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# Some Common Fixed Point Theorems in Fuzzy Metric Spaces for Sequence of Self Mapping Using the Property (E.A)

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*Abstract.* In this paper, we establish some common fixed point theorems in fuzzy metric spaces for sequence of self mappings using implicit relation and the property (E.A). Our results extend, generalize and improve several results of metric spaces and menger spaces to fuzzy metric spaces.

*Keywords:* Fuzzy metric space, t-norm, weakly commuting mappings, (E. A) property, implicit relation

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## 1. Introduction

The idea of fuzzy set was first introduced by Iranian mathematician Zadeh [18]. Fuzzy set is specifically designed to mathematically represent uncertainty and vagueness. Fuzzy set is defined by a membership function which assigns each object to a grade of membership between zero and one. Adak et al. [1] and Pradhan and Pal [14] have done remarkable work on fuzzy set theory. Kramosil and Michalek [10] applied the concept of fuzziness to the classical metric space and compare the fuzzy metric space with probabilistic metric space, the generalization of metric space. George and Veeramani [4] modified the concept of fuzzy metric space by imposing some stronger conditions using continuous t-norm and defined the hausdorff topology of fuzzy metric spaces. Gregori and Sapena [6] defined the concepts of convergent sequence, Cauchy sequence, completeness and compactness in sense of fuzzy metric space. Grabiec [5] introduced the fuzzy version of Banach contraction principle.

Jungck [7] introduced commuting mappings in metric space. Sessa [16] generalized commuting mappings in metric space as weakly commuting mappings. Pant [13] introduced R weak commutativity in metric space. Vasuki (2010) defined R weak commutativity in fuzzy metric space. Jungck [8] enlarged the class of noncommuting mappings by compatible mappings. Also the concept of compatible mappings was further improved by Jungck et al. [9] with the notion of weakly compatible mappings which

merely commute at coincidence points. Mishra et al. [12] defined compatible mappings in fuzzy metric spaces and proved some common fixed point theorems.

Aamri and El Moutawakil [2] defined the (E.A) property for self mappings whose class contains the class of noncompatible as well as compatible mappings. Common property (E.A) is introduced by Ali et al. [3]. It is observed that (E.A) property and common property (E.A) require the closedness of the subspaces for the existence of fixed point. Mihet [11] defined (E.A) property in fuzzy metric spaces.

In this paper, we prove some common fixed point theorems in fuzzy metric spaces using implicit relation and the common property (E. A).

#### 2. Preliminaries

Definition 2.1. (Schweizer B. and Sclar A., 1986) A binary operation

\*:  $[0,1] \times [0,1] \rightarrow [0,1]$  is called continuous t-norm if it satisfies the following conditions:

- 1) \* is commutative and associative;
- 2) \* is continuous;
- 3) a \* 1 = a, for all  $a \in [0, 1]$ ;
- 4)  $a * b \le c * d$  whenever  $a \le c$  and  $b \le d$ , for all  $a, b, c, d \in [0, 1]$ .

**Remark 2.2.** (Schweizer B. and Sclar A., 1986) The concept of t-norm can be considered as fuzzy union.

**Definition 2.3.** (George A. and Veeramani P., 1994) The 3-tuple (X, M, \*) is said to be a fuzzy metric space (FMS) if, X is a non empty set, \* is a continuous t-norm, M is a fuzzy set on  $X \times X \times (0, \infty)$  satisfying the following conditions:

- 1) M(x, y, t) > 0;
- 2) M(x, y, t) = 1 if and only if x=y;
- 3) M(x, y, t) = M(y, x, t);
- 4)  $M(x, y, t) * M(y, z, s) \le M(x, z, t + s);$

 $M(x, y, \cdot): (0, \infty) \rightarrow [0, 1]$  is continuous, for all x, y,  $z \in X$  and s, t > 0.

M(x, y, t) is considered as the degree of nearness of x and y with respect to t.

#### Example 2.4. (George A. and Veeramani P., 1994)

Let (X, d) be a metric space  $a * b = min \{a, b\}$  and  $\forall x, y \in X$  and t > 0.

$$M_{d}(x, y, t) = \frac{t}{t + d(x, y)},$$

then (X, M, \*) is a Fuzzy metric space.

#### Definition 2.5. (Gregori V. and Sapena A., 2002)

- I. A sequence  $\{x_n\}$  is said to convergent to x in X, if and only if  $\lim_{n\to\infty} M(x_n, x, t) = 1$  for all t>0.
- II. A sequence  $\{x_n\}$  is said to M cauchy, if and only if for each  $\mathcal{E} \in (0, 1), t > 0$ , there exist  $n_0 \in N$  such that  $\lim_{n \to \infty} M(x_m, x_n, t) > 1 \mathcal{E}$  for any m,  $n \ge n_0$  for all t>0.

Some Common Fixed Point Theorems in Fuzzy Metric Spaces for Sequence of Self Mapping Using the Property (E.A)

III. The fuzzy metric space M(x, y, t) is called M-complete if every M-cauchy sequence is convergent.

**Definition 2.6.** (Vasuki R., 1999) Two self mappings f and g of a fuzzy metric space (X, M, \*) are said to be weakly commuting if,

M (fgx, gfx, t)  $\ge$  M(fx, gx t), for all x  $\in$  X, t > 0.

**Definition 2.7. (Vasuki R., 1999)** Two self mappings f and g of a fuzzy metric space (X, M, \*) are said to be R weakly commuting if,

M ( fgx, gfx, t)  $\geq$  M(fx, gx,  $\frac{t}{R}),$  for all x  $\in$  X, t > 0.

**Definition 2.8.** (Mihet D., 2010) Two self mappings f and g of a fuzzy metric space (X, M, \*) are said to satisfy the (E.A) property if there exist a sequence  $\{x_n\}$  in X such that for all t > 0,

 $\lim_{n\to\infty} M(fx_n, gx_n, t) = 1.$ 

**Definition 2.9.** (Ali J. et al., 2010) Two pairs (A, S) and (B, T) of self mappings of a fuzzy metric space (X, M, \*) are said to satisfy the (E.A) property if there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in X such that for all t > 0,

 $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z, \text{ for some } z \text{ in } X.$ 

### 3. Main results

**Definition 3.1. (Implicit relation)** Let  $\Phi$  be the class of all real valued, non-decreasing and continuous functions,  $\phi: (R^+)^4 \to R$ , satisfying the following condition:  $\phi(x, y, x, y) \ge 0$  or  $\phi(x, y, y, x) \ge 0$  or  $(x, x, y, y) \ge 0$  implies  $x \ge y$ , for all  $x, y \ge 0$ ;

**Theorem 3.2.** Let A, B and  $\langle F_i \rangle$  where  $i \Box N \cup \{0\}$ , be self-maps of fuzzy metric space (X, M, \*), satisfying the following conditions,

I.  $F_i \subseteq B$  and  $F_0 \subseteq A$ ;

II.  $\phi(M(F_ix, F_0y, kt), M(Ax, By, t), M(F_ix, Ax, t), M(F_0y, By, kt)) \ge 0;$ 

III. The pairs  $(F_i, A)$  and  $(F_0, B)$  share the common property (E.A.);

IV. The pairs  $(F_i, A)$  and  $(F_0, B)$  are R-weakly commuting,

for all x,  $y \in X$ , t > 0,  $k \in (0, 1)$ . If range of one of A and B is closed subspace of X, then  $\langle F_i \rangle$  where  $i \square N \cup \{0\}$ , A and B have a unique common fixed point.

**Proof:** Since The pairs ( $F_i$ , A) and ( $F_0$ , B) share the common property (E.A.), there exist two sequences { $x_n$ } and { $y_n$ } such that,

$$\lim_{n \to \infty} F_0 \mathbf{x}_n = \lim_{n \to \infty} B \mathbf{x}_n = \lim_{n \to \infty} F_i \mathbf{y}_n = \lim_{n \to \infty} A \mathbf{y}_n = \mathbf{z}$$

Suppose that A(X) is a closed subspace of X, then there exists some  $u \square X$  such that z = Au.

Now we show that  $F_i u = z$ . using condition II with x = u and  $y = x_n$ .

 $\phi(M(F_iu,F_0x_n,kt),M(Au,Bx_n,t),M(F_iu,Au,t),M(F_0x_n,Bx_n,kt)) \ge 0$ Taking limit as  $n \to \infty$ 

> $\phi(M(F_{i}u, z, kt), M(z, z, t), M(F_{i}u, Au, t), M(z, z, kt)) \ge 0$  $\phi(M(F_{i}u, z, kt), 1, M(F_{i}u, Au, t), 1) \ge 0$

Since  $\phi$  is non decreasing therefore,

$$\phi(M(F_iu, z, t), 1, M(F_iu, Au, t), 1) \ge 0$$

From the definition 3.1

 $M(F_i u, z, t) \geq 1$ 

Therefore,  $F_i u = z = Au$ .

Since  $F_i \subseteq B$ , there exists some  $v \in X$ , such that  $F_i u = z = Bv$ . Now we show that  $F_0v = z$ . using condition II with y = v and  $x = y_n$ .  $\phi(M(F_{i}y_{n}, F_{0}v, kt), M(Ay_{n}, Bv, t), M(F_{i}y_{n}, Ay_{n}, t), M(F_{0}v, Bv, kt)) \geq 0$ Taking limit as  $n \rightarrow \infty$  $\phi(M(z, F_0 v, kt), M(z, z, t), M(z, z, t), M(F_0 v, z, kt)) \ge 0$  $\phi(M(z, F_0v, kt), 1, 1, M(F_0v, z, kt)) \ge 0$ From the definition 3.1  $M(F_0v,z,kt) > 1$ Therefore,  $F_0 v = z = B v$ ,

Hence,  $F_i u = Au = F_0 v = Bv = z$ .

Since  $F_i$  and A are pointwise R-weakly commutting, there exists R > 0 such that, M ( $F_iAu, AF_iu, t$ )  $\geq M(F_iu, Au, \frac{t}{R}) = 1$ , hence  $F_iAu = AF_iu = F_iF_iu = AAu$ . Similarly  $F_0$  and B are pointwise R-weakly commuting, there exists R > 0 such that,

M ( $F_0Bv$ ,  $BF_0v$ , t)  $\geq$  M( $F_0v$ , Bv,  $\frac{t}{B}$ ) = 1 hence  $F_0Bv = BF_0v = F_0F_0v = BBv$ . using condition II with y = v and  $x = F_i u$ ,  $\phi(M(F_iF_iu, F_0v, kt), M(AF_iu, Bv, t), M(F_iF_iu, AF_iu, t), M(F_0v, Bv, kt)) \geq 0$  $\phi(M(F_iF_iu, F_iu, kt), M(F_iF_iu, F_iu, t), 1, 1) \ge 0$ Since  $\phi$  is non decreasing therefore,  $\phi(M(F_iF_iu, F_iu, t), M(F_iF_iu, F_iu, t), 1, 1) \ge 0$ From the definition 3.1,  $M(F_iF_iu, F_iu, t) \geq 1$ It is possible only when,  $F_i F_i u = F_i u$ Hence,  $F_i z = z$ Therefore,  $F_i z = z = A z$ Therefore z is a fixed point of  $F_i$  and A. Now using condition II with  $y = F_0 v$  and x = u,  $\phi(M(F_iu, F_0F_0v, kt), M(Au, BF_0v, t), M(F_iu, Au, t), M(F_0F_0v, BF_0v, kt)) \ge 0$  $\phi(M(F_0v, F_0F_0v, kt), M(F_0v, F_0F_0v, t), 1, 1) \ge 0$ Since  $\phi$  is non decreasing therefore,  $\phi(M(F_0v, F_0F_0v, t), M(F_0v, F_0F_0v, t), 1, 1) \ge 0$ 

From the definition 3.1,

 $M(F_0v, F_0F_0v, t) \ge 1$ It is possible only when,  $F_0 v = F_0 F_0 v = z$ Hence,  $F_0 z = z$ Therefore,  $F_0 z = B z = z$ Therefore z is a fixed point of  $F_0$  and B. This gives,  $F_i z = A = F_0 z = B z = z$ . Hence z is a common fixed point of  $\langle F_i \rangle$ , A and B.

Some Common Fixed Point Theorems in Fuzzy Metric Spaces for Sequence of Self Mapping Using the Property (E.A)

Now to show that z is unique common fixed point of  $\langle F_i \rangle$ , where  $i \Box N \cup \{0\}$ , A and B. Suppose  $z_1$  is another common fixed point of  $\langle F_i \rangle$ , where  $i \square N \cup \{0\}$ , A and B. Now using condition II with  $y = z_1$  and x = z,

$$\phi(M(F_iz, F_0z_1, kt), M(Az, Bz_1, t), M(F_iz, Az, t), M(F_0z_1, Bz_1, kt)) \ge 0$$
  
$$\phi(M(z, z_1, kt), M(z, z_1, t), 1, 1) \ge 0$$

Since  $\phi$  is non decreasing therefore,

$$\phi(M(z, z_1, t), M(z, z_1, t), 1, 1) \ge 0$$

From the definition 3.1,

$$M(z, z_1, t) \ge 1$$

It is possible only when,  $z = z_1$ 

Hence z is unique common fixed point of  $\langle F_i \rangle$  where i  $\Box N \cup \{0\}$ , A and B.

**Theorem 3.3.** Let A, B, F and G be self-maps of fuzzy metric space (X, M, \*), satisfying the following conditions,

I.  $F \subseteq B$  and  $G \subseteq A$ ;

II.  $\phi(M(Fx, Gy, kt), M(Ax, By, t), M(Fx, Ax, t), M(Gy, By, kt)) \ge 0;$ 

III. The pairs (F, A) and (G, B) share the common property (E.A.);

IV. The pairs (F, A) and (G, B) are R-weakly commuting,

for all x,  $y \in X$ , t > 0,  $k \in (0, 1)$ . If range of one of A and B is closed subspace of X, then F, G, A and B have unique common fixed point.

**Proof:** Since The pairs (F, A) and (G, B) share the common property (E.A.), there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  such that,

 $\lim_{n \to \infty} G x_n = \lim_{n \to \infty} B x_n = \lim_{n \to \infty} F y_n = \lim_{n \to \infty} A y_n = z$ Suppose that A(X) is a closed subspace of X, then there exists some u  $\Box$  X such that z = Au.

Now we show that Fu = z. using condition II with x = u and  $y = x_n$ .

 $\phi(M(Fu, Gx_n, kt), M(Au, Bx_n, t), M(Fu, Au, t), M(Gx_n, Bx_n, kt)) \ge 0$ Taking limit as  $n \rightarrow \infty$ 

$$\phi(M(Fu, z, kt), M(z, z, t), M(Fu, Au, t), M(z, z, kt)) \geq 0$$

 $\phi(M(Fu, z, kt), 1, M(Fu, Au, t), 1) \ge 0$ 

Since  $\phi$  is non decreasing therefore,

$$\phi(M(Fu, z, t), 1, M(Fu, Au, t), 1) \ge 0$$

From the definition 3.1,

$$M(Fu, z, kt) \geq 1$$

Therefore Fu = z = Au.

Since  $F \subseteq B$ , there exists some  $v \in X$ , such that Fu = z = v.

Now we show that, Gv = z. using condition II with y = v and  $x = y_n$ .

 $\phi(M(Fy_n, Gv, kt), M(Ay_n, Bv, t), M(Fy_n, Ay_n, t), M(Gv, Bv, kt)) \ge 0$ Taking limit as  $n \rightarrow \infty$ 

$$\phi(M(z,Gv,kt),M(z,z,t),M(z,z,t),M(Gv,z,kt)) \geq 0$$

Since  $\phi$  is non decreasing therefore,

$$M(Fv, z, kt) \ge 1$$

Therefore, Gv = z = Bv,

Hence, Fu = Au = Gv = Bv = z.

Since F and A are pointwise R-weakly commutting, there exists R > 0 such that,

 $M(FAu, AFu, t) \ge M(Fu, Au, \frac{t}{p}) = 1$  that is, FAu = AFu = FFu = AAu. Similarly, G and B are pointwise R-weakly commutting, there exists R > 0 such that, M (GBv, BGv, t)  $\geq$  M(Gv, Bv,  $\frac{t}{R}$ ) = 1 that is, GBv = BGv = GGv = BBv. using condition II with y = v and  $x = F_i u$ ,  $\phi(M(FFu, Gv, kt), M(AFu, Bv, t), M(FFu, AFu, t), M(Gv, Bv, kt)) \geq 0$  $\phi(M(FFu, Fu, kt), M(FFu, Fu, t), 1, 1) \ge 0$ Since  $\phi$  is non decreasing therefore,  $\phi(M(FFu, Fu, t), M(FFu, Fu, t), 1, 1) \geq 0$ From the definition 3.1,  $M(FFu, Fu, t) \geq 1$ It is possible only when, FFu = FuHence, Fz = zTherefore, Fz = z = AzTherefore z is a fixed point of F and A. Now using condition II with y = Gv and x = u,  $\phi(M(Fu, GGv, kt), M(Au, BGv, t), M(Fu, Au, t), M(GGv, BGv, kt)) \geq 0$  $\phi(M(GGv, Gv, kt), M(GGv, Gv, t), 1, 1) \geq 0$ Since  $\phi$  is non decreasing therefore,  $\phi(M(GGv, Gv, t), M(GGv, Gv, t), 1, 1) \geq 0$ From the definition 3.1.  $M(GGv, Gv, t) \geq 1$ It is possible only when, Gv = GGv = zHence, Gz = zTherefore, Gz = Bz = zTherefore z is a fixed point of G and B. This gives, Fz = A = Gz = Bz = z. Hence z is a common fixed point of F, G, A and B. Now to show that z is unique common fixed point of F, G, A and B. Suppose  $z_1$  is another common fixed point of F, G, A and B. Now using condition II with  $y = z_1$  and  $x = z_2$ ,  $\phi(M(Fz,Gz_1,kt),M(Az,Bz_1,t),M(Fz,Az,t),M(Fz_1,Bz_1,kt)) \geq 0$  $\phi(\,M(z,z_1,kt),M(z,z_1,t),1,1\,)\geq 0$ Since  $\phi$  is non decreasing therefore,  $\phi(M(z, z_1, t), M(z, z_1, t), 1, 1) \ge 0$ From the definition 3.1,  $M(z, z_1, t) \ge 1$ It is possible only when,  $z = z_1$ . Hence z is unique common fixed point of F, G, A and B.

**Corollary 3.4.** Let A, B and  $\langle F_i \rangle$  where  $i \Box N \cup \{0\}$ , be self-maps of fuzzy metric space (X, M, \*), satisfying the following conditions,

- I.  $F_i \subseteq B$  and  $F_0 \subseteq A$ ;
- II.  $\phi(M(F_ix, F_0y, kt), M(Ax, By, t), M(F_ix, Ax, t), M(F_0y, By, kt)) \ge 0;$
- III. The pair  $(F_0, B)$  satisfies property (E.A.);
- IV. The pairs  $(F_i, A)$  and  $(F_0, B)$  are R-weakly commuting,

Some Common Fixed Point Theorems in Fuzzy Metric Spaces for Sequence of Self Mapping Using the Property (E.A)

for all x,  $y \in X$ , t > 0,  $k \in (0, 1)$ . If range of one of A and B is closed subspace of X, then  $\langle F_i \rangle$  where  $i \in \mathbb{N} \cup \{0\}$ , A and T have a unique common fixed point.

**Proof:** Since the pair  $(F_0, B)$  share the common property (E.A), there exist a sequence  $\{x_n\}$  such that,

 $\lim_{n\to\infty} F_0 x_n = \lim_{n\to\infty} B x_n = z, \text{ for some } z \in X.$ Since  $F_0(X) \subseteq A(X)$ , there exists  $\{y_n\}$  in X such that,  $F_0 x_n = A y_n$  and  $\lim_{n\to\infty} A y_n = z$ . Now we show that,  $\lim_{n\to\infty} F_i y_n = z$ .

Using condition II with  $x = y_n$  and  $y = x_n$ .

 $\phi(M(F_{i}y_{n}, F_{0}x_{n}, kt), M(Ay_{n}, Bx_{n}, t), M(F_{i}y_{n}, Ay_{n}, t), M(F_{0}x_{n}, Bx_{n}, kt)) \geq 0$ 

Taking limit as  $n \rightarrow \infty$ 

$$\phi(M(F_iy_n, z, kt), M(z, z, t), M(F_iy_n, z, t), M(z, z, kt)) \ge 0$$
  
$$\phi(M(F_iy_n, z, kt), 1, M(F_iy_n, z, t), 1) \ge 0$$
  
Since  $\phi$  is non decreasing therefore,  
$$\phi(M(F_iy_n, z, t), 1, M(F_iy_n, z, t), 1) \ge 0$$

$$\phi(M(F_i y_n, z, t), 1, M(F_i y_n, z, t), 1) \ge 0$$

From the definition 3.1,

$$M(F_i y_n, z, t) \ge 1$$

It is possible only when,  $\lim_{n\to\infty} F_i y_n = z_i$ Therefore,

$$\lim F_0 x_n = \lim B x_n = \lim F_i y_n = \lim A y_n = z$$

The pairs  $(F_i, A)$  and  $(F_0, B)$  share the common property (E.A). Therefore all the conditions of theorem 3.1 are satisfied hence  $\langle F_i \rangle$  where  $i \Box N \cup \{0\}$ , A and B have a unique common fixed point.

**Corollary 3.5.** Let A and B be self mappings of fuzzy metric space (X, M, \*), satisfying the following conditions:

- $\phi(M(Ax, Ay, kt), M(Bx, By, t), M(Ax, Bx, t), M(Ay, By, kt)) \ge 0;$ I.
- II. The pair (A, B) satisfies property (E.A.);
- III. The pair A and B are R-weakly commuting;
- IV. Range of B is closed subspace of X,
  - for all x, y  $\in$  X, t > 0, k  $\in$  (0, 1), then A and B have unique common fixed point in X.

**Proof:** The proof can be obtained by putting  $F_i = F_0 = A$  and A = B in theorem 3.1.

Corollary 3.6. Let A and I be self mappings of fuzzy metric space (X, M, \*), satisfying the following conditions,

- I.  $\phi(M(Ax, Ay, kt), M(x, y, t), M(Ax, x, t), M(Ay, y, kt)) \ge 0;$
- II. The pair (A, I) satisfies property (E.A.),

for all x,  $y \in X$ , t > 0,  $k \in (0, 1)$ , then A has unique common fixed point in X. **Proof:** The proof can be obtained by putting  $F_i = F_0 = A$  and A = B = I in theorem 3.1.

**Example 3.7.** Let (X, M, \*) be a fuzzy metric space, X = [1, -1] with  $M(x, y, t) = \frac{t}{t + |x - y|}$ , for all x,  $y \in X$ ,  $a * b = \min\{a, b\}$  for all  $a, b \in [0, 1]$ , t > 0, let

$$F_i x = \frac{x}{10i}$$
 for  $i \in N$ ,  $F_0 x = 0$ ,  $Ax = -x$ ,  $Bx = x$ ,  $< x_n > = \frac{1}{n}$ ,  $< y_n > = \frac{-1}{n}$ ,

Since,

 $\lim_{n\to\infty} F_0 \mathbf{x}_n = \lim_{n\to\infty} B \mathbf{x}_n = \lim_{n\to\infty} F_i \mathbf{y}_n = \lim_{n\to\infty} A \mathbf{y}_n$ and  $Ai(X) \in T(X), AO(X) \in S(X).$ 

Also M(F<sub>i</sub>Ax, AF<sub>i</sub>x, t)  $\geq$  M(F<sub>i</sub>x, Ax,  $\frac{t}{R}$ ), M(F<sub>0</sub>Bx, BF<sub>0</sub>x, t)  $\geq$  M(F<sub>i</sub>x, Sx,  $\frac{t}{R}$ ) for all

x∈X.

Therefore,  $(F_i, A)$  and  $(F_0, B)$  are R weakly commuting.

Let  $\phi: R^+ \to R$  be defined as,

$$\phi(a,b,c,d) = a - b$$

Therefore all the conditions of theorem 3.2 are satisfied and 0 is the common fixed point.

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