Annals of Pure and Applied Mathematics
Vol. 13, No. 1, 2017, 1-7
ISSN: 2279-087X (P), 2279-0888(online)
Published on 4 January 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v13n1a1

Annals of
Pure and Applied Mathematics

Two New Multiplicative Atom Bond Connectivity Indices

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

Received 20 November 2016; accepted 31 December 2016

Abstract. In this paper, we introduce second and fourth multiplicative atom bond connectivity indices of a graph. A topological index is a numeric quantity from structural graph of a molecule. In this paper, we determine the fourth multiplicative atom bond connectivity index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of $TUC_4C_8\,[p,q]$.

Keywords: molecular graph, fourth multiplicative atom bond connectivity index, nanostructures.

AMS mathematics subject classification (2010): 05*C*05, 05*C*12, 05*C*35

1. Introduction

Let G be a finite, simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. We refer to [1, 2] for undefined term and notation.

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numerical parameter mathematically derived from the graph structure. These indices are useful for establishing correlation between the structure of a molecular compound and its physico-chemical properties, see [3].

The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The subdivision graph $S(G)$ of G is the graph obtained from G by replacing each of its edges by a path of length two.

We need the following results

Lemma 1. [1] Let G be a (p, q) graph. Then $L(G)$ has q vertices and $\frac{1}{2}\sum_{i=1}^{p} d_G(u_i)^2 - q$ edges.

Lemma 2. [1] Let G be a (p, q) graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

One of the well-known and widely used topological index is the product connectivity index or Randić index introduced by Randić in [4].

Motivated by the definition of the product connectivity index and its wide applications, Kulli [5] introduced the first multiplicative atom bond connectivity index of a graph G and it is defined as

$$ABC_1II(G)=\prod_{uv\in E(G)}\sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}.$$

Recently many other multiplicative indices were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14].

Motivated by the definition of the first multiplicative atom bond connectivity index and by previous research on topological indices, we now introduce the second multiplicative atom bond connectivity index and the fourth multiplicative atom bond connectivity index of a graph as follows:

The second multiplicative atom bond connectivity index of a graph G is defined as

$$ABC_2II(G)=\prod_{uv\in E(G)}\sqrt{\frac{n_u+n_v-2}{n_un_v}}.$$

where the number n_u of vertices of G lying closer to the vertex u than to the vertex v for the edge uv of a graph G.

The fourth multiplicative atom bond connectivity index of a graph G is defined as

$$ABC_4II(G)=\prod_{uv\in E(G)}\sqrt{\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}}$$

where $S_G(u)$ is the sum of the degrees of all vertices adjacent to a vertex u.

In this paper, the fourth multiplicative atom bond connectivity index of the line graphs of the subdivision graphs of 2-D lattice, nanotube and nanotorus of $TUC_4C_8\,[p, q]$ are determined.

2. 2. D-lattice, nanotube, nanotorus of $TUC_4C_8\,[p,q]$

We consider the graph of 2D-lattice, nanotube and nanotorus of $TUC_4C_8\,[p,q]$ where p and q denote the number of squares in a row and the number of rows of squares respectively. These graphs are shown in Figure 1.

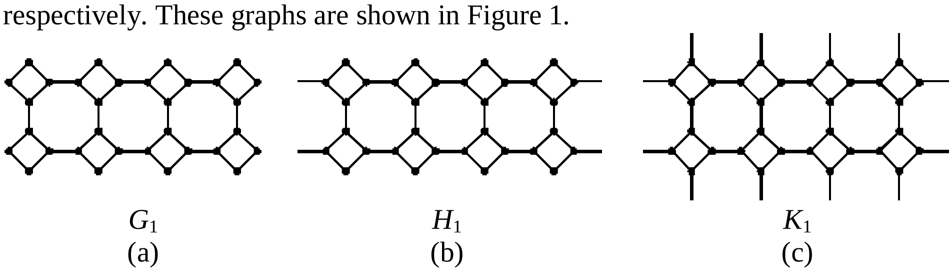
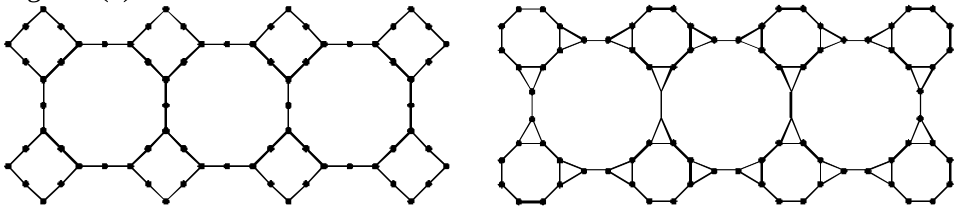


Figure 1: (a)2D-lattice of $TUC_4C_8[4, 2]$ (b) $TUC_4C_8[4,2]$ nanotube (c) $TUC_4C_8\,[4, 2]$ nanotorus

By algebraic method, we get $|V(G_1) = 4pq$, $|E(G_1)| = 6pq - p - q$; $|V(H_1)|= 4pq$, $|E(H_1)| = 6pq - p$, $|V(K_1)| = 4pq$, $|E(K_1)| = 6pq$.

3. Results for 2D-Lattice of TUC_4C_8 [p,q]

The line graph of the subdivision graph of 2D- lattice of $TUC_4C_8[p, q]$ is depicted in Figure 2(b).



(a) (b)

Figure 2: (a)subdivision graph of 2D-lattice of $TUC_4C_8[4,2]$ (b) line graph of the subdivision graph of TUC_4C_8 [4, 2]

Theorem 1. Let G be the line graph of the subdivision graph of 2D-lattice of TUC_4C_8 $[p,q]$. Then

$$ABC_4II(G) = \left(\frac{3}{8}\right)^2 \times \left(\frac{7}{20}\right)^4 \times \left(\frac{8}{25}\right)^{p+q-4} \times \left(\frac{11}{40}\right)^{2(p+q-4)} \times \left(\frac{15}{72}\right)^{4(p+q-2)} \times \left(\frac{4}{9}\right)^{2(9pq+10)-19(p+q)}$$

if $p>1$, $q>1$,

$$= \left(\frac{3}{8}\right)^3 \times \left(\frac{7}{20}\right)^2 \times \left(\frac{8}{25}\right)^{p-2} \times \left(\frac{11}{40}\right)^{2(p-1)} \times \left(\frac{14}{64}\right)^{p-1} \times \left(\frac{15}{72}\right)^{2(p-1)} \times \left(\frac{4}{9}\right)^{p-1}, \text{if } p>1,\ q=1.$$

Proof: The 2D-lattice of TUC_4C_8 $[p,q]$ is a graph G with 4 pq vertices and $6pq - p - q$ edges. By Lemma 2, the subdivision graph of 2D-lattice of TUC_4C_8 $[p,q]$ is a graph with 10 $pq - p - q$ vertices and $2(6pq - p - q)$ edges. Thus by Lemma 1, G has $2(6pq - p - q)$ vertices and 18 $pq - 5p - 5q$ edges. It is easy to see that the vertices of G are either of degree 2 or 3, see Figure 2. Therefore we have partition of the edge set of G as follows.

$S_G(u), S_G(v)\backslash uv \in E(G)$	(4, 4)	(4, 5)	(5, 5)	(5, 8)	(8, 9)	(9, 9)
Number of edges	4	8	$2(p+q-4)$	$4(p+q-2)$	$8(p+q-2)$	$2(9pq+10) - 19(p+q)$

Table 1: Edge partition of G with $p>1$ and $q>1$.

$S_G(u), S_G(v)\backslash uv \in E(G)$	(4, 4)	(4, 5)	(5, 5)	(5, 8)	(8, 8)	(8, 9)	(9, 9)
Number of edges	6	4	$2(p-2)$	$4(p-1)$	$2(p-1)$	$4(p-1)$	$p-1$

Table 2: Edge partition of G with $p>1$ and $q = 1$.

Case 1. Suppose $p>1$ and $q>1$.

By algebraic method, we obtain $|V_4|=8$, $|V_5|=4(p+q-2)$, $|V_8|=4(p+q-2)$ and $|V_9| = 2(6pq - 5p - 5q + 4)$ in G. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 1.

$$ABC_4II(G) = \prod_{uv\in E(G)} \sqrt{\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}}$$

$$= \left(\frac{4+4-2}{4\times4}\right)^{\frac{1}{2}4} \times \left(\frac{4+5-2}{4\times5}\right)^{\frac{1}{2}8} \times \left(\frac{5+5-2}{5\times5}\right)^{\frac{1}{2}2(p+q-4)} \times \left(\frac{5+8-2}{5\times8}\right)^{\frac{1}{2}4(p+q-2)}$$

$$\times \left(\frac{8+9-2}{8\times9}\right)^{\frac{1}{2}8(p+q-2)} \times \left(\frac{9+9-2}{9\times9}\right)^{\frac{1}{2}(2(9pq+10)-19(p+q))}$$

$$= \left(\frac{3}{8}\right)^{2} \times \left(\frac{7}{20}\right)^{4} \times \left(\frac{8}{25}\right)^{p+q-4} \times \left(\frac{8}{25}\right)^{2(p+q-2)} \times \left(\frac{15}{72}\right)^{4(p+q-2)} \times \left(\frac{4}{9}\right)^{2(9pq+10)-19(p+q)}$$

Case 2. Suppose $p>1$ and $q = 1$.

The edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 2.

$$ABC_4II(G) = \prod_{uv\in E(G)} \sqrt{\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}}$$

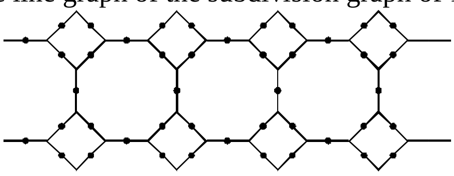
$$= \left(\frac{4+4-2}{4\times4}\right)^{\frac{1}{2}6} \times \left(\frac{4+5-2}{4\times5}\right)^{\frac{1}{2}4} \times \left(\frac{5+5-2}{5\times5}\right)^{\frac{1}{2}2(p-2)} \times \left(\frac{5+8-2}{5\times8}\right)^{\frac{1}{2}4(p-1)}$$

$$\times \left(\frac{8+8-2}{8\times8}\right)^{\frac{1}{2}2(p-1)} \times \left(\frac{8+9-2}{8\times9}\right)^{\frac{1}{2}4(p-1)} \times \left(\frac{9+9-2}{9\times9}\right)^{\frac{1}{2}(p-1)}$$

$$= \left(\frac{3}{8}\right)^{3} \times \left(\frac{7}{20}\right)^{2} \times \left(\frac{8}{25}\right)^{p-2} \times \left(\frac{11}{40}\right)^{2(p-1)} \times \left(\frac{14}{64}\right)^{p-1} \times \left(\frac{15}{72}\right)^{2(p-1)} \times \left(\frac{4}{9}\right)^{p-1}$$

4. Results for $TUC_4C_8\,[p,q]$ nanotube

The line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotube is shown in Figure 3(b)



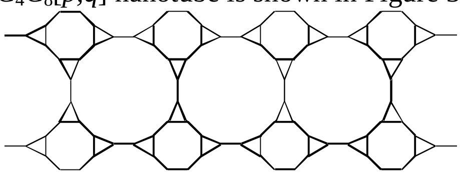


Figure 3: (a) Subdivision graph of $TUC_4C_8\,[4, 2]$ nanotube (b) line graph of subdivision graph of $TUC_4C_8\,[4, 2]$ nanotube.

Theorem 2. Let H be the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotube. Then

$$ABC_4II(H) = \left(\frac{8}{25}\right)^{p} \times \left(\frac{11}{40}\right)^{2p} \times \left(\frac{15}{72}\right)^{4p} \times \left(\frac{4}{9}\right)^{18pq-19p}, \text{ if } p>1 \text{ and } q>1,$$

$$= \left(\frac{8}{25}\right)^{p} \times \left(\frac{11}{40}\right)^{2p} \times \left(\frac{14}{64}\right)^{p} \times \left(\frac{15}{72}\right)^{2p} \times \left(\frac{4}{9}\right)^{p}, \text{ if } p>1 \text{ and } q=1.$$

Proof: The $TUC_4C_8\,[p,q]$ nanotube is a graph H with $4pq$ vertices and $6pq - p$ edges. By Lemma 2, the subdivision graph of $TUC_4C_8[p,q]$ nanotube is a graph with $10pq - p$

vertices and $12pq - 2p$ edges. Thus by Lemma 1, H has $12pq-2p$ vertices and $18pq - 5p$ edges. We see that in H, there are $4p$ vertices, are of degree 2 and remaining all vertices are of degree 3. Therefore we have partition of the edge set of H as follows:

$S_H(u), S_H(v)\backslash uv\in E(H)$	(5, 5)	(5, 8)	(8, 9)	(9, 9)
Number of edges	$2p$	$4p$	$8p$	$18pq - 19p$

Table 3: Edge partition of H with $p>1$ and $q>1$.

$S_H(u), S_H(v)\backslash uv\in E(H)$	(5, 5)	(5, 8)	(8, 8)	(8, 9)	(9, 9)
Number of edges	$2p$	$4p$	$2p$	$4p$	p

Table 4: Edge partition of H with $p>1$ and $q = 1$

Case 1: Suppose $p>1$ and $q > 1$.
By algebraic method, we obtain $|V_5| = 4p$, $|V_8| = 4p$ and $|V_9|= 2(6pq - 5p)$ in H. Thus the edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 3.

$$ABC_4II(H) = \prod_{uv\in E(H)} \sqrt{\frac{S_H(u)+S_H(v)-2}{S_H(u)S_H(v)}}$$

$$= \left(\frac{5+5-2}{5\times 5}\right)^{\frac{1}{2}2p} \times \left(\frac{5+8-2}{5\times 8}\right)^{\frac{1}{2}4p} \times \left(\frac{8+9-2}{8\times 9}\right)^{\frac{1}{2}8p} \times \left(\frac{9+9-2}{9\times 9}\right)^{\frac{1}{2}(18pq-19p)}$$

$$= \left(\frac{8}{25}\right)^{p} \times \left(\frac{11}{40}\right)^{2p} \times \left(\frac{15}{72}\right)^{4p} \times \left(\frac{4}{9}\right)^{18pq-19p}.$$

Case 2. Suppose $p>1$ and $q = 1$.
The edge partition based on the degree sum of neighbor vertices of each vertex is obtained as given in Table 4.

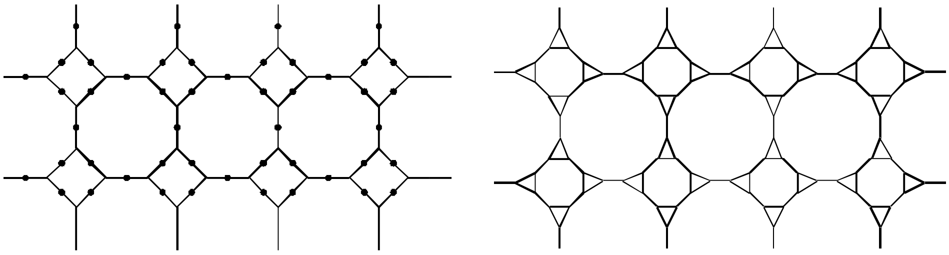
$$ABC_4II(H) = \prod_{uv\in E(H)} \sqrt{\frac{S_H(u)+S_H(v)-2}{S_H(u)S_H(v)}}$$

$$= \left(\frac{5+5-2}{5\times 5}\right)^{\frac{1}{2}2p} \times \left(\frac{5+8-2}{5\times 8}\right)^{\frac{1}{2}4p} \times \left(\frac{8+8-2}{8\times 8}\right)^{\frac{1}{2}2p} \times \left(\frac{8+9-2}{8\times 9}\right)^{\frac{1}{2}4p} \times \left(\frac{9+9-2}{9\times 9}\right)^{\frac{1}{2}p}$$

$$= \left(\frac{8}{25}\right)^{p} \times \left(\frac{11}{40}\right)^{2p} \times \left(\frac{14}{64}\right)^{p} \times \left(\frac{15}{72}\right)^{2p} \times \left(\frac{4}{9}\right)^{p}.$$

5. Results for $TUC_4C_8[p,q]$ nanotorus

The line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotorus is shown in Figure 4 (b).



(a) (b)

Figure 4: (a) subdivision graph of $TUC_4C_8[4,2]$ nanotorus (b) line graph of subdivision graph of $TUC_4C_8[4,2]$ nanotorus.

Theorem 3. Let K be the line graph of the subdivision graph of $TUC_4C_8[p,q]$ nanotorus. Then

$$ABC_4II(K) = \left(\frac{4}{9}\right)^{18pq}.$$

Proof: The $TUC_4C_8\,[p, q]$ nanotorus is a graph K with $4pq$ vertices and $6pq$ edges. Then Lemma 2, the subdivision graph of $TUC_4C_8\,[p, q]$ nanotorus is a graph with $10pq$ vertices and $12pq$ edges. Thus by Lemma 1, K has $12pq$ vertices and $18pq$ edges. We see easily that in K, $|V_9| = 9$ and we have edge partition based on the degree sum of neighbor vertices of each vertex as given in Table 5.

$S_K(u)\ S_K(v)\backslash uv\in E(K)$	(9, 9)
Number of edges	$18pq$

Table 5: Edge partition of K.

$$ABC_4II(K) = \prod\sqrt{\frac{S_K(u)+S_K(v)-2}{S_K(u)S_K(v)}} = \left(\frac{9+9-2}{9\times 9}\right)^{\frac{1}{2}18pq} = \left(\frac{4}{9}\right)^{18pq}.$$

6. Conclusion

In this paper, we have introduced second and fourth multiplicative atom bond connectivity indices of a graph. We have computed the fourth multiplicative atom bond connectivity index of line graphs of subdivision graphs of 2D-lattice, nanotube and nanotorus of TUC4C8[p,q].

REFERENCES

1. F.Harary, *Graph Theory*, Addison Wesley, Reading, MA (1969).
2. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
3. N.Trinajestić, *Chemical Graph Theory*, CRC Press, Boca Raton, FL. 1992.
4. M.Randić, On characterization of molecular branching, *Journal of the American Chemical Society*, 97(23) (1975), 6609-6615.
5. V.R.Kulli, Multiplicative connectivity indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 12(2) (2016) 169-176. DOI: http://dx.doi.org/10.22457/apam.v12n2a8.

6. V.R.Kulli, First multiplicative K Banhatti index and coindex of graphs, *Annals of Pure and Applied Mathematics*, 11(2) (2016), 79-82.
7. V.R.Kulli, Second multiplicative K Banhatti index and coindex of graphs, *Journal of Computer and Mathematical Sciences*, 7(5) (2016), 254-258.
8. V.R.Kulli, Multiplicative K hyper-Banhatti indices and coindices of graphs, *International Journal of Mathematical Archive*, 7(6) (2016), 60-65
9. V.R.Kulli, On multiplicative K Banhatti and multiplicative K hyper-Banhatti indices of V-Phenylenic nanotubes and nanotorus, *Annals of Pure and Applied Mathematics*, 11(2) (2016), 145-160.
10. V.R.Kulli, General multiplicative Zagreb indices of $TUC_4C_8[m,n]$ and $TUC_4[m,n]$ nanotubes, *International Journal of Fuzzy Mathematical Archive*, 11(1) (2016) 39-43. http://dx.doi.org/10.22457/ijfma.v11n1a6.
11. V.R.Kulli, Multiplicative connectivity indices of nanostructures, *Journal of Ultra Scientist of Physical Sciences*, A 29(1) (2017) 1-10.
12. V.R.Kulli, Multiplicative connectivity indices of $TUC_4C_8[m,n]$ and $TUC_4[m,n]$ nanotubes, *Journal of Computer and Mathematical Sciences*, 7(11) (2016) 599-605.
13. V.R.Kulli, Some new multiplicative geometric-arithmetic indices, *Journal of Ultra Scientist of Physical Sciences-A*, 29(2) (2017) 52-57.
14. V.R.Kulli, Computation of general topological indices for titania nanotubes, *International Journal of Mathematical Archive*, (2016)
15. S.M.Hosamani, Computing sanskruti index of nanostructures, *J. Appl. Math. Comput.* DoI: 10.1007/s12190-016-1016-9.