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Some Results on Decompositions of M-Continuity

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Abstract. In this paper, we obtain some important results of decomposition of M-continuity in minimal spaces. In most of the occasions, our ideas are illustrated and substantiated by suitable examples.

Keywords: m-A set, m-B set, m-t set, M-A continuity, M-B continuity, M-C continuity.

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1. Introduction

Njastad [13] initiated the concept of nearly open sets in topological spaces. Following this initiation, many research papers were introduced in this area [1, 5, 10, 11, 12, 16, 17]. Many researchers like Hatir [6, 7, 8, 9], Dontchev [3] and Ganster [4] proposed decompositions of continuity in topological spaces. It is an effort based on them to bring out a work in the name of decompositions of M-continuity in minimal spaces using the new sets like m-A sets, m-B sets and m-C sets and the new mappings like M-A continuous, M-B continuous and M-C continuous. In this paper, we obtain some important results in minimal spaces.

2. Some basic results

Definition 2.1. A minimal space (X, m_x) has the property I if the any finite intersection of m-open sets is m-open.

Remark 2.1. For subsets A and B of a minimal space (X, m_x) satisfying property I, the following holds:

$$\text{m-Int}(A \cap B) = \text{m-Int}(A) \cap \text{m-Int}(B).$$

Definition 2.2. [15] A minimal structure m_x on a nonempty set X is said to have property $\mathscr{B}$ if the union of any family of subsets belonging to m_x belongs to m_x.

Lemma 2.1. [15] The following are equivalent for the minimal space (X, m_x).

(i) m_x have property $\mathscr{B}$.
(ii) If m_x-Int(E) = E, then $E \in m_x$.
(iii) If m_x-Cl(F) = F, then $F^c \in m_x$.

Example 2.1. For subsets A and B of a minimal space (X, m_x) satisfying property $\mathscr{B}$, the following does not hold:

m-Int($A \cap B$) = m-Int(A) $\cap$ m-Int(B).

Let X= { a, b, c, d}, m_x = {ϕ, X, {a}, {a, b}, {a, c}, {b, c}, {a, b, c}}
Let A ={a, b} and B = {b, c}. Then $A \cap B$ = {b}.
We have m-Int(A) = {a, b}; m-Int(B) = {b, c} and m-Int(A) $\cap$ m-Int(B) = {b}.
But m-Int($A \cap B$) = ϕ. Hence m-Int($A \cap B$) ≠ m-Int(A) $\cap$ m-Int(B).

3. m-C sets

We introduce the following sets [2]:

Definition 3.1. A subset S of X is said to be
(i) regular m-open if S = m-Int(m-Cl(S)),
(ii) regular m-closed if S = m-Cl(m-Int(S)).
The family of all regular m-closed sets of X is denoted m-RC(X).

Definition 3.2. A subset S of X is said to be
(i) a m-A set if $S = M \cap N$ where M is m-open and $N \in$ m-RC(X),
(ii) a m-t set if m-Int(m-Cl(S)) = m-Int(S),
(iii) a m-B set if $S = M \cap N$ where M is m-open and N is a m-t set,
(iv) a m-h set if m-Int(m-Cl(m-Int(S))) = m-Int(S),
(v) a m-C set if $S = M \cap N$ where M is m-open and N is a m-h set.

Example 3.1. Let X = {a, b, c} and m_x = {ϕ, X, {a}, {b}, {a, b}}.
Then the sets in {ϕ, X, {b, c}, {a, c}, {c}} are called m_x-closed.

Example 3.2. Let X = {a, b, c} and m_x = {ϕ, X, {a}, {c}, {a, b}, {a, c}}. Then the sets in {ϕ, X, {b, c}, {a, b}, {c}, {b}} are called m_x-closed.

Example 3.3. Let X = {a, b, c} and m_x = {ϕ, X, {a, b}, {b, c}}. Then the sets in {ϕ, X, {c}, {a}} are called m_x-closed.

Remark 3.1. It is evident that any m-open set of X is an m-α-open and each m- α -open set of X is both m-semi-open and m-preopen. But the separate converses are not true.

Theorem 3.1. If A and B are two m-t sets of a space X satisfying property I, then $A \cap B$ is a m-t set in X.
Proof: Since $A \subseteq$ m-Cl(A),

$$\text{m-Int}(A \cap B) \subseteq \text{m-Int}(\text{m-Cl}(A \cap B))$$

$$\begin{aligned}&\subseteq \text{m-Int}(\text{m-Cl}(A) \cap \text{m-Cl}(B))\\ &= \text{m-Int}(\text{m-Cl}(A)) \cap \text{m-Int}(\text{m-Cl}(B))\\ &= \text{m-Int}(A) \cap \text{m-Int}(B) \text{(since A, B are m-t sets)}\\ &= \text{m-Int}(A \cap B).\end{aligned}$$

Thus m-Int(m-Cl($A \cap B$)) = m-Int($A \cap B$) and $A \cap B$ is m-t set.

Theorem 3.2. If A is a m-t set of X and $B \subseteq X$ with $A \subseteq B \subseteq$ m-Cl(A) then B is a m-t set.

Proof: We note that m-Cl(B) $\subseteq$ m-Cl(A).

So we have

$$\begin{aligned}\text{m-Int}(B) &\subseteq \text{m-Int}(\text{m-Cl}(B))\\ &\subseteq \text{m-Int}(\text{m-Cl}(A))\\ &= \text{m-Int}(A) \subseteq \text{m-Int}(B).\end{aligned}$$

Thus m-Int(B) = m-Int(m-Cl(B)) and therefore B is a m-t set.

Remark 3.2. The union of two m-h sets need not be a m-h set. Refer Example 3.1, {a} and {b} are m-h sets but {a, b} is not m-h set.

Remark 3.3. Let (X, m_x) have property I. Then the intersection of any two m-h sets is a m-h set.

4. Comparison

Theorem 4.1. Any m-open set is an m-A set.

Proof: $S = X \cap S$ where $X \in$ m-RC(X) and S is m-open. The proof is completed.

Remark 4.1. The converse of Theorem 4.1 is not true. Refer Example 3.1, {b, c} is m-A set but not m-open.

Theorem 4.2. Any m-closed set is a m-t set.

Proof: Since A = m-Cl(A), m-Int(A) = m-Int(m-Cl(A)). The proof is completed.

Remark 4.2. The converse of Theorem 4.2 is not true. Refer Example 3.1, {a} is m-t set but not m-closed.

Theorem 4.3. A regular m-open set is a m-t set.

Proof: Since S = m-Int(m-Cl(S)), m-Int(S) = m-Int(m-Cl(S)).

The proof is completed.

Remark 4.3. The converse of Theorem 4.3 is not true. Refer Example 3.1, {c} is a m-t set but not regular m-open.

Theorem 4.4. Let (X, m_x) have property $\mathfrak{B}$. Then every regular m-open set is m-open.

Proof: Suppose S = m-Int(m-Cl(S)).

Then m-Int(S) = m-Int(m-Cl(S)) and we have S = m-Int(S).

Thus, S is m-open.

Remark 4.4. The converse of Theorem 4.4 is not true. Refer Example 3.1, {a, b} is m-open but not regular m-open.

Theorem 4.5. Any m-t set is m-B set.
Proof: $S = X \cap S$ where X is m-open and S is m-t set. The proof is completed.

Remark 4.5. The converse of Theorem 4.5 is not true. Refer Example 3.2, {a} is a m-B set but not m-t set.

Theorem 4.6. Any m-open set is a m-B set.
Proof: Since $S = X \cap S$ where S is m-open and X is regular m-open, by Theorem 4.3, X is m-t set. The proof is completed.

Remark 4.6. The converse of Theorem 4.6 is not true. Refer Example 3.1, {c} is m-B set but not m-open.

Theorem 4.7. A m-closed set is a m-B set.
Proof: It follows from Theorem 4.2 and Theorem 4.5.

Theorem 4.8. Let (X, m_x) have property $\mathscr{B}$. Then every m-A set is a m-B set.
Proof: $S = X \cap S$ where X is m-open and S is regular m-closed.
Since S is m-closed, by Theorem 4.2, S is m-t set. The proof is completed.

Remark 4.7. The converse of Theorem 4.8 is not true. Refer Example 3.1, {c} is m-B set but not m-A set.

Theorem 4.9. Any m-t set is m-h set.
Proof: Since m-Int(S) = m-Int(m-Cl(S)),
m-Cl(m-Int(S)) = m-Cl(m-Int(m-Cl(S)))
implies m-Int(m-Cl(m-Int(S))) = m-Int(m-Cl(S)) = m-Int(S).
The proof is completed.

Remark 4.8. The converse of Theorem 4.9 is not true. Refer Example 3.3, {b} is m-h set but not m-t set.

Theorem 4.10. Any m-h set is m-C set.
Proof: $S = X \cap S$ where X is m-open and S is m-h set. The proof is completed.

Remark 4.9. The converse of Theorem 4.10 is not true. Refer Example 3.2, {a} is m-C set but not m-h set.

Theorem 4.11. Any m-open set is m-C set.
Proof: $S = X \cap S$ where X is m-h set and S is m-open. The proof is completed.

Remark 4.10. The converse of Theorem 4.11 is not true. Refer Example 3.1, {c} is m-C set but not m-open.

Theorem 4.12. A m-B set is m-C set.
Proof: S = X ∩ S where X is m-open and S is m-t set.
By Theorem 4.9, S is m-h set. The proof is completed.

Remark 4.11. The converse of Theorem 4.12 is not true. Refer Example 3.3, {b} is m-C set but not m-B set.

Remark 4.12. A m-A set need not be m-semi-open as shown in the following example.
Let X = {a, b, c} and m_x = {ϕ, X, {a}, {b}, {a, b}, {b, c}, {a, c}}. Then the sets in {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} are called m-closed. We have {c} is m-A set but not m-semi-open.

Remark 4.13. A m-semi-open set need not be m-A set as shown in the following example.
Let X = {a, b, c} and m_x = {ϕ, X, {a}}. Then the sets in {ϕ, X, {b, c}} are called m-closed. We have {a, b} is m-semi-open but not m-A set.

Remark 4.14. By the previous Theorems, Examples and Remarks, we obtain the following diagram:

$$\begin{tikzcd}
 & & (5) \arrow[dl] & (6) \\
(1) & (2) \arrow[l] & (3) \arrow[l] \arrow[r] \arrow[u] & (4) \arrow[u] \arrow[d] \\
(7) \arrow[u] & (8) \arrow[l] \arrow[u] & (9) \arrow[l] \arrow[r] \arrow[u] & (10)
\end{tikzcd}$$

Here (1) = m-C set, (2) = m-B set, (3) = m-open set,
(4) = m-α-open set, (5) = m-A set, (6) = m-semi-open set,
(7) = m-h set, (8) = m-t set, (9) = regular m-open set,
(10) = m-preopen set.

5. Decompositions of m-continuity

Definition 5.1. Let f : X → Y be a mapping where X has property $\mathcal{B}$. Then f is said to be M-continuous [14] if $f^{-1}(V)$ is m_x-open in X for every m_y-open set V in Y.

We introduce new classes of mappings as follows:

Definition 5.2. Let $f : X \to Y$ be a mapping. Then f is said to be
(i) M-A continuous if $f^{-1}(V)$ is M-A set in X for every m_y-open set V in Y,
(ii) M-B continuous if $f^{-1}(V)$ is M-B set in X for every m_y-open set V in Y,
(iii) M-C continuous if $f^{-1}(V)$ is m-C set in X for every m_y-open set V in Y.

Theorem 5.1. Let (X, m_x) have property $\mathscr{B}$. Then a subset S of X is regular m-open if and only if it is both m-preopen and m-t set.
Proof: Let S be a regular m-open.
By Theorem 4.3, S is m-t set. Also by Theorem 4.4, S is m-open.
Thus, S is m-preopen.
Conversely, let S be both m-preopen and m-t set.
Since m-Int(S) $\subseteq$ S $\subseteq$ m-Int(m-Cl(S)) = m-Int(S), S = m-Int(m-Cl(S)).
Hence, S is regular m-open.

Theorem 5.2. Let (X, m_x) have property $\mathscr{B}$ and property I. Then a subset S of X is m-open if and only if it is both m-α-open and m-A set.
Proof: Let S be an m-open. Then S is m-α-open and by Theorem 4.1, S is m-A set.
Conversely, let S be both m-α-open and m-A set.
Since S is m-A set, S = M $\cap$N where M is m-open and N $\in$ m-RC(X).
Since S is m-α-open,

$$\begin{aligned} M \cap N &\subseteq \text{m-Int}(\text{m-Cl}(\text{m-Int}(M \cap N))) \\ &\subseteq \text{m-Int}(\text{m-Cl}(\text{m-Int}(M) \cap \text{m-Int}(N))) \\ &= \text{m-Int}(\text{m-Cl}(M \cap \text{m-Int}(N))) \\ \text{as M is m-open} &\subseteq \text{m-Int}(\text{m-Cl}(M) \cap \text{m-Cl}(\text{m-Int}(N))) \\ &= \text{m-Int}(\text{m-Cl}(M) \cap N) \\ \text{As } N \in \text{m-RC}(X) &\subseteq \text{m-Int}(\text{m-Cl}(M)) \cap \text{m-Int}(N) \end{aligned} \tag{1}$$

Since M $\subset$ m-Int(m-Cl(M)), by (1)

$$\begin{aligned} S &= M \cap N = (M \cap N) \cap M \\ &\subseteq (\text{m-Int}(\text{m-Cl}(M)) \cap \text{m-Int}(N)) \cap M \\ &= M \cap \text{m-Int}(N) \\ &= \text{m-Int}(M \cap N) \text{ by property I} = \text{m-Int}(S). \end{aligned}$$

Therefore, S $\subseteq$ m-Int(S). But m-Int(S) $\subseteq$ S.
Hence, S is m-open.

Theorem 5.3. Let (X, m_x) have property $\mathscr{B}$ and property I. Then a subset S of X is m-open if and only if it is both m-α-open and m-B set.
Proof: Let S be an m-open. Then S is m-α-open. Also, by Theorem 4.6, S is m-B set.
Conversely, let S be both m-α-open and m-B set.
Since S is m-B set, S = X $\cap$ S where X is m-open and S is m-t set.
Then S = X $\cap$ S $\subseteq$ X $\cap$ m-Int(m-Cl(S)) (as S is m-preopen) = X $\cap$ m-Int(S) (as S is m-t set).
We have S $\subseteq$ X $\cap$ m-Int(S).

Hence $S \subseteq$ m-Int($X \cap S$) by property I and $S \subseteq$ m-Int(S).
But always m-Int(S) $\subseteq$ S. Thus S = m-Int(S) and by property $\mathcal{B}$, S is m-open.

Theorem 5.4. Let (X, m_x) have property $\mathcal{B}$ and property I. Then a subset S of X is m-open if and only if it is both m-α-open and m-C set.

Proof: Let S be an m-open in X. Then S is m-α-open and by Theorem 4.11, S is m-C set.

Conversely, let S be both m-α-open and m-C set.
Since S is m-C set, $S = M \cap N$ where M is m-open and N is m-h set.
Since S is m-α-open set,

$$\begin{aligned} S &\subseteq \text{m-Int(m-Cl(m-Int(S)))} \\ &= \text{m-Int(m-Cl(m-Int(M)))} \cap \text{m-Int(m-Cl(m-Int(N)))} \\ &= \text{m-Int(m-Cl(M))} \cap \text{m-Int(N) (as M is m-open and N is m-h set).} \end{aligned}$$

$$\begin{aligned} \text{Now } S &= M \cap N \\ &= M \cap (M \cap N) \\ &= M \cap S \\ &\subset M \cap (\text{m-Int(m-Cl(M))} \cap \text{m-Int(N)}) \\ &= M \cap \text{m-Int(N) (as } M \subseteq \text{m-Int(m-Cl(M)))} \\ &= \text{m-Int}(M \cap N) \text{ (by property I)} = \text{m-Int(S)}. \end{aligned}$$

Thus, $S \subseteq$ m-Int(S) and m-Int(S) $\subseteq$ S.
Hence, by property $\mathcal{B}$, S is m-open.

Theorem 5.5. Let (X, m_x) have property $\mathcal{B}$ and property I and $f : X \to Y$ be a mapping. Then f is M-continuous if and only if

(i) it is M-α-continuous and M-A continuous.
(ii)it is M-α-continuous and M-B continuous.
(iii) it is M-α-continuous and M-C continuous.

Proof: It is the decompositions of M-continuity from Theorems 5.2, 5.3 and 5.4.

Remark 5.1. In the above four theorems, both properties are used and so the above four theorems are nothing but topological results.

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