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On Contra p*gα-Continuous Functions and Strongly p*gα-Closed Spaces

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Abstract. The aim of this paper is to introduce and study the concept of contra p*gα continuous function and strongly p*gα closed function. Already Jafari and Noiri introduced new generalization of centre continuity, contra–α-continuity, contra precontinuity. Here, we introduce a new study and a new class of contra continuous functions.

Keywords: contra p*gα continuous function, strongly p*gα closed function

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1. Introduction

Dontcher and Noiri are investigated the notions of Contra-continuity and contra semi continuity respectively. Caldas and Jafari introduced the notion of Contra β – continuous functions in topological spaces. Dass and Rodrigo investigated and give more properties in contra d. continuous functions. The aim of this paper to introduce a new class of functions called contra p*gα - continuous functions.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) will always denote the topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let A be a Subset of (x, τ), cl (A) and Int (A) denote the closure and interior of A, respectively, Now we recall some definitions which we needed in this paper.

Definition 2.1. Let (X,τ) be a topological space. A subset A of the space X is said to be
1. Preopen if $A \subseteq$ int(cl(A)) and preclosed if cl(int(A)) $\subseteq$ A.
2. Semi open if $A \subseteq$ cl(int(A)) and semi closed if int(cl(A)) $\subseteq$ A .
3. Regular open if A = int(cl(A)) and regular closed if A = cl(int(A)).

Definition 2.2. Let (X,τ) be a topological space. A subset $A \subseteq X$ is said to be
1. g-closed if cl(A) $\subseteq$ U whenever $A \subseteq U$ and Uis open in X.
2. ∗gα-closed if cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is gα-open in X.

3. p*gα-closed if pcl(A) ⊆ (U) whenever A ⊆ U and U is*gα-open in X.
The complements of above mentioned sets are called their respective open sets.

Definition 2.3. A function f : (X,τ) → (Y,σ) is called

1. g-continuous if f^{-1}(V) is g-closed in (X,τ) for every closed set V in (Y,σ).
2. *gα-continuous if f^{-1}(V) is gα -closed in (X,τ) for every closed set V in (Y,σ).
3. p*gα -continuous if f^{-1} (V) is p*gα -closed in (X,τ) for every closed set V in (Y,σ).
4. p*gα -irresolute if f^{-1}(V) is p*gα -closed in (X,τ) for every p*gα -closed set V in (Y,σ).
5. Strongly p*gα continuous if f^{-1}(V) is closed in (X,τ) for every p*gα closed set V in (Y,σ).
6. Pre-p*gα continuous if f^{-1}(V) is p*gα-closed in (X,τ) for every pre-closed set V in (Y,σ).
7. Perfectly p*gα-continuous if f^{-1}(V) is clopen in (X,τ) for every p*gα -closed set V in (Y,σ).
8. Super continuous if f^{-1}(V) is regular open in (X,τ) for every open set V in (Y,σ).
9. Contra-continuous if f^{-1}(V) is closed in (X,τ) for every open set V in (Y,σ).
10. Contra pre-continuous if f^{-1}(V) is preclosed in (X,τ) for every open set V in (Y,σ).
12. Contra g-continuous if f^{-1}(V) is g-closed in (X,τ) for every open set V in (Y,σ).
13. Contra semi-continuous if f^{-1}(V) is semiclosed in (X,τ) for every open set V in (Y,σ).
14. RC-continuous if f^{-1}(V) is regular closed in (X,τ) for every open set V in (Y,σ).
15. p*gα -open if f(V) is p*gα -open in (Y,σ) for every p*gα -open set V in (X,τ).

Definition 2.4. A space (X,τ) is called

1. A gT**α space [21] if every g-closed set is p*gα closed.
2. A P-Tsspace [3] if every p*gα -closed set is closed.

Theorem 2.5. [1] Let (X,τ) be a topological space.
(1) A subset A of (X,τ) is regular open ⇔ A is opened p*gα closed.
(2) A subset A of (X,τ) is open and regular closed then A is p*gα closed.

Theorem 2.6. [2] Every closed set in a topological space (X,τ) is p*gα−closed.

3. Contra-p*gα-Continuous Functions

Definition 3.1. A function f: (X,τ)→(Y,σ) is called contra- p*gα-continuous if f^{-1} (V) is p*gα-open (respectively p*gα closed) in (X,τ) for every closed (respectively open) set V in (Y,σ).

Example 3.2.Let X = {a, b, c} = Y, τ = {ϕ, {a}, X} and σ = {ϕ, {b, c}, Y}. Then the identi ty function f : (X, τ) →(Y, σ) is contra-p*gα continuous function, since for the closed (respectively open) set V = {b} in (Y,σ), f^{-1}(V) = {b} is p*gα- open (respectively p*gα-closed) in (X, τ).

Definition 3.3. Let A be a subset of a topological space (X, τ). The set $\cap\{U \in \tau / A \subset U\}$ is called the kernel of A and is denoted by Ker(A).

Lemma 3.4. The following properties hold for subsets A, B of a space X :

1. x∈ Ker(A) if and only if $A \cap F \neq \phi$ for any F∈C(X, x).
2. $A \subset$ Ker(A) and A = Ker(A) if A is open in X.
3. If $A \subset B$ then Ker(A) $\subset$ Ker(B).

Theorem 3.5. Every contra-continuous function is a contra-p*gα-continuous function.
Proof: Let f : (X,τ) → (Y,σ) be a function. Let V be an open set in (Y,σ). Since f is contra-continuous, f^{-1}(V) is closed in (X, τ). Hence by theorem 2.6, f^{-1}(V) is p*gα-closed in (X, τ). Thus f is a contra-p*gα-continuous function.

Remark 3.6. Converse of this theorem need not be true as seen from the following example.

Example 3.7. Let X = {a, b, c} = Y, τ = {ϕ, {a}, X} and σ = {ϕ, {a}, {b, c}, Y}.
Define f : (X, τ) → (Y, σ) by f(a) = b; f(b) = c and f(c) = a.Then f is contrap*gαconti nuous but not contracontinuous, since for the open (resp.closed) set V = {b,c}, f^{-1}(V) = {a, b} is p*gα-closed (resp. p*gα-open) but it isnot closed.

Remark 3.8. Contra-p*gα-continuous and contra-p*gα-continuous (respectively contra semi-continuous, contra-semi pre-continuous, contra semi-continuous) are independent concepts.

Remark 3.10. The composition of two contra p*gα-continuous functions need not be contra p*gα continuous and this is shown by the following example.

Example 3.1. Let X = {a, b, c} = Y = Z, τ = {ϕ,{a},X}, σ = {ϕ, {b, c},Y} and η = {ϕ,{a, c} ,Z}. Define
f : (X, τ) →(Y, σ) by f(a) = a; f(b) = b and f(c) = b. Then f is contrap*gα continuous, since f or the closed set V = {a}, f^{-1}(V) ={a}isp*gαopen in (X, τ).
Define g : (Y, σ) → (Z, η) by g(x) = x. Then g is contra-p*gα continuous, since for the closed set V = {b} in (Z, η), g^{-1}(V) ={b} is p*gα-open in (Y,σ). But their composition is not a contra-p*gα-continuous, since for the closed set V={b}in (Z,η),$f^{-1}(g^{-1}(V))$ =f^{-1}({b})={b, c} is not a p*gα-open in (X, τ).

Theorem 3.12. The following are equivalent for a function f : (X, τ) → (Y, σ) : Assume that p*gα(X) (respectively p*gαC(X)) is closed under any union (resp. intersection)
1. f is contra-p*gα-continuous

2. The inverse image of a closed set V of Y is p*gα-open
3. For each x$\in$X and each V$\in$C(Y, f(x)), there exists U$\in$p*gαO(X, x) such that f(U) $\subseteq$ V.
4. f(p*gα-cl(A)) $\subseteq$ Ker (f(A)) for every subset A of X.
5. p*gα-cl(f^{-1}(B)) $\subseteq$ f^{-1} (Ker (B)) for every subset B of Y.

Proof: The implications (1) $\Rightarrow$ (2), (2) $\Rightarrow$ (3), is true obvious.

(3) $\Rightarrow$ (2)

Let V be any closed set of Y and x $\in$ f^{-1}(V). Then f(x) $\in$ V and there exists $U_x\in$ p*gαO(X, x) such that f(U_x) $\subset$V. Hence we obtain f^{-1}(V) = $\cup$\{U_x / x $\in$ f^{-1}(V)) and by assumption f^{-1}(V) is p*gα-open.

(2) $\Rightarrow$ (4)

Let A be any subset of X. Suppose that y $\notin$ Ker (f(A)).Then by **Lemma 3.4**, there exists V $\in$ C (X, x) such that f(A) $\cap$ V = ϕ. Thus we have A $\cap$ f^{-1}(V) = ϕ and p*gα-cl(A) $\cap$ f^{-1}(V) = ϕ. Hence we obtain f (p*gα -cl(A)) $\cap$ V = ϕ and y $\notin$ f(p*gα – cl(A)).Thus f(p*gα-cl(A)) $\subseteq$ Ker(f(A)).

(4) $\Rightarrow$ (5)

Let B be any subset of Y. By (4) and **Lemma 3.4,** we have
f(p*gα -cl(f^{-1}(B))) $\subset$ Ker(f(f^{-1}(B))) $\subset$ ker(B) andp*gα– cl(f^{-1}(B)) $\subset$ f^{-1}(Ker (B)).

(5) $\Rightarrow$ (1)

Let U be any open set of Y by **Lemma 3.4** we have f(p*gα-cl(f^{-1}(U)) $\subset$ f^{-1}(ker(U)) = f^{-1}(U) and p*gα-cl (f^{-1}($\cup$)) = f^{-1}(U). By assumption f^{-1}(U) is p*gα-closed in X. Hence f is a contra-p*gα-continuous.

Theorem 3.13. If f: (X, τ) → (Y, σ) is a p*gα-irresolute (resp. contra p*gα-contiuous) and g: (Y, σ) → (Z, η) in contra p*gα-continuous (respectively continuous). Then their composition gof: (X, τ) → (Z, η) is contra p*gα-continuous.

Proof: Let U be any open set in (Z, η). Since g is contra-p*gα -continuous (respectively continuous) then g^{-1}(V) in is p*gα -in (Y, σ) and since f is p*gα-irresolute (respectively contra p*gα -continuous) then f^{-1}(g^{-1}(V)) is p*gα –closed in (X,τ). Hence gof is contra-p*gα -continuous.

Theorem 3.14. If f : (X,τ) → (Y,σ) is contra-continuous and g : (Y,σ) → (Z, η) is continuous then their composition gof : (X, τ) → (Z, η) is contra-p*gα -continuous.

Proof: Let U be any open set in (Z, η).Since g is this p*gα continuous, g^{-1}(U) is open in (Y,σ).

Since f is contra-continuous, f^{-1}(g^{-1}(U)) is closed in (X, τ). Hence by theorem 2.6, $(gof)^{-1}$(U) is p*gα -closed in (X, τ). Hence gof is contra-p*gα -continuous.

Theorem 3.15. If f:(X, τ) → (Y, σ) is contra-continuous and super-continuous and g:(Y,σ) → (Z, η) is contra-continuous then their composition gof : (X, τ) → (Z, η) is contra-p*gα -continuous.

Proof: Let U be any open set in (Z, η).Since g is contra-continuous, $g^{-1}(U)$ is closed in (Y, σ) and since f is contra-continuous and super-continuous $f^{-1}(g^{-1}(U))$ is both open and regular closed in (X, τ). Hence by theorem 2.5, $(gof)^{-1}(U)$ is p*gα-closed in (X, τ). Hence gof is contra-p*gα-continuous.

Theorem 3.16. Let (X,τ), (Y,σ) be any topological spaces and (Y,σ) be $T_{1/2}$ space (respectively gT_α^{**} space). Then the composition gof : (X, τ) → (Z, η) of contra-p*gα -continuous function f : (X, τ) → (Y, σ) and the g-continuous (respectively p*g-continuous) function g : (Y, σ) → (Z, η) is contra-p*gα -continuous.
Proof: Let V be any closed set in (Z, η). Since g is g-continuous (resp*gα -continuous), $g^{-1}(V)$ is g-closed (respectively p*gα -closed) in (Y, σ) and (Y, σ) is $T_{1/2}$ space (respectively gT_α^{**}-space), hence $g^{-1}(V)$ is closed in (Y, σ). Since f is contra-p*gα -continuous, $f^{-1}(g^{-1}(V))$ is p*gα -open in (X, τ). Hence gof is contra-p*gα -continuous.

Theorem 3.17. If f : (X, τ) → (Y, σ) is a surjective p*gα -open function and g : (Y, σ) → (Z, η) is a function such that gof : (X, τ) → (Z, η) is contra-p*gα -continuous then g is contra-p*gα -continuous.
Proof: Let V be any closed subset of (Z, η). Since gof is contra-p*gα continuous then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is p*gα -open in (X, τ) and since f is surjective and p*gα -open, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is p*gα -open in (Y, σ). Hence g is contra-p*gα -continuous.

Theorem 3.18: Let $\{X_i \,/\, i\in I\}$ be any family of topological spaces. If $f : X \to \Pi X_i$ is a contra-p*gα-continuous function. Then $\pi_I\circ f : X \to X_i$ is contra-p*gα -continuous for each $i \in I$, where π_i is the projection of ΠX_i onto X_i.

Theorem 3.19. If f : (X, τ) → (Y, σ) is strongly p*gα -continuous and g : (Y, σ) → (Z, η) is contra-p*gα -continuous then gof: (X, τ) → (Z, η) is contra-continuous
Proof: Let U be any open set in (Z, η). Since g is contra-p*gα -continuous, then $g^{-1}(U)$ is p*gα -closed in (Y, σ). Since f is strongly p*gα -continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is closed in (X, τ). Hence gof is contra-continuous.

Theorem 3.20: If f: (X, τ) → (Y, σ) is pre p*gα-continuous and g: (Y, σ) → (Z, η) is contra –pre continuous then gof: (X, τ) → (Z, η) is contra-p*gα continuous.
Proof : Let U be any open set in (Z, η). Since g is contra pre continuous, then $g^{-1}(U)$ is pre-closed in (Y, σ). Since f is pre p*gα-continuous then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is p*gα-closed in (X, τ). Hence gof is contra p*gα-continuous.

Theorem 3.21. If f: (X, τ) → (Y, σ) is strongly p*gα-continuous and g: (Y, σ) → (Z, η) is contra p*gα-continuous then gof: (X, τ) → (Z, η) is contra p*gα-continuous.
Proof: Let U be any open set in (Z, η). Since g is contra-p*gα-continuous, then $g^{-1}(U))$ isp*gα-closed in (Y, σ) and since f is strongly-p*gα -continuous, then $f^{-1}(g^{-1}(U)) = (gof)^{-1}(U)$ is closed in (X, τ). By theorem 2.6, $(gof)^{-1}(U)$ is p*gα-closed in (X, τ). Hence gof is contra-p*gα-continuous.

Theorem 3.22. Let f : (X, τ) → (Y, σ) be surjective p*gα-irresolute and p*gα-open and g:$(Y,\sigma) \to (Z, \eta)$ be any function. Then gof : $(X, \tau) \to (Z, \eta)$ is contra-p*gα-continuous if and only if g is contra-p*gα-continuous.

Proof: The necessary part obvious. To prove the converse part, let V be any closed set in (Z, η). Since gof is contra-p*gα-continuous, then $(gof)^{-1}(V)$ is p*gα-open in (X, τ) and since f is p*gα-open surjection, then $f((gof)^{-1}(V)) = g^{-1}(V)$ is p*gα-open in (Y,σ). Hence g is contra-p*gα-continuous.

Theorem 3.23. Let f : $(X,\tau) \to (Y,\sigma)$ be a contra-p*gα-continuous function and H is an open p*gα-closed subset of (X,τ). Assume that p*gαC(X,τ) (the class of all p*gα-closed sets of (X,τ)) is p*gα-closed under finite intersections. Then the restriction $f_H : (H,\tau_H) \to (Y,\sigma)$ is contra-p*gα-continuous.

Proof: Let U be any open set in (Y,σ). By hypothesis and assumption, $f^{-1}(U)\cap H = H_1$(say) is p*gα-closed in (X,τ). Since $(f_H)^{-1}(U) = H_{1,}$ it is sufficient to show that H_1 is p*gα-closed in H. By hypothesis, H_1 is p*gα-closed in H. Thus f_H is contra p*gα-continuous.

Theorem 3.24. Let f : $(X,\tau) \to (Y,\sigma)$ be a function and g : $X \to X \times Y$ the graph function given by $g(x) = (x,f(x))$ for every $x\in X$. Then f is contra-p*gα-continuous if g is contra-p*gα-continuous

Proof: Let V be a closed subset of Y. Then $X \times V$ is a closed subset of $X \times Y$. Since g is contra-p*gα continuous ,then $g^{-1}(X \times V)$ is a p*gα-open subset of X. Also $g^{-1}(X \times V) = f^{-1}(V)$. Hence f is contra-p*gα continuous.

Theorem 3.25. If a function f: $(X, \tau) \to (Y, \sigma)$ is contra-p*gα-continuous and Y is regular, then f is p*gα continuous.

Proof: Let x be an arbitrary point of X and N be an open set of Y containing f(x). Since Y is regular, there exists an open set U in Y containing f(x) such that $cl(U) \subseteq N$. Since f is contra-p*gα-continuous, by **theorem 3.12**, there exists $Z \in$ p*gαO(X, x) such that $f(Z) \subseteq cl(U)$. Then $f(Z) \subseteq N$. Hence by **theorem 4.13,** f is p*gα-continuous.

Theorem 3.26. Every continuous and RC-continuous function is contra-p*gα-continuous.

Proof: Let f : $(X, \tau) \to (Y, \sigma)$ be a function. Let U be an open set in (Y, σ). Since f is continuous and RC-continuous, $f^{-1}(U)$ is open and regular closed in (X, τ). Hence by theorem 2.5, f is contra-p*gα-continuous.

Theorem 3.27. Every continuous and contra-p*gα-continuous (respectively contra-continuous and p*gα-continuous) function is a super-continuous function.

Proof: Let f : $(X, \tau) \to (Y, \sigma)$ be a function. Let U be an open (resp. closed) set in (Y, σ). Since f is continuous and contra-p*gα-continuous (respectively contra-continuous and p*gα-continuous), $f^{-1}(U)$ is open and p*gα-closed in (X, τ). Hence by **theorem 2.5,** $f^{-1}(U)$ is regular open in (X, τ). This shows that f is a super-continuous function.

Theorem 3.28. Let f : $(X, \tau) \to (Y, \sigma)$ be a function and X a pT_s space. Then the following are equivalent.

1. f is contra-p*gα-continuous.

2. f is contra-continuous

Proof:

(1) $\Rightarrow$ (2).

Let U be an open set in (Y, σ). Since f is contra- p*gα-continuous, $f^{-1}(U)$ is p*gα-closed in (X, τ) and since X is p-Ts space, $f^{-1}(U)$ is closed in (X, τ). Hence f is contra-continuous.

(2) $\Rightarrow$ (1)

Let U be an open set in (Y, σ). Since f is contra-continuous, $f^{-1}(U)$ is closed in (X, τ). Hence by theorem 2.6, $f^{-1}(U)$ is p*gα-closed in (X, τ). Hence f is contra p*gα-continuous.

4. Contra-p*gα-**closed and strongly** p*gα-**closed**

Definition 4.1. The graph G(f) of a function f: (X, τ) $\rightarrow$ (Y, σ) is said to be contra-p*gα-closed in $X \times Y$ if for each (x, y) $\in$ ($X \times Y$) – G(f) there exist U $\in$ p*gα O(X, x) and V $\in$ C(Y, y) such that ($U \times V$) $\cap$G(f) = ϕ.

Lemma 4.2. The graph G(f) of a function f : (X, τ) $\rightarrow$ (Y, σ) is contra-p*gα-closed if and only if for each (x, y) $\in$ ($X \times Y$) – G(f), there exists U$\in$ p*gα O(X, x) and V $\in$ C(Y, y) such that f(U) $\cap$ V =ϕ.

Theorem 4.3. If f : (X, τ) $\rightarrow$ (Y, σ) is contra-p*gα-continuous and Y is Urysohn then G(f) is contra-p*gα-closed in $X \times Y$.

Proof: Let (x, y) $\in$ $X \times Y$ – G(f).Then y $\neq$ f(x) and there exist open sets A,B such that f(x)$\in$ A, y $\in$ B and cl(A) $\cap$ cl(B) = ϕ. Since f is contra-p*gα-continuous and by theorem 3.12 there exists U$\in$p*gαO(X, x) such that f(U) $\subseteq$A.Hence f(U) $\cap$ cl(B) = ϕ.Thus by lemma 4.2, G(f) is contra p*gα-closed in $X \times Y$.

Definition 4.4. A topological space (X,τ) is said to be

1. Strongly S-closed if every closed cover of X has a finite subcover.
2. S-closed if every regular closed cover of X has a finite subcover.
3. Strongly compact if every preopen cover of X has a finite subcover.
4. Locally indiscrete if every open set of X is closed in X
5. Midly Hausdorff if the δ-closed sets form a network for its topology τ,where a δ-closed set is the intersection of regular closed sets.
5. Ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets
6. Nearly compact if every regular open cover of X has a finite subcover.
7. p*gα-compact if every p*gα-open cover of X has a finite subcover.
8. p*gα-connected if X cannot be written as the disjoint union of two non-empty p*gα-open sets.

Definition 4.5. A topological space (X,τ) is said to be strongly p*gα-closed if every p*gα-closed cover of X has a finite sub cover.

Example 4.6. A p-T_s strongly S-closed space is strongly p*gα-closed.

Theorem 4.7. Let (X, τ) be p-Ts space. If f : (X, τ) $\rightarrow$ (Y, σ) has a contra-p*gα-closed graph, then the inverse image of a strongly S-closed set K of Y is closed in (X, τ).
Proof: Let K be a strongly S-closed set of Y and $x \in f^{-1}(K)$. For each $k \in K$, $(x, k) \notin G(f)$. By Lemma 4.2, there exist $U_k \in p^*g\alpha O(X, x)$ and $V_k \in C(Y, k)$ such that $f(U_k) \cap V_k = \phi$. Since $\{K \cap V_k \,/\, k \in K\}$ is a closed cover of the subspace K, there exists a finite subset $K_0 \subset K$ such that $K \subset \cup\{V_k / k \in K_0\}$. Set $U = \cap \{U_k \,/\, k \in K_0\}$. Then U is open, since X is a P-Ts space. Therefore, $f(U) \cap K = \phi$ and $U \cap f^{-1}(K) = \phi$. This shows that $f^{-1}(K)$ is closed in (X, τ).

Theorem 4.8. If a space (X,τ) is strongly p*gα-closed then the space is strongly S-closed.
Proof: We get the result from the definitions of 4.4 and 4.5 and theorem 2.6.

Theorem 4.9. Let (X,τ) be p*gα-connected and (Y, σ) be a T_1-space. If f : (X,τ) $\rightarrow$ (Y,σ) is contra-p*gα-continuous then f is constant.
Proof: Since (Y, σ) is a T_1space, $\eta = \{ f^{-1} (y)/y \in Y\}$ is a disjoint p*gα – open partition of X.
If $|\eta| >=2$, then X is the union of two non-empty p*gα -open sets. Since (X,τ) is p*gα – connected,
$|\eta| =1$. Hence f is constant.

Theorem 4.10. Let f : (X, ,τ) ->(Y, σ) be a contra-p*gα-continuous and pre-closed surjection. If (X, τ) is a P-Ts, then (X,,τ) is a locally indiscrete space.
Proof: Let U be any open set in (Y, σ). Since f is contra p*gα-continuous and (X,,τ) is a P-Ts space, f^{-1} (U) is closed in (X,τ). Since f is a pre-closed surjection, then U is pre-closed in (Y, σ). Therefore cl(U) = cl (Int(U)) $\subset$ U. Hence U is closed in (Y, σ). Thus (Y, σ) is a locally indiscrete space.

Theorem 4.11. If every closed subset of a space X is p*gα-open then the following are equivalent.
1. X is S-closed
2. X is strongly S-closed

Proof:
(1) $\Rightarrow$ (2)
Let $\{A_\alpha \,/\, \alpha \in I\}$ be a closed cover of X. Then by hypothesis and by **theorem 2.5,** $\{A_\alpha \,/\, \alpha \in I\}$ is a regular closed cover of X. Since X is S-closed, then we have a finite sub cover of X. Hence X is strongly S-closed.

(2) $\Rightarrow$ (1)
Let $\{A_\alpha \,/\, \alpha \in I\}$ be a regular closed cover of X. Since every regular closed is closed and X is strongly S-closed, then we have a finite subcover of X. Hence X is S-closed.

Definition 4.12. A topological space (X, τ) is said to be
1. P-Hausdorff if for each pair of distinct points x and y in X there exist disjoint p*gα open sets A and B of x and y respectively.

2. P-Ultra Hausdorff if for each pair of distinct points x and y in X there exist disjoint P-clopen sets A and B of x and y respectively.

Theorem 4.13. If $f : (X, \tau) \to (Y, \sigma)$ is contra-p*gα-continuous injection, where Y is Urysohn then the topological space (X, τ) is a P-Hausdorff.

Proof: Let x_1 and x_2 be two distinct points of (X, τ). Suppose $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is injective and $x_1 \neq x_2$ then $y_1 \neq y_2$. Since the space Y is Urysohn, there exist open sets A and B such that $y_1 \in A, y_2 \in B$ and $cl(A) \cap cl(B) = \phi$. Since f is contra-p*gα-continuous and by **theorem 3.12**, there exist p*gα-open sets $Ux_1 \in p^*g\alpha O(X, x_1)$ and $Ux_2 \in p^*g\alpha O(X, x_2)$ such that $f(Ux_1) \subset cl(A)$ and $f(Ux_2) \subset cl(B)$. Thus we have $Ux_1 \cap Ux_2 = \phi$, since $cl(A) \cap cl(B) = \phi$. Hence X is a P-Hausdorff.

Theorem 4.14. If f: $(X, \tau) \to (Y, \sigma)$ is a contra p*gα-continuous injection, where Y is P-ultra Hausdorff then the topological space (X, τ) is P-Hausdorff

Proof: Let x_1 and x_2 be two distinct points of (X, τ). Since f is injection and Y is P-ultra Hausdorff, then $f(x_1) \neq f(x_2)$ and also there exist clopen sets U and V in Y such that $f(x_1) \in U$ and $f(x_2) \in V$, where $U \cap V = \phi$.

Since f is contra-p*gα-continuous, x_1 and x_2 belong top*gα-open sets $f^{-1}(U)$ and $f^{-1}(V)$ respectively, where $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence X is P-Hausdorff.

Lemma 4.15. Every mildly Hausdorff strongly S-closed space is locally indiscrete.

Theorem 4.16. If a function $f : (X, \tau) \to (Y, \sigma)$ is continuous and (X, τ) is a locally indiscrete space, then f is contra-p*gα-continuous.

Proof: Let U be any open set in (Y, σ).Since f is continuous, $f^{-1}(U)$is open in (X, τ) and since (X, τ) is locally indiscrete, $f^{-1}(U)$ closed in (X, τ).

Hence by **theorem 2.6,** $f^{-1}(U)$ is p*gα -closed in (X, τ). Thus f is contra- p*gα–continuous.

Corollary 4.17. If a function f: $(X,\tau) \to (Y,\sigma)$ is a continuous and (X,τ) is mildly Hausdorff strongly S-closed space then f is contra- p*gα **–**continuous.

Proof: It follows from **Lemma 4.15** and **theorem 4.16**.

Theorem 4.18. A contra -p*gα-continuous image of a p*gα-connected space is connected.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be a contra-p*gα-continuous function of p*gα-connected space onto a topological space Y. If possible, assume that Y is not connected. Then $Y = A \cup B$, $A \neq \phi$, $B \neq \phi$ and $A \cap B = \phi$, where A and B are clopen sets in Y. Since f is contra-p*gα-continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty p*gα-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \phi$.

Hence X is not p*gα-connected, which is a contradiction. Therefore Y is connected.

Definition 4.19. A topological space (X, τ) is said to be P-normal if each pair of non-empty disjoint closed sets can be separated by disjoint p*gα-open sets.

Theorem 4.20. If f : $(X, \tau) \to (Y, \sigma)$ is a closed contra-p*gα-continuous injection and Y is ultra-normal, then X is P-normal.

Proof: Let A_1 and A_2 be non-empty disjoint closed subsets of X. Since f is closed and injective, then $f(A_1)$ and $f(A_2)$ are non-empty disjoint closed subsets of Y. Since Y is ultra-normal, then $f(A_1)$ and $f(A_2)$ can be separated by disjoint clopen sets B_1 and B_2 respectively.

Hence, $V_1 \subset f^{-1}(B_1)$ and $V_2 \subset f^{-1}(B_1)$. Since f is contra-p*gα-continuous, then $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are p*gα-open subsets of X and $f^{-1}(B_1) \cap f^{-1}(B_2) = \phi$. Hence X is P-normal.

Theorem 4.21. The image of a strongly p*gα-closed space under a contra-p*gα continuous surjective function is compact.

Proof: Suppose that f: $(X, \tau) \to (Y, \sigma)$ is a contra- p*gα-continuous surjection. Let$\{V_\alpha / \alpha \in I\}$ be any open cover of Y. Since f is contra- p*gα-continuous, then $\{f^{-1}(V_\alpha)/ \alpha \in I\}$ is a p*gα-closed cover of X. Since X is strongly p*gα -closed, then there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) / \alpha \in I_0\}$. Thus we have $Y = \cup\{V\alpha / \alpha \in I_0\}$. Hence Y is compact.

Theorem 4.22. Every strongly p*gα -closed space (X, τ) is a compact S-closed space.

Proof: Let $\{V_\alpha / \alpha \in I\}$ be a cover of X such that for every $\alpha \in I$, V_α is open and regular closed by assumption. Then by **theorem 2.5,** each V_α is p*gα -closed in X. Since X is strongly p*gα-closed, there exists a finite subset I_0 of I such that $X = \cup\{V_\alpha / \alpha \in I_0\}$. Hence (X, τ) is a compact S-closed space.

Theorem 4.23. The image of a p*gα -compact space under a contra-p*gα -continuous surjective function is strongly S-closed.

Proof: Suppose that f : $(X, \tau) \to (Y, \sigma)$ is a contra-p*gα -continuous surjection. Let $\{V_\alpha /\alpha \in I\}$ be any closed cover of Y. Since f is contra-p*gα -continuous, then $\{f^{-1}(V_\alpha) /\alpha \in I\}$ is a p*gα -open cover of X. Since X is p*gα -compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) / \alpha \in I_0\}$. Thus we have $Y = \cup\{V_\alpha / \alpha \in I_0\}$. Hence Y is strongly S-closed.

Theorem 4.24. The image of a p*gα -compact space in any P -Ts space under a contra-p*gα -continuous surjective function is strongly p*gα -closed.

Proof: Suppose that f : $(X, \tau) \to (Y, \sigma)$ is a contra-p*gα -continuous surjection. Let $\{V_\alpha / \alpha \in I\}$ be any p*gα -closed cover of Y. Since Y is P-Ts space, then $\{V_\alpha / \alpha \in I\}$ is a closed cover of Y. Since f is contra-p*gα -continuous, then $\{f^{-1}(V_\alpha) / \alpha \in I\}$ is a p*gα -open cover of X. Since X is p*gα -compact, there exists a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) / \alpha \in I_0\}$. Thus we have $Y = \cup\{V_\alpha / \alpha \in I_0\}$. Hence Y is strongly p*gα -closed.

Theorem 4.25. The image of strongly p*gα -closed space under a p*gα-irresolute surjective function is strongly p*gα-closed.

Proof: Suppose that f : $(X, \tau) \to (Y,\sigma)$ is an p*gα irresolute surjection. Let $\{V_\alpha / \alpha \in I\}$ be any p*gα -closed cover of Y. Since f is p*gα–irresolute then $\{f^{-1}(V_\alpha) / \alpha \in I\}$ is a p*gα closed cover of X. Since X is strongly p*gα–closed then there exist a finite subset

I_0f I such that X=$\cup$ {f $^{-1}(V_\alpha)$ / $\alpha\in I_0$}. Thus we have Y = $\cup\{V_\alpha$ / $\alpha\in I_0\}$. Hence Y is strongly p*gα -closed.

Lemma 4.26. The product of two p*gα-open sets is p*gα-open

Theorem 4.27. Let f : $(X_1, \tau) \to (Y, \sigma)$ and g : $(X_2, \tau) \to (Y, \sigma)$ be two functions where Y is a Urysohn space and f and g are contra-p*gα -continuous function. Then $\{(x_1,x_2)$ $/f(x_1) = g(x_2)\}$ is p*gα -closed in the product space $X_1\times X_2$.

Proof: Let V denote the set $\{(x_1,x_2) / f(x_1) = g(x_2)\}$. In order to show that V is p*gα-closed, we show that $(X_1\times X_2) - V$ is p*gα-open. Let $(x_1,x_2) \notin V$. Then $f(x_1) \neq g(x_2)$. Since Y is Urysohn, there exist open sets U_1 and U_2 of $f(x_1)$ and $g(x_2)$ such that $cl(U_1)\cap cl(U_2) = \phi$. Since f and g are contra-p*gα-continuous, $f^{-1}(cl(U_1))$ and $g^{-1}(cl(U_2))$ are p*gα-open sets containing x_1 and x_2 in X_1 and X_2. Hence by **Lemma 4.26,** $f^{-1}(cl(U_1))$ $\times\, g^{-1}(cl(U_2))$ is p*gα-open.

Further $(x_1,x_2)\in f^{-1}(cl(U_1)) \times g^{-1}(cl(U_2)) \subset ((X_1\times X_2) - V)$. If follows that $(X_1\times X_2) -$ V is p*gα-open. Thus V is p*gα-closed in the product space $X_1\times X_2$.

Corollary 4.28. If f : $(X, \tau) \to (Y, \sigma)$ is contra-p*gα-continuous and Y is a Urysohn space, then $V = \{(x_1, x_2) / f(x_1) = f(x_2)\}$ is p*gα-closed in the product space $X_1\times X_2$.

Theorem 4.29. Let f : $(X, \tau) \to (Y, \sigma)$ be a continuous function. Then f is RC-continuous if and only if it is contra-p*gα-continuous.

Proof: Suppose that f is RC-continuous. Since every RC-continuous function is contra-continuous. Therefore by **Theorem 3.5,** f is contra p*gα continuous.

Conversely, Let V be any open set in (Y, σ). Since f is continuous and contra-p*gα-continuous, $f^{-1}(V)$ is open and p*gα-closed in (X, τ). By **theorem 2.5,** $f^{-1}(V)$ is regular open in (X, τ).

That is, $Int(cl(f^{-1}(V))) = f^{-1}(V)$. Since $f^{-1}(V)$ is open, $Int(cl(f^{-1}(V))) = Int(f^{-1}(V))$ and so $cl(Int(f^{-1}(V))) = f^{-1}(V)$. Therefore V is regular closed in (X, τ). Hence f is RC-continuous.

Theorem 4.30. Let f: $(X, \tau) \to (Y, \sigma)$ be perfectly p*gα-continuous function, X be locally indiscrete space and connected. Then Y has an indiscrete topology.

Proof: Suppose that there exists a proper open set U of Y. Since Y is locally indiscrete, U is a closed set of Y.

Therefore by **theorem 2.6,** U is a p*gα-closed set of Y. Since f is perfectly p*gα-continuous, $f^{-1}(U)$ is a proper clopen set of X. This shows that X is not connected, which is a contradiction. Therefore Y has an indiscrete topology.

Theorem 4.31. If f : $(X, \tau) \to (Y, \sigma)$ is a function and (X, τ) a P-Ts space, then the following statements are equivalent:

1. f is perfectly continuous.
2. f is continuous and contra-continuous
3. f is continuous and contra-p*gα-continuous.
4. f is super-continuous.

Proof:
(1)$\Rightarrow$ (2) is obvious.
(2)$\Rightarrow$ (3) by **theorem 2.6 ,** it is clear.
(3)$\Rightarrow$ (4) by **theorem 3.27,** it is clear
(4)$\Rightarrow$ (1)
Let U be any open set in (Y, σ). By assumption, $f^{-1}(U)$ is regular open in (X,τ). By **theorem 2.5,** $f^{-1}(U)$ is open and p*gα-closed in (X, τ). Since (X, τ) is a P-Ts space, $f^{-1}(U)$ is clopen in (X, τ). Hence f is perfectly continuous.

Theorem 4.32. Let f : (X, τ) $\rightarrow$ (Y, σ) be a contra -p*gα-continuous function. Let A be an open p*gα-closed subset of X and let B be an open subset of Y. Assume that p*gαC (X, τ)(the class of all p*gα –closed sets of (X,τ) be p*gα-closed under finite intersections. Then, the restriction f|A:(A, τ_A) –>(B, σ_B). Is a contra p*gα-continuous function.
Proof: Let V be an open set in (B, σ_B). Then V=B$\cap$K for some open set K in (Y, σ). Since B is an open set of Y, V is an open set in (Y, σ). By hypothesis and assumption, $f^{-1}(V)\cap A=H_1$(say) is a p*gα-closed set in (X, τ). Since (f|A)-1 (V)= (H_1), it is sufficient to show that H1 is a p*gα-closed set in (A, τ_A). Let $G_{1 \text{ be}}$ p*gα-open in (A, τ_A) such (H_1) $\subseteq$ (G_1). then by hypothesis and **theorem 4.21.** $G_1$is p*gα-open in (X, τ).
Since H1 is a p*gα-closed set in (X, τ),we have $pcl_X(H_1) \subseteq Int(G_1)$. Since A is open $pcl_A(H_1) = pcl_X(H_1) \cap A \subseteq Int(G_1) \cap Int(A) = Int(G_1 \cap A) \subseteq Int(G_1)$ and so $H_1 = (f|A)^{-1}(V)$ is a p*gα-closed set in (A, τ_A). Hence f | A is contra-p*gα-continuous function.

Theorem 4.33. A topological space (X, τ) is nearly compact if and only if it is compact and strongly p*gα-closed.

Theorem 4.34. A topological space (X, τ) is S-closed if and only if it is strongly S-closed and p*gα-compact.

Theorem 4.35. If a topological space (X, τ) is locally indiscrete space then compactness and strongly p*gα-closedness are the same.
Proof: Let (X, τ) be a compact space. Since (X, τ) is a locally indiscrete space, then every open set is closed and by **theorem 2.6,** compactness and strongly p*gα-compactness are the same in a locally indiscrete topological space.

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