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Triple Connected Edge Domination Number of Graphs and its Relation with other Dominations

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Abstract. In this paper, we define a triple connected edge dominating set and a triple connected edge domination number of graphs. We obtain some bounds on triple connected edge domination number and also exact values for some standard graphs.

Keywords: Edge domination number, connected edge domination number, triple connected edge dominating set and triple connected edge domination number.

AMS mathematics subject classification (2010): 05C72

1. Introduction

The study of dominating sets in graph theory began around 1960. In 1958, domination was formalized as a theoretical area in graph theory by Berge. He referred to the domination number as a coefficient of external stability. In 1962, Ore was the first to use the term dominating set and domination number for undirected graphs. In 1977 Cockayne and Hedetniemi made an interesting and extensive survey of the results about dominating sets in graphs. Hedetniemi and Laskar [10] published their bibliography on domination in graphs and some basic definitions of domination parameters. The publication of the first large two volume text books on domination, fundamentals of domination in graphs and domination in graphs advanced topics were edited by Haynes, Hedetniemi and Slater [11] and [12]. The concept of edge domination number was introduced by Gupta [3] and Mitchell and Hedetniemi [6]. Sampathkumar and Walikar [9] established the new concept of domination called the connected domination number of a graph. The connected edge domination in graphs was introduced by Arumugam and Velammal [2]. Recently the concept of triple connected graphs has been introduced by Joseph et.al. [8]. They have studied the properties of triple connected graphs and established many results on them. The concept of triple connected domination number of a graph has been introduced by Mahadevan et.al., [4]. Paulson and Lilly [5] highlighted applications of domination in graphs in several fields and the importance of graph theoretical ideas in various areas of science and engineering. This motivated us to work on this paper. We introduce the concept of triple connected edge domination number of graphs and we analyze some bounds on it. Further we develop its relation with other different dominations. Here we consider a finite, simple connected and undirected graph $G(V,E)$, where $m = |V|$ and $n = |E|$ denote the number of vertices and number of edges of a graph G respectively.

2. Preliminaries

Definition 2.1. [10] A set $D \subseteq V$ of vertices in a graph $G = (V,E)$ is called a dominating set if every vertex $v \in V$ is either any element of D or is adjacent to an element of D. The minimum cardinality of a dominating set of G is called a domination number and is denoted by $\gamma(G)$.

Definition 2.2. [9] A dominating set $D \subseteq V$ is said to be a connected dominating set of G if the sub graph $< D >$ is connected. The minimum of the cardinalities of the connected dominating sets of G is called the connected domination number of G and is denoted by $\gamma_c(G)$.

Definition 2.3. [1] A subset $D'$of E is called an edge dominating set of G if every edge not in D' is adjacent to some edge in D'. The edge domination number is the minimum cardinality taken over all edge dominating sets of G and is denoted by $\gamma'(G)$.

Definition 2.4. [2] An edge dominating set D' is called a connected edge dominating set, if the sub graph $< D' >$ is connected. The minimum cardinality of a connected edge dominating set of G is called a connected edge domination number and is denoted by $\gamma_c'(G)$.

Definition 2.5. [4] A subset $D$of V of a nontrivial connected graph G is said to be triple connected dominating set, if D is a dominating set and the sub graph $< D >$ is triple connected. The minimum cardinality taken over all triple connected dominating sets of G is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$.

3. Main results

3.1. Triple connected edge domination

Definition 3.1. An edge dominating set $D' \subseteq V(L(G))$ is a triple connected edge dominating set of G, if the sub graph $< D' >$ is triple connected. The triple connected edge domination number of G is the minimum cardinality taken over all triple connected edge dominating sets of G and is denoted by $\gamma_{tc}'(G)$.

Example 3.2.

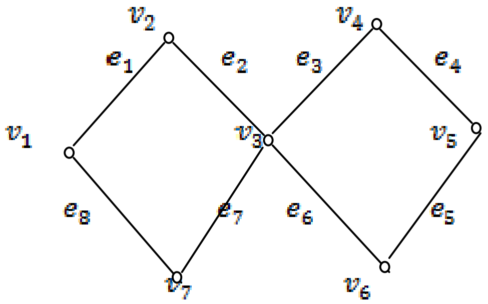

Figure 1: Graph G

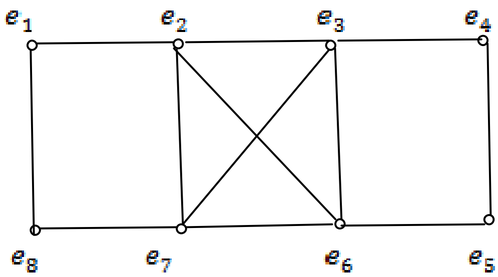

Figure 2: $L(G)$ - Edge graph of G.

Theorem 3.3. For any connected graph $L(G)$ with $n \geq 3$, $\gamma'(G) \leq \gamma_c'(G) \leq \gamma_{tc}'(G)$.

Proof: Let $L(G)$ be a graph with no isolated vertices and let D' be a minimum dominating set of $L(G)$.

Since for every graph $L(G)$, a triple connected dominating set is also a connected dominating set.

Therefore, $\gamma_c'(G) \leq \gamma_{tc}'(G)$.

It is clear that, every connected dominating set is also a dominating set.

Hence, $\gamma'(G) \leq \gamma_c'(G) \leq \gamma_{tc}'(G)$.

Theorem 3.4. Let $L(G)$ be a line graph with $n \geq 9$ vertices and maximum degree $\Delta'(G)$. Then

$$\left\lceil \frac{n}{\Delta'(G)+1} \right\rceil \leq \gamma'(G) \leq \gamma_c'(G) \leq \gamma_{tc}'(G) \leq n - \Delta'(G) + 2.$$

Proof: First we consider the lower bound. Each vertex in graph $L(G)$ can dominate at most $\Delta'(G)$ vertices and itself. Hence $\left\lceil \frac{n}{\Delta'(G)+1} \right\rceil \leq \gamma'(G)$.

By theorem 3.3, $\gamma'(G) \leq \gamma_c'(G) \leq \gamma_{tc}'(G)$

For the upper bound, let v be a vertex with maximum degree $\Delta'(G)$ in $L(G)$. Form a spanning tree $L(T)$ of $L(G)$ such that every neighbour of v in $L(G)$ is also a neighbour of v in $L(T)$. This will result in a tree $L(T)$ with $N(v)$ branches in it and hence with at least $\Delta'(G)$ leaves. Hence the triple connected domination number is at most $n - \Delta'(G) + 2$.

Theorem 3.5. Let $L(G)$ be a line graph with $n \geq 3$. Then $\gamma'(G) = \gamma_c'(G) = 1$ and $\gamma_{tc}'(G) = 3$ if $G = K_3, K_{1,n-1}$.

Proof: If $G = K_3$, then $L(K_3) = K_3$, we get $\gamma'(G) = \gamma_c'(G) = 1$ and $\gamma_{tc}'(G) = 3$. If $G = K_{1,n-1}$, then all the edges of $K_{1,n-1}$ are incident to a common vertex.
All the vertices are adjacent to each other vertex in the line graph of $K_{1,n-1}$ and thus $L(K_{1,n-1}) = K_{n-1}$. Therefore $\gamma'(G) = \gamma_c'(G) = 1$ and $\gamma_{tc}'(G) = 3$.

Theorem 3.6. For any path P_m with $m \geq 6$, $\gamma_{tc}'(P_m) = m - 3$.

Proof: If $G = P_m$, then all the edges of P_m are incident to corresponding two vertices. The line graph of P_m is P_{m-1}. For obtaining a triple connected domination for a path with $m - 1$ vertices ,we must select atleast $m - 3$ vertices by omitting the end points of p_{m-1}.
Therefore, $\gamma_{tc}'(P_m) = m - 3$.

Theorem 3.7. For any cycle c_m with $m \geq 5$,$\gamma_{tc}'(c_m) = m - 2$.

Proof: If $G = c_m$, then $L(c_m) = c_m$. Since the number of vertices and edges are equal, select at least $m - 2$ vertices to obtain a triple connected domination for a cycle with m vertices.
Therefore, $\gamma_{tc}'(c_m) = m - 2$.

Theorem 3.8. For any $G = F_p$ where $p \geq 3$,
$\gamma'(G) = \gamma_c'(G) = \gamma_{tc}'(G) = n - \Delta'(G)$.

Proof: If $G = F_p$, then all the vertices of F_p are adjacent to a common vertex. Therefore maximum degree of $L(F_p)$ is same as the maximum degree of F_p .
Let $A \subseteq V(L(G))$ be the set of all vertices have the maximum degree $\Delta'(G)$.
Each vertex in the subset A can dominate at most $\Delta'(G)$ vertices and itself.
There are $n - \Delta'(G) - 1$ non adjacent and non dominate vertices.
Select those $n - \Delta'(G) - 1$ vertices and a vertex which has the maximum degree to form a minimal dominating set.
Clearly, $n - \Delta'(G)$ vertices form a minimal triple connected dominating set of $L(G)$.

Therefore, $\gamma'(G) = \gamma_c'(G) = \gamma_{tc}'(G) = n - \Delta'(G)$ for any F_p with $p \geq 3$.

Theorem 3.9. For any connected graph $L(G)$ with $n \geq 3$, $\gamma'(G) \leq \gamma_c'(G) \leq \gamma_{tc}'(G) \leq 3\gamma'(G)$.

Proof: Let D' be the line dominating set and $|D'| = \gamma'(G)$ be the line domination number .Let $\omega'(G)$ be the number of components of $< D' >$. It is clear that $\gamma'(G) \geq \omega'(G)$.

We shall show that there exists two components (say c_i' and c_j' where $i \neq j$) of $< D' >$ such that the length of the shortest path between c_i' and c_j' is at most 3 in G. Suppose for the purpose of contradiction, the shortest path between any pair of disjoint components have length at most 4. Let P' be the shortest of all the shortest paths between any two distinct components of $< D' >$. Then there exists a vertex v in the path such that v is at a distance of at least two from the end points of P'.

Since D' is a line dominating set. The vertex v must be at a distance of at most one from a component. This gives us that v lies on a path P'' between two components such that P'' is shorter than P', thus contradicting the assumption on P'.

Thus there exists two components c_i' and c_j' with a path of length at most 3. Adding the vertices in the path to D' decreases the number of components in $< D' >$ by one.

This procedure can be repeated until there is only one component in D'. This resulting in a triple connected line dominating set. Note that, at most $2(\omega'(G))$ vertices are added to D' to from a triple connected line dominating number.

$$\begin{aligned} \gamma_{tc}'(G) &\leq |D'| + 2(\omega'(G)) \\ &\leq \gamma'(G) + 2(\gamma'(G)) \\ &\leq 3\gamma'(G). \end{aligned}$$

Theorem 3.10. If $G = k_m, k_{m,m}$, $m \geq 2$ then $L(G)$ is a regular graph.

Proof: Let $G = (V, E)$ be a complete graph k_m or a complete bipartite graph $k_{m,m}$ where $m \geq 2$.

Let $e = uv$ be an edge of G. All the edges of G are adjacent to $2d(u) - 2$ edges of G for all u in V. In $L(G)$, degree of all the vertices are equal to $2d(u) - 2$. Therefore degree of all the vertices of $L(G)$ are equal. Hence $L(G)$ is a regular graph.

4. Conclusion

In this paper, we have discussed about triple connected domination number over line graphs. This concept can further be generalized for fuzzy graphs in our future work.

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