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m∗-Operfect Sets and α-m∗-Closed Sets

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Abstract. In this paper, we introduce the notions of m∗-operfect sets, m∗-clopen sets, α-m∗-closed sets, strongly α-m∗-closed sets, pre-m∗-closed sets, m-clopen sets, α*-m-I-sets and obtain a diagram to show their relationships between these sets and related sets. Also we investigate some properties and characterizations of these sets. Suitable examples are given to establish the results.

Keywords: m∗-operfect set, α-m∗-closed set, pre-m∗-closed set, α*-m-I-set.

AMS Mathematics Subject Classification (2010): 54A05, 54C10, 54D15

1. Introduction and preliminaries

The study of ideal topological spaces was initiated by Kuratowski [5] and Vaidyanathaswamy [14]. Several authors studied and developed the properties of topological spaces and ideal minimal spaces [2,3,6,8,11,12,13,15,16]. Jankovic and Hamlett [4] developed the study in local and systematic manner and offered some new results in the field of ideal topological spaces and established some applications. Bhattacharya [1] introduced regular closed sets. In [7] Maki introduced the notions of minimal structures and minimal spaces. Popa and Noiri introduced a new idea of M-continuous function as a function defined between sets, satisfying some minimal conditions. The concept of ideal minimal spaces was introduced by Ozbakir and Yildirim [9] by combining a minimal space and ideals. In this paper, we define m∗-operfect and α-m∗-closed sets and investigate some of the properties of the above sets. The relationships among these sets are discussed.

Example 1.1. [9] Let (X, m_x) be a minimal space with an ideal I on X.

(i) If $I = \{\phi\}$, then $A_m^*(\phi)$ = m-Cl(A),

(ii) If $I = \wp(X)$, then $A_m^*(\wp(X)) = \phi$.

Lemma 1.1. [9] Let (X, m_x, I) be an ideal minimal space and $A \subseteq X$. If A is m∗-dense in itself, then $A_m^* =$ **m-Cl(** A_m^* **)** = **m-Cl(A)** = **m-Cl*(A).**

2. Some new subsets

Definition 2.1. [10] A subset A of an ideal minimal space (X, m_x, I) is said to be

(i) regular-m-I-closed if A = $(\text{m-Int}(A))^*_m$,

(ii) t-m-I-set if m-Int(A) = m-Int(m-Cl*(A)),

(iii) semi-m-I-regular if A is both semi-m-I -open and a t-m-I-set.

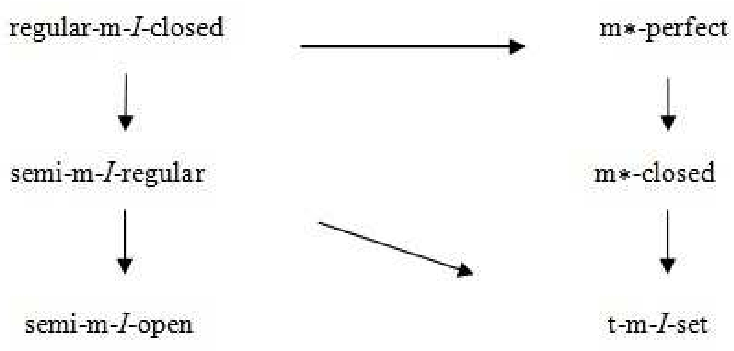

Figure 2.1:

Remark 2.1. None of the implications in Diagram 2.1 is reversible as seen in the following Examples.

Example 2.1. Let (X, m_x, I) be an ideal minimal space such that

X = {a, b, c, d},

m_x = {ϕ, X, {c}, {b, c, d}}

and I = {ϕ}.

Then A = {a, b, d} is an m*-perfect set but not a regular-m-I-closed set.

Example 2.2. Let (X, m_x, I) be an ideal minimal space such that

X = {a, b, c, d},

m_x = {ϕ, X, {a}, {c, d}, {b, c, d}}

and I = {ϕ, {b}}.

Then A = {a, b} is an m∗-closed set but not an m∗-perfect set.

Example 2.3. Let (X, m_x, I) be an ideal minimal space such that

X = {a, b, c, d},

m_x = {ϕ, X, {b}, {d}}

and I = {ϕ}.

Then A = {a, d} is a t-m-I-set but not an m∗-closed set.

Example 2.4. Let (X, m_x, I) be an ideal minimal space such that

X = {a, b, c, d},

$m_x = \{\phi, X, \{a, b\}\}$
and $I = \{\phi, \{a\}, \{c\}, \{a, c\}\}$.
Then A = {a, c} is a t-m-*I*-set but not a semi-m-*I* -regular set.

Example 2.5. Let (X, m_x, *I*) be an ideal minimal space such that
X = {a, b, c, d},
$m_x = \{\phi, X, \{a\}, \{a, d\}, \{a, c, d\}\}$
and $I = \{\phi, \{a\}\}$.
Then A = {a, d} is a semi-m-*I*-open set but not a semi-m-*I*-regular set.

Example 2.6. Let (X, m_x, *I*) be an ideal minimal space such that
X = {a, b, c, d},
$m_x = \{\phi, X, \{b\}, \{d\}\}$
and $I = \{\phi\}$.
Then A = {a, d} is a semi-m-*I* -regular set but not a regular-m-*I* -closed set.

3. m∗-operfect sets and α- m∗-closed sets

Definition 3.1. A subset A of an ideal minimal space (X, m_x, *I*) is said to be
(i) m∗-operfect if A is m-open and m∗-perfect,
(ii) m∗-clopen if A is m-open and m∗-closed,
(iii) α- m∗-closed if m-Cl*(m-Int(m-Cl*(A))) ⊆ A,
(iv) strongly α- m∗-closed if m-Cl(m-Int(m-Cl*(A))) ⊆ A,
(v) pre-m∗-closed if $(\text{m} - \text{Int}(A))^*_m \subseteq A$,
(vi) α*-m-*I*-set if m-Int(A) = m-Int(m-Cl*(m-Int(A))) and
(vii) m-clopen if A is m-open and m-closed.

Remark 3.1. To avoid confusion we will denote the family of all α-m∗-closed sets by αm∗C(X), strongly α-m∗-closed sets by sαm∗C(X) and pre-m∗-closed sets by pm∗C(X).

Proposition 3.1. For a subset A of an ideal minimal space (X, m_x, *I*), the following properties hold.
(i) Every m∗-operfect set is a regular-m-*I*-closed set.
(ii) Every m∗-clopen set is an m-open set.
(iii) Every m∗-clopen set is an m∗-closed set.

Proof:
(i) Let A be a m∗-perfect set.
Since A is both m-open and m∗-perfect, we have

$$(\text{m} - \text{Int}(A))^*_m = A^*_m = A.$$

This shows that A is regular-m-*I*-closed.
(ii) and (iii) are obvious from the Definition 3.1 that A is m-open and m∗-closed.

Remark 3.2. The converses of Proposition 3.1 need not be true as seen in the following examples.

Example 3.1. Let (X, m_x, I) be an ideal minimal space such that

$X = \{a, b, c, d\}$,

$m_x = \{\phi, X, \{b\},\{c\},\{b, c, d\}\}$

and $I = \{\phi\}$.

Then A = {a, b, d} is a regular-m-I-closed set which is not m-open. Therefore, A is neither an m∗-clopen set nor an m∗-operfect set.

Example 3.2. Let (X, m_x, I) be an ideal minimal space such that

$X = \{a, b, c, d\}$,

$m_x = \{\phi, X, \{a\}, \{a, b\}\}$

and $I = \{\phi\}$.

Then A = {a, b} is an m-open set, but not an m∗-closed set and hence not an m∗-clopen set. Moreover, A is not a pre-m∗-closed set.

Example 3.3. Let (X, m_x, I) be an ideal minimal space such that

$X = \{a, b, c, d\}$,

$m_x = \{\phi, X, \{a\}, \{c, d\}, \{b, c, d\}\}$

and $I = \{\phi, \{b\}\}$.

Then A = {a, b} is an m∗-closed set, but not an m-open set and hence not an m∗-clopen set. Moreover, A is not an m∗-perfect set.

Proposition 3.2. A subset A of an ideal minimal space (X, m_x, I) which satisfies property B, then every m∗-operfect set is an m-clopen set.

Proof: Let A be an m∗-operfect set.

Then A is m-open and m∗-perfect.

By Lemma 1.1, we have m-Cl(A) = m-Cl(A_m^*) = A_m^* = A.

Hence A is m-open and m-closed. Therefore A is m-clopen.

Remark 3.3. The converse of Proposition 3.2 need not be true as seen in the following example.

Example 3.4. Let (X, m_x, I) be an ideal minimal space such that

$X = \{a, b, c, d\}$,

$m_x = \{\phi, X, \{a\}, \{d\}, \{b, c\}, \{a, d\}, \{b, c, d\}, \{a, b, c\}\}$

and $I = \{\phi, \{a\}\}$.

Then A = {a, b, c} is an m-clopen set, but not an m∗-operfect set.

Proposition 3.3. For a subset A of an ideal minimal space (X, m_x, I), the following properties hold.

(i) Every m∗-perfect set is a strongly α-m∗-closed set.

(ii) Every α-m∗-closed set is a pre-m∗-closed set.

(iii) Every α-m∗-closed set is a t-m-*I* -set.

(iv) Every m-preclosed set is a pre-m∗-closed set.

(v) Every pre-m∗-closed set is an α*-m-*I*-set.

(vi) Every m-clopen set is an m∗-clopen set.

Proof: Let A be subset of an ideal minimal space (X, m_x, I)

(i) Let A be an m∗-perfect set, then we have $A^*_m = A$.

Thus we obtain that

$$\begin{aligned} m\text{-}Cl(m\text{-}Int(m\text{-}Cl^*(A))) &= m\text{-}Cl(m\text{-}Int(A \cup A^*_m)) \\ &= m\text{-}Cl(m\text{-}Int(A^*_m)) \\ &\subseteq m\text{-}Cl(A^*_m) = A^*_m = A. \end{aligned}$$

Hence A is a strongly α-m∗-closed set.

(ii) Let A be an α-m∗-closed set.

Therefore,
$$\begin{aligned} (m-\mathrm{Int}(A))^*_m &\subseteq m\text{-}Cl^*(m\text{-}Int(A)) \\ &\subseteq m\text{-}Cl^*(m\text{-}Int(m\text{-}Cl^*(A))) \\ &\subseteq A. \end{aligned}$$

Hence A is a pre-m∗-closed set.

(iii) Let A be an α-m∗-closed set.

Then we obtain that,

$$\begin{aligned} m\text{-}Int(m\text{-}Cl^*(A)) &= m\text{-}Int(m\text{-}Cl^*(m\text{-}Int(m\text{-}Cl^*(A)))) \\ &\subseteq m\text{-}Int(A). \end{aligned}$$

Since $A \subseteq m\text{-}Cl^*(A)$, $m\text{-}Int(A) \subseteq m\text{-}Int(m\text{-}Cl^*(A))$.

This shows that A is a t-m-*I* -set.

(iv) Let A be a m-preclosed set.

Then we have $(m-\mathrm{Int}(A))^*_m \subseteq m\text{-}Cl(m\text{-}Int(A)) \subseteq A$.

This shows that A is a pre-m∗-closed set.

(v) Let A be a pre-m∗-closed set.

Then we have $(m-\mathrm{Int}(A))^*_m \subseteq A$.

Then we obtain that

$$(m\text{-}Int(A)) \cup (m-\mathrm{Int}(A))^*_m \subseteq (m\text{-}Int(A)) \cup A \subseteq A$$

and $m\text{-}Int(m\text{-}Cl^*(m\text{-}Int(A))) \subseteq (m\text{-}Int(A))$.

On the other hand, it is obvious that

$$(m\text{-}Int(A)) \subseteq m\text{-}Int(m\text{-}Cl^*(m\text{-}Int(A))).$$

This shows that A is an α*-m-*I* -set.

(vi) Let A be an m-clopen set.

Then A is m-open and m-closed and we obtain that $A^*_m \subseteq$ m-Cl(A)=A.
Hence A is m-open and m*-closed and hence m$*$-clopen.

Proposition 3.4. For a subset A of an ideal minimal space (X, m_x, *I*) satisfying property I, then every m$*$-closed set is an α-m$*$-closed set.

Proof: Let A be an m$*$-closed set, then we have $A^*_m \subseteq$ A.
Therefore,

$$\begin{aligned} \text{m-Cl*(m-Int(m-Cl*(A)))} &\subseteq \text{m-Cl*(m-Cl*(A))} \\ &= \text{m-Cl*(A)} = \text{A}. \end{aligned}$$

This shows that A is an α-m$*$-closed set.

Remark 3.4. The converses of Propositions 3.3 and 3.4 need not be true as shown in the following examples.

Example 3.5. Let (X, m_x,*I*) be an ideal minimal space such that X = {a, b, c},
m_x = {ϕ, X, {a}, {a, c}} and *I* = {ϕ, {a}}.
Then A = {a} is an m$*$-closed set and hence A is an m$*$-clopen set, but it is neither an m-clopen set nor an m$*$-perfect set.

Example 3.6. Let (X, m_x,*I*) be an ideal minimal space such that
X = {a, b, c, d},
m_X = {ϕ, X, {a}, {b, c}}
and *I* = {ϕ, {a}, {b}, {a, b}}.
Then A = {a, b, d} is a strongly α-m$*$-closed set, but not an m$*$-perfect set.

Example 3.7: Let (X, m_x, *I*) be an ideal minimal space such that
X = {a, b, c, d},
m_X = {ϕ, X, {a}, {c, d}}
and *I* = {ϕ, {a}}.
Then A = {a, d} is a pre-m$*$-closed set, but it is neither a t-m-*I* -set nor an α-m$*$-closed set. Moreover, A is not an m-open set.

Remark 3.5. By Examples 3.2 and 3.7, m-open sets and pre-m$*$-closed sets are independent.

Example 3.8. Let (X, m_x, *I*) be an ideal minimal space such that
X = {a, b, c, d},
m_x = {ϕ, X, {b}, {d}}
and *I* = {ϕ}.
Then A = {a, d} is a t-m-I-set, but it is neither an α-m$*$-closed set nor a pre-m$*$-closed *set.* Moreover, it is an $\alpha*$-m-*I* -set.

Example 3.9. Let (X, m_x, I) be an ideal minimal space as in Example 3.7. Then it is clear that
A = {a, d} is a pre-m∗-closed set, but not an m-preclosed set.

Example 3.10. Let (X, m_x, I) be an ideal minimal space such that

$$X = \{a, b, c, d\},$$
$$m_x = \{\phi, X, \{a\}, \{c\}\}$$
and $$I = \{\phi, \{a\}\}.$$

Then A = {a, d} is an α-m∗-closed set, but not an m∗-closed set.

Remark 3.6. By example 3.11, m-clopen sets and m∗-perfect sets are independent.

Example 3.11.

(i) Let (X, m_x, I) be an ideal minimal space such that

$$X = \{a, b, c, d\},$$
$$m_x = \{\phi, X, \{a\}, \{d\}, \{b, c\}, \{a, d\}, \{b, c, d\}, \{a, b, c\}\}$$
and $$I = \{\phi, \{a\}\}.$$

Then A = {a, b, c} is an m-clopen set, but not an m∗-perfect set.

(ii) Let (X, m_x, I) be an ideal minimal space such that

$$X = \{a, b, c, d\},$$
$$m_x = \{\phi, X, \{b\}, \{c\}, \{b, c, d\}\}$$
and $$I = \{\phi\}.$$

Then A = {a, b, d} is an m∗-perfect set, but not an m-open set and hence not an m-clopen set.

Remark 3.7. It follows from Examples 3.7 and 3.8 that pre-m∗-closed sets and t-m-*I*-sets are independent.

Proposition 3.5. For a subset A of an ideal minimal space (X, m_x, I), the following properties hold.

(i) Every m-α-closed set is strongly α-m∗-closed.

(ii) Every strongly α-m∗-closed set is α-m∗-closed.

Proof:

(i) Let A be an m-α-closed set.
Then, m-Cl(m-Int(m-Cl*(A))) $\subseteq$ m-Cl(m-Int(mCl(A))) $\subseteq$ A.
This shows that A is strongly α-m∗-closed.

(ii) Let A be a strongly α-m∗-closed.
Then, m-Cl*(m-Int(m-Cl*(A))) $\subseteq$ m-Cl(m-Int(m-Cl*(A))) $\subseteq$ A.
This shows that A is α-m∗-closed.

Remark 3.8. The converses of Proposition 3.5 need not be true as seen in the following Examples.

Example 3.12. Let (X, m_x, I) be the same ideal minimal space as in Example 3.6.
Then it is clear that A = {a, b, d} is a strongly α-m*-closed.
But since m-Cl(m-Int(m-Cl(A))) = X $\not\subset$ A, it is not an m-α-closed set.

Example 3.13. Let (X, m_x, I) be the same ideal minimal space as in Example 3.10.
Then it is obvious that A = {a, d} is an α-m∗-closed set.

Since m-Cl(m-Int(m-Cl*(A))) = {a, b, d} $\not\subset$ A, A is not a strongly α-m∗-closed set.

Moreover, it is not an m∗-closed set.

From the above propositions and diagram 2.1 we obtain the following diagram.

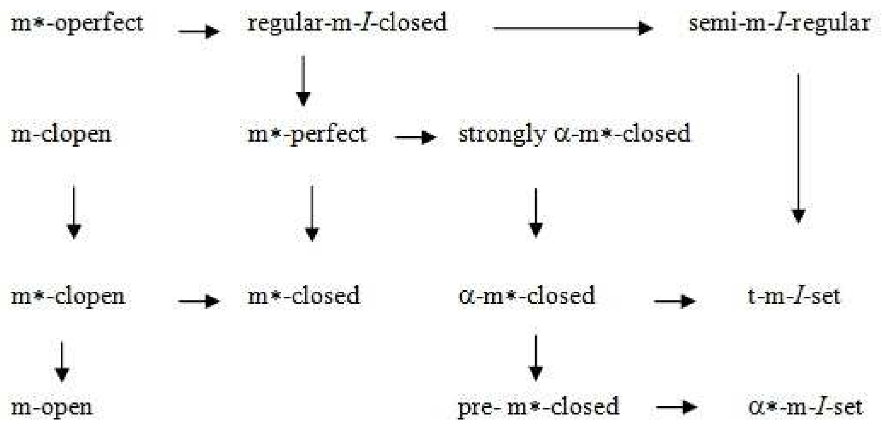

Figure 3.1:

Theorem 3.1. For a pre-m-I-open set A of an ideal minimal space (X, m_x, I) satisfying property B, the following property holds.
A is strongly α-m∗-closed if and only if A is m-closed.

Proof: Let A be a m-closed set.
Then m-Cl(m-Int(m-Cl*(A))) ⊆ m-Cl(m-int(m-Cl(A)))
⊆ m-Cl(A) = A.
Therefore, A is strongly α-m∗-closed.

Let A be a strongly α-m∗-closed set.

Since A is pre-m-I-open,

m-Cl(A) ⊆ m-Cl(m-Int(m-Cl*(A)))
⊆ A.

Hence A is an m-closed set.

Theorem 3.2: For a pre-m-I-open set A of an ideal minimal space (X, m_x, I) satisfying property I, the following property holds. A is α-m∗-closed if and only if A is m∗-closed.

Proof: By Proposition 3.4, every m∗-closed set is an α-m∗-closed set.

Let A be an α-m∗-closed set. Then

m-Cl*(A) ⊆ m-Cl*(m-Int(m-Cl*(A)))
⊆ A since A is pre-m-I-open.

Hence we obtain that A is an m∗-closed set.

Theorem 3.3. For a subset A of an ideal minimal space (X, m_x, *I*), the following are equivalent.

(i) A is an α-m∗-closed set.

(ii) A is a pre- m∗-closed set and a t-m-*I*-set.

Proof:

(i) ⇒ (ii) According to diagram 3.1, it is obvious.

(ii) ⇒ (i) Let A be a pre-m∗-closed and t-m-*I*-set.

Thus, m-Cl*(m-Int(m-Cl*(A))) = m-Cl*(m-Int(A))

$= (\text{m-Int(A)}) \cup (\text{m} - \text{Int(A)})^*_{\text{m}} \subseteq \text{A}.$

This shows that A is an α-m∗-closed set.

Theorem 3.4. For a subset A of an ideal minimal space (X, m_x, *I*), the following are equivalent.

(i) A is a regular-m-I-closed set.

(ii) A is a semi-m-I-regular set and a strongly α-m∗-closed set.

(iii) A is a semi-m-I-regular set and an α-m∗-closed set.

(iv) A is a semi-m-I-open set and an α-m∗-closed set.

(v) A is a semi-m-I-open set and a pre-m*-dosed set.

Proof:

(i) ⇒ (ii), (ii) ⇒ (iii), (iii) ⇒ (iv) and (iv) ⇒ (v) are easily seen by diagram 2.1 and diagram 3.1.

(v) ⇒ (i) Let A be a semi-m-*I*-open and pre-m∗-closed set.

Then, we have $(\text{m} - \text{Int(A)})^*_{\text{m}} \subseteq$ A since A is pre-m∗-closed.

Also since $m_x \subseteq m_x$*, m-Int(A) ⊂ $(\text{m} - \text{Int(A)})^*_{\text{m}}$.

Since A is semi-m-*I*-open, we obtain that

$$\begin{aligned} \text{A} &\subseteq \text{m-Cl*(m-Int(A))} \\ &= (\text{m-Int(A)}) \cup (\text{m} - \text{Int(A)})^*_{\text{m}} \\ &= (\text{m} - \text{Int(A)})^*_{\text{m}} \subseteq \text{A}. \end{aligned}$$

This implies that A is a regular-m-I-closed set.

Example 3.14.

(i) Let (X, m_x, *I*) be an ideal minimal space such that

X = {a, b, c, d},

m_x = {ϕ, X, {b}, {d}}

and *I* = {ϕ}.

Then A= {a, d} is a semi-m-*I*-open set and a t-m-*I*-set and hence a semi-m-*I*-regular set but not a pre-m∗-closed set.

(ii) Let (X, m_x, *I*) be an ideal minimal space such that

$X = \{a, b, c, d\}$,

$m_x = \{\phi, X, \{a,b\}\}$

and $I = \{\phi, \{a\},\{c\},\{a, c\}\}$.

Then A = {a, c} is a pre-m∗-closed set and a t-m-*I*-set but not a semi-m-*I*-open set and hence not a semi-m-*I*-regular set.

From the above example 3.14 that the semi-m-*I*-open (hence semi-m-*I*-regular) sets and pre-m∗-closed sets are independent.

Example 3.15.

(i) Let (X, m_x, *I*) be the same ideal minimal space as in Example 3.14 (ii). Then A = {a, c} is a strongly α-m∗-closed set and hence an α-m∗-closed set but not a semi-m-*I*-regular set.

(ii) Let (X, m_x, *I*) be the same ideal minimal space as in Example 3.14 (i). Then A = {a, d} is semi-m-*I*-regular set but it is neither a strongly α-m∗-closed set nor an α-m∗-closed set.

From the above example 3.15 that the semi-m-*I*-open (hence semi-m-*I*-regular) sets and strongly α-m∗-closed (hence α-m∗-closed) sets are independent.

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