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# ADCSS-Labeling for Some Middle Graphs

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***Abstract.*** A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The graph for which every edge label is the absolute difference of the sum of the cubes of the end vertices and the sum of the squares of the end vertices is called adcss labeling. It is also observed that the weights of the edges are found to be multiples of 2. Here we characterize middle graphs of paths, cycles, stars, bistars, centipede graphs, comb graphs for adcss labeling.

***Keywords:*** Graph labeling, sum square graph, square sum graphs, cubic graphs, middle graphs.

***AMS Mathematics Subject Classification (2010):*** 05C72

## 1. Introduction

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.
A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1-7]. Some basic concepts are taken from Frank Harary [3]. We introduced the new concept, an absolute difference of cubic and square sum labeling of a graph in [8]. In [8-15] it is shown that planar grid, web graph, kayak paddle graph, snake graphs, friendship graph, armed crown, fan graph, cycle graphs, wheel graph, 2 tuple graph of some graphs and total graph of some graphs have an adcss labeling. In this paper we investigated ADCSS labeling of some middle graphs.

## 2. Preliminaries

**Definition 1. [5]** Let G = (V(G), E(G)) be a graph. A graph G is said to be absolute difference of the sum of the cubes of the vertices and the sum of the squares of the vertices, if there exist a bijection f : V(G) → {1,2, …, p} such that the induced function $f^*_{adcss}$: E(G) → multiples of 2 is given by $f^*_{adcss}(uv)$ = $|\{f(u)^3 + f(v)^3\} - (f(u)^2 + f(v)^2)|$ is injective.

**Definition 2.** A graph in which every edge associates distinct values with multiples of 2 is called the sum of the cubes of the vertices and the sum of the squares of the vertices. Such a labeling is called an absolute difference of cubic and square sum labeling or an absolute difference css-labeling.

## 3. Main results

**Definition 1.** Let V(G) and X(G) denote the vertex set and the edge set of G, respectively. The middle graph M(G) of G whose vertex set is V(G) union X(G) where two vertices are adjacent if and only if

(i) They are adjacent edges of G or
(ii) One is a vertex and other is an edge incident with it.

**Theorem 2.** The middle graph $M(P_n)$ of a path $P_n$ admits ADCSS - labeling.
**Proof :** Let $G = M(P_n)$ and let $v_1, v_2, \ldots, v_{2n-1}$ are the vertices of G.
Here $|V(G)| = 2n-1$ and $|E(G)| = 3n-4$.
Define a function $f : V \rightarrow \{1,2,3,\ldots, 2n-1\}$ by
$f(v_i) = i$ , $i = 1,2,\ldots,2n-1$.
For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) = (i+1)^2 i + i^2(i-1)$, $\quad i = 1,2,3, \ldots, 2n-2$
$f^*_{adcss}(v_{2i}\, v_{2i+2}) = (2i+2)^2(2i+1) + (2i)^2(2i-1)$ , $\quad i = 1,2,3, \ldots, n-2$.

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $M(P_n)$ admits adcss-labeling.

**Example 3.** The following illustration shows, the middle graph of path $P_5$ admits adcss labeling.

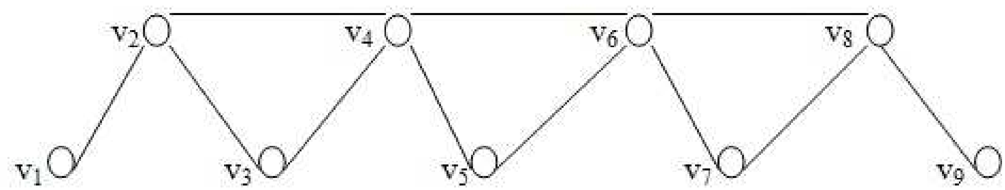


**Figure 1:**

**Theorem 4.** The middle graph $M(C_n)$ of a cycle $C_n$ admits ADCSS - labeling.
**Proof:** Let $G = M(C_n)$ and let $v_1, v_2, \ldots, v_{2n}$ are the vertices of G.
Here $|V(G)| = 2n$ and $|E(G)| = 3n$.
Define a function $f : V \rightarrow \{1,2,3, \ldots, 2n\}$ by
$f(v_i) = i$ , $i = 1,2, \ldots, 2n$.
For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\, v_{i+1}) = (i+1)^2 i + i^2(i-1)$, $\quad i = 1,2,3, \ldots, 2n-1$
$f^*_{adcss}(v_{2i}\, v_{2i+2}) = (2i+2)^2(2i+1) + (2i)^2(2i-1)$ , $\quad i = 1,2,3, \ldots, n-1$.
$f^*_{adcss}(v_2\, v_{2n}) = (2n)^2(2n-1) + 4$
$f^*_{adcss}(v_1\, v_{2n}) = (2n)^2(2n-1)$

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $M(C_n)$ admits adcss-labeling.

**Example 5.** The following illustration shows, the middle graph of cycle $C_3$ admits adcss labeling.

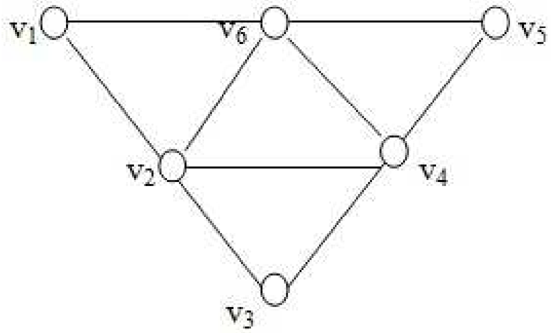


**Figure 2:**

**Theorem 6.** The middle graph $M(K_{1,n})$ of star graph $K_{1,n}$ admits ADCSS - labeling.
**Proof:** Let G = $M(K_{1,n})$ and let $v_1, v_2, \ldots, v_{2n+1}$ are the vertices of G.
Here $|V(G)| = 2n+1$ and $|E(G)| = 2n + \frac{(n-1)(n)}{2}$.
Define a function $f : V \to \{1,2,3, \ldots, 2n+1\}$ by
$f(v_i) = i$, $i = 1,2,\ldots, 2n+1$.
For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$f^*_{adcss}(v_{2n+1}\ v_i) = i^2(i-1) + (2n+1)^2(2n), \quad i = 1,2, \ldots, n$$
$$f^*_{adcss}(v_i\ v_{i+n}) = (i+n)^2(i+n-1) + (i)^2(i-1), \quad i = 1,2, \ldots, n$$
$$f^*_{adcss}(v_j\ v_{i+j}) = (j)^2(j-1) + (i+j)^2(i+j-1), \quad j = 1,2,\ldots, n-1,\ i = 1,2,\ldots,n-j$$

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence $M(K_{1,n})$ admits adcss-labeling.

**Example 7.** The following illustration shows, the middle graph of star $K_{1,4}$ admits adcss labeling.

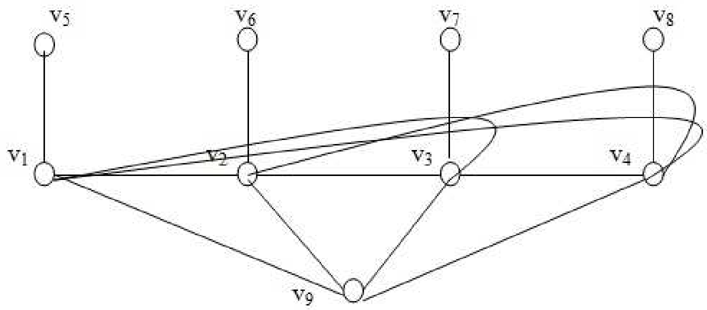


**Figure 3:**

**Definition 8.** The graph obtained from $K_{1,n}$ and $K_{1,m}$ by joining their centers with an edge is called a Bistar. It is denoted by B(m,n).

**Theorem 9.** The middle graph M{B(m,n)} of Bistar B(m,n) admits ADCSS - labeling.
**Proof:** Let G = M{B(m,n)} and let $v_1, v_2, \ldots, v_{2m+2n+3}$ are the vertices of G.
Here, $|V(G)| = 2m+2n+3$ and $|E(G)| = 2(m+n+1) + \frac{n(n+1)}{2} + \frac{m(m+1)}{2}$.
Define a function $f : V \to \{1,2,3, \ldots, 2m+2n+3\}$ by
$f(v_i) = i$, $i = 1,2, \ldots, 2m+2n+3$.
For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$f^*_{adcss}(v_1\, v_{m+n+2+i}) = (m+n+2+i)^2(m+n+1+i), \quad i = 1,2,3, \ldots, m+1.$$
$$f^*_{adcss}(v_{m+2}\, v_{2m+n+2+i}) = (m+2)^2(m+1) + (2m+n+2+i)^2(2m+n+1+i), \quad i = 1,2,3, \ldots, n+1.$$
$$f^*_{adcss}(v_{i+1}\, v_{m+n+2+i}) = (i+1)^2(i) + (m+n+2+i)^2(m+n+1+i), \quad i = 1,2, \ldots, m$$
$$f^*_{adcss}(v_{m+2+i}\, v_{2m+n+3+i}) = (m+2+i)^2(m+1+i) + 2m+n+3+i)^2(2m+n+2+i), \quad i = 1,2, \ldots, n$$
$$f^*_{adcss}(v_{m+n+3+j}\, v_{m+n+3+j+i}) = (m+n+3+j)^2(m+n+2+j) + (m+n+3+j+i)^2(m+n+2+j+i), \quad j = 0,1,2, \ldots, m-1, \; i = 1,2,3, \ldots, m-j$$
$$f^*_{adcss}(v_{2m+n+3+j}\, v_{2m+n+3+j+i}) = (2m+n+3+j)^2(2m+n+2+j) + (2m+n+3+j+i)^2(2m+n+2+j+i), \quad j = 0,1,2, \ldots, n-1, \; i = 1,2,3, \ldots, n-j$$

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence M{B(m,n)} admits adcss-labeling.

**Definition 10.** The (n,2)-centipede tree, $C_{n,2}$, is the graph with $V(C_{n,2}) = \{v_1, v_2, \ldots, v_{3n}\}$, and $E(C_{n,2}) = \{v_{3k-1} v_{3k-2}, v_{3k-1} v_{3k}, k = 1,2, \ldots, n\} \cup \{v_{3k-1} v_{3k+2}, k = 1,2, \ldots, n-1\}$.

**Theorem 11.** The middle graph $M\{C_{n,2}\}$ of (n,2) –centipede tree $C_{n,2}$ admits ADCSS - labeling.
**Proof:** Let G = $M\{C_{n,2}\}$ and let $v_1, v_2, \ldots, v_{6n-1}$ are the vertices of G.
Here $|V(G)| = 6n-1$ and $|E(G)| = 12n-8$.
Define a function $f : V \to \{1,2,3, \ldots, 6n-1\}$ by
$f(v_i) = i$, $i = 1,2, \ldots, 6n-1$.
For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$$f^*_{adcss}(v_i\, v_{i+1}) = (i+1)^2(i) + i^2(i-1), \quad i = 1,2,3, \ldots, 2n-2$$
$$f^*_{adcss}(v_{2i-1}\, v_{2n-1+i}) = (2i-1)^2(2i-2) + (2n-1+i)^2(2n-2+i), \quad i = 1,2,3, \ldots, n.$$
$$f^*_{adcss}(v_{2n-1+i}\, v_{3n-1+i}) = (2n-1+i)^2(2n-2+i) + (3n-1+i)^2(3n-2+i), \quad i = 1,2, \ldots, n$$
$$f^*_{adcss}(v_{2i-1}\, v_{4n-1+i}) = (2i-1)^2(2i-2) + (4n-1+i)^2(4n-2+i), \quad i = 1,2, \ldots, n$$

$f^*_{adcss}(v_{4n-1+i}\ v_{5n-1+i})$ = (4n-1+i)$^2$(4n-2+i) + (5n-1+i)$^2$(5n-2+i), i = 1,2,…, n

$f^*_{adcss}(v_{2n-1+i}\ v_{4n-1+i})$ = (2n-1+i)$^2$(2n-2+i) + (4n-1+i)$^2$(4n-2+i), i = 1,2,…., n

$f^*_{adcss}(v_{2i}\ v_{2n-1+i})$ = (2i)$^2$(2i-1) + (2n-1+i)$^2$(2n-2+i), i = 1,2,…,n-1

$f^*_{adcss}(v_{2i}\ v_{4n-1+i})$ = (2i)$^2$(2i-1) + (4n-1+i)$^2$(4n-2+i), i = 1,2,…,n-1

$f^*_{adcss}(v_{2i}\ v_{2n+i})$ = (2i)$^2$(2i-1) + (2n+i)$^2$(2n-1+i), i = 1,2,…,n-1

$f^*_{adcss}(v_{2i}\ v_{4n+i})$ = (2i)$^2$(2i-1) + (4n+i)$^2$(4n-1+i), i = 1,2,…,n-1

$f^*_{adcss}(v_{2i}\ v_{2i+2})$ = (2i)$^2$(2i-1) + (2i+2)$^2$(2i+1), i = 1,2,…,n-2

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence M{$C_{n,2}$} admits adcss-labeling.

**Definition 12.** A graph obtained by adding a single pendant edge to each vertex of a path $P_n$ is called a comb graph and is denoted by Comb($P_n$)

**Theorem 13.** The middle graph M{ Comb($P_n$)} of comb graph Comb($P_n$) admits ADCSS - labeling.

**Proof:** Let G = M{ Comb($P_n$)} and let $v_1,v_2,\ldots,v_{4n-1}$ are the vertices of G.
Here |V(G)| = 4n-1 and |E(G)| = 7n-6.
Define a function f : V → {1,2,3,…, 4n-1} by

f($v_i$) = i , i = 1,2, …, 4n-1.

For the vertex labeling f, the induced edge labeling $f^*_{adcss}$ is defined as follows

$f^*_{adcss}(v_i\ v_{i+1})$ = (i+1)$^2$(i) + i$^2$(i-1), i = 1,2,3,…,2n-2

$f^*_{adcss}(v_{2i-1}\ v_{2n-1+i})$ = (2i-1)$^2$(2i-2) + (2n-1+i)$^2$(2n-2+i) , i = 1,2,3,…, n.

$f^*_{adcss}(v_{2n-1+i}\ v_{3n-1+i})$ = (2n-1+i)$^2$(2n-2+i) + (3n-1+i)$^2$(3n-2+i), i = 1,2,…, n

$f^*_{adcss}(v_{2i}\ v_{2n-1+i})$ = (2i)$^2$(2i-1) + (2n-1+i)$^2$(2n-2+i), i = 1,2,…, n-1

$f^*_{adcss}(v_{2i}\ v_{2n+i})$ = (2i)$^2$(2i-1) + (2n+i)$^2$(2n-1+i), i = 1,2,…, n-1

$f^*_{adcss}(v_{2i}\ v_{2i+2})$ = (2i)$^2$(2i-1) + (2n+i)$^2$(2n-1+i), i = 1,2,….,n-2.

All edge values of G are distinct, which are multiples of 2.That is the edge values of G are in the form of an increasing order. Hence M{ Comb($P_n$)} admits adcss-labeling.

**Example14.** The following illustration shows, the middle graph of comb(4) admits adcss labeling.

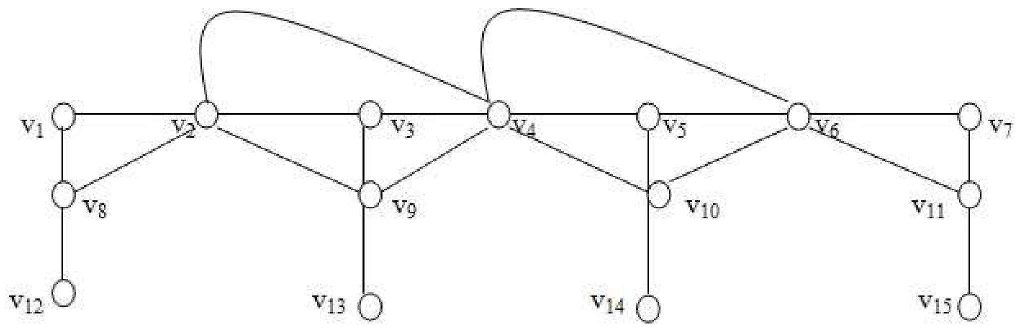


**Figure 4:**

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