing for both the types of nanofluids.

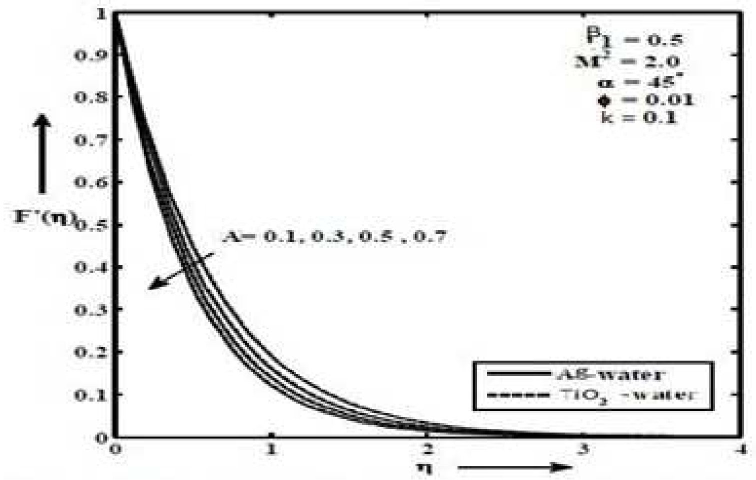

Figure 7: Dimensionless velocity profiles for different values of A

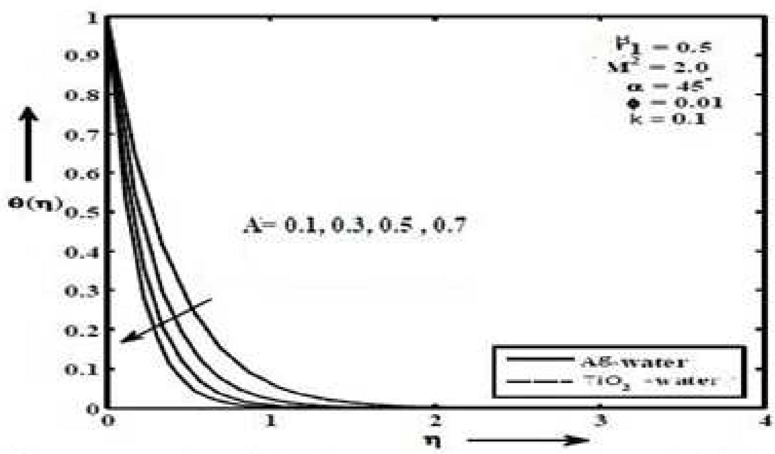

Figure 8: Temperature distribution for different values of A

Fig.9 shows the various values of volume fraction over the dimensionless velocity for both the silver -water and Titanium oxide-water nanofluids. The fluid velocity is found to decreases with increase innumber of copper nanoparticles. Whereas, the nanofluid velocity increases by increasing the volume fraction of Titanium oxide nanoparticles. It is alsonoted that the nanofluid momentum boundary layer thickness decreases slightly by adding the number of silver nanoparticles whereas the reverse is true for the momentum boundary layer in the case of Titanium oxide nanoparticles. The temperature distribution for different values of ϕ for both the silver-water and Titanium oxide -water nanofluids is shown in Fig.10. Increasing values of volume fraction lead to both the enhancement of temperature and the thermal boundary layer thickness for both the silver-water and Titanium oxide -water nanofluids.

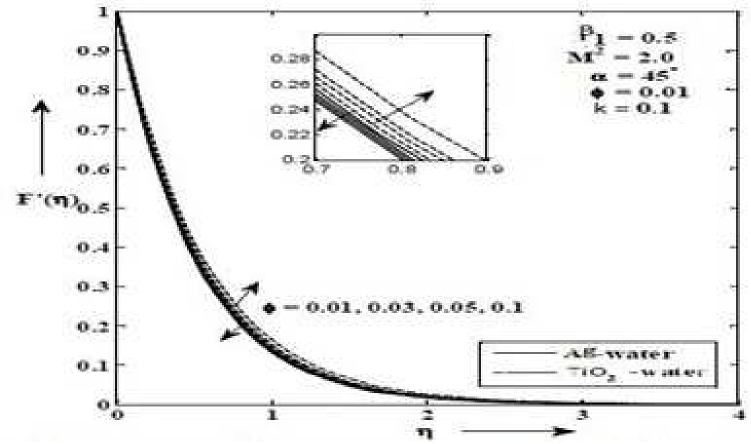

Figure 9: Dimensionless velocity profiles for different ϕ

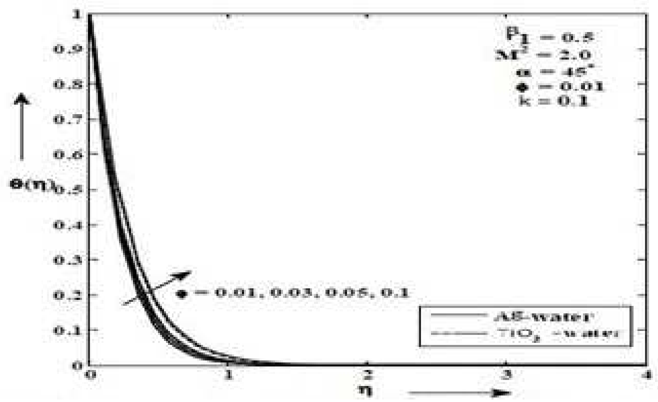

Figure 10: Temperature distribution for different ϕ

Figs. 11, 12 illustrate the velocity and temperature distributions for different values of the slip parameter k for both types Nanofluids. It is also seen that the effect of slip parameter together with the presence of nanoparticles is more significant in the case of Silver – Water Nanofluid than that of Titanium oxide – Water Nanofluid. The effect of slip parameter k is seen to increase the temperature.

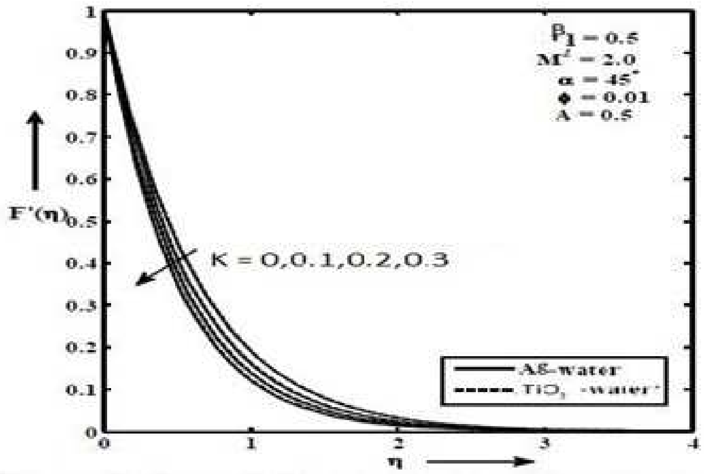

Figure 11: Dimensionless velocity profiles for different values of K

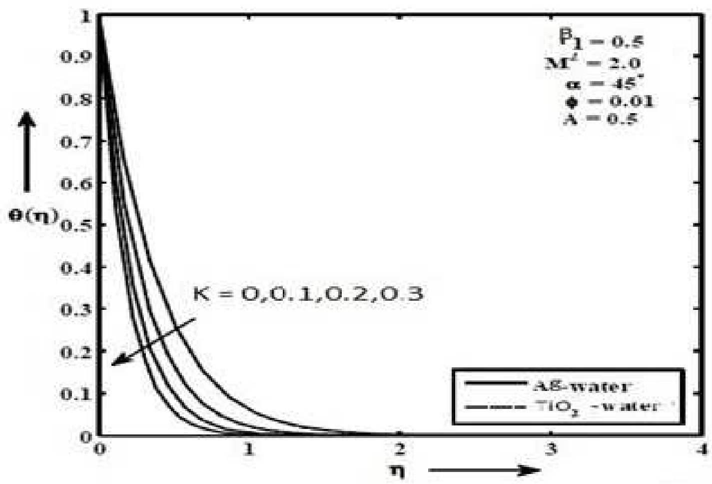

Figure 12: Temperature distribution for different K

ϕ	M^2	α	k	A	β_1	Ag $(\frac{1}{(1-\emptyset)^{2.5}}f''(0)$	TiO_2 $(\frac{1}{(1-\emptyset)^{2.5}}f''(0)$
0.01 0.03 0.05 0.1	2	45^0	0.1	0.5	0. 5	-1.882491 -2.019789 -2.149876 -2.492271	-1.866731 -1.940001 -2.018998 -2.221124
0.01	0 1 2 4	45^0	0.1	0.5	0. 5	-1.176284 -1.569889 -1.882491 -2.391517	-1.1389967 -1.539929 -1.866731 -2.376892

0.01	2	0^0 30^0 45^0 60^0	0.1	0.5	0.5	-1.823112 -1.855624 -1.882491 -1.939989	-1.785437 -1.824268 -1.866731 -1.915930
0.01	2	45^0	0 0.1 0.2 0.3	0.5	0.5	-1.678921 -1.882491 -2.011729 -2.034671	-1.658941 -1.866731 -2.009729 -2.02346
0.01	2	45^0	0.1	0.1 0.3 0.5 0.7	0.5	-1.586177 -1.736721 -1.872491 -2.037892	-1.576891 -1.732912 -1.866731 -2.037892
0.01	2	45^0	0.1	0.5	0.0 0.5 0.7 1.0	-1.906789 -1.882491 -1.876542 -1.869212	-1.870999 -1.866731 -1.849937 -1.836392

Table 2: Skin friction coefficient for silver water and titanium oxide nanofluids for various parameters (taking λ=1.5 and Pr=6.2)

ϕ	M^2	α	k	A	β_1	Ag($-\frac{k_{nf}}{K_f}\theta'(0)$)	TiO_2 ($(-\frac{k_{nf}}{K_f}\theta'(0))$)
0.01 0.03 0.05 0.1	2	45^0	0.1	0.5	0.5	4.352261 4.365769 4.380012 4.408958	4.351628 4.369065 4.384122 4.429125
0.01	0 1 2 4	45^0	0.1	0.5	0.5	4.516251 4.423689 4.352261 4.237845	4.364127 4.358602 4.351628 4.237904
0.01	2	0^0 30^0 45^0 60^0	0.1	0.5	0.5	4.362131 4.358299 4.352261 4.345163	4.364719 4.358710 4.351628 4.100378
0.01	2	45^0	0 0.1 0.2 0.3	0.5	0.5	4.607891 4.352261 4.201789 4.109981	4.606789 4.351628 4.200021 4.100378

0.01	2	45^0	0.1	0.1 0.3 0.5 0.7	0.5	2.421512 3.349972 4.352261 5.427891	2.421780 3.348169 4.351628 5.412010
0.01	2	45^0	0.1	0.5	0.0 0.5 0.7 1.0	4.885642 4.352261 4.107892 3.608172	4.883642 4.351628 4.107691 3.686021

Table 3: Rate of heat transfer for silver water and titanium oxide nanofluids for various parameters (taking λ=1.5 and Pr=6.2)

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