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Intuitionistic Q_1-Fuzzy k-ideals of Semi-Ring

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Abstract. A Q-fuzzy set is a mapping from $X \times Q \to [0,1]$ where X is the universe of discourse and Q is a non-empty set. Some works has been emanated for this Q-fuzzy set. In all the above work the set 'Q' is treated as a non-empty set without any algebraic structure. This article presents an algebraic structure for Q-fuzzy set over a semiring named as Q_1-fuzzy set and provide some properties and results.

Keywords: Semi-ring, right (left) k-subsemigroup, Q_1-fuzzy right (left) k-ideal, intuitionistic Q_1- fuzzy right (left) k-ideal, Q-fuzzy set, intuitionistic Q-fuzzy set.

AMS Mathematics Subject Classification (2010): 03E72, 20M12

1. Introduction

Following the introduction of fuzzy set theory by Zadeh [18], the fuzzy set theory proclaimed by Zadeh himselfand others have found many applications in various field of Science, Engineering, Medical science etc. Pu and Liu [13] deduced the notion of fuzzy points, the fuzzy points of a semi-group G_S are the key tools to describe the algebraic system of G_S. Wang et al., [17] characterized fuzzy ideals as fuzzy points of semi-group. Liu [10] exposed fuzzy subrings as well as fuzzy ideals in rings. Malik and Mordeson [11] investigated some properties of fuzzy ideals in semi-ring. Rosenfeld [14], Das [15] and Bhattacharya [2] found the connection between fuzzy groups and so called level subgroups. Henriksen [4] defined a more restricted class of ideals in semiring, which is called the class of k-ideals, with the property that if the semiring S is a ring then a complex in S is a k-ideal iff it is a ring ideal. Dutta and Biswas [3] studied fuzzy k-ideals of semiring. Kar and Purkait [6] characterized the regularity of some semiring by special fuzzy k-ideals and general properties of fuzzy k-ideals. Notions of fuzzy k-ideals, prime fuzzy k-ideals and semi regularity of semiring are described by Ahsen et al., [5]. Atanassov [1] introduced intuitionistic fuzzy sets which is a generalization of the notion of fuzzy sets. The fuzzy sets give the degree of membership of an element in a given set, while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. Muhammad and dudek [12] put forth the concept of IFS to semirings and intuitionistic fuzzy left k-ideals of semirings and analyzed their properties. Solairaja and Nagarajan [16] constructed Q-fuzzy group. Kim [7] studied intuitionistic Q-fuzzy semiprime ideal in semigroups. Lekkoksung [8,9] investigated some properties of a Q-fuzzy interior ideal of a semigroup and further applied the concept of intuitionitic Q-fuzzy set to semiring and intuitionistic Q-fuzzy right k-idealin semiring.

In this article, the universe of discouse is taken as a semiring and the set Q is considered as a semigroup. In section 2, basic definitions needed for the study are recalled. In section 3, right (left) k-subsemigroup of semigroup is introduced with an example. Strongness between Q-fuzzy (semiring, ideals) and Q_1-fuzzy (semiring, ideals) are analyzed with suitable examples. Level subsets for the Q_1-fuzzy set has been defined and the level set properties are discussed. Further Q_1-fuzzy k-ideal has been introduced and an interesting results with k-subsemigroup is studied. The same has been extended to intuitionistic fuzzy semiring.

2. Preliminaries

In this section we recall some definitions needed for our study.

Definition 2.1. A non-empty set S together with two binary operation '+' and '.' is said to be a semiring if

1) $(S,+)$ is a commutative semigroup,
2) $(S,.)$ is a semigroup,
3) $a(b+c)=ab+ac$ and $(a+b)c=ac+bc\ \forall\, a,b,c\in S.$

We say that a semiring S has a zero if there exists an element $0\in S$ such that $0x=x0=0$ and $0+x=x+0=x$ for all $x\in S.$

Definition 2.2. A non-empty subset A of S is said to be a subsemiring of S if A is closed under the operation of addition and multiplication in S.

Definition 2.3. A subsemiring of S is called a right (*left*) ideal of S if for all $r\in S, x\in I, xr\in I(rx\in I).$ A subsemiring I of a semiring S is called an ideal of S if it is both left and right ideal.

Definition 2.4. An (A right (left)) ideal I of a semiring S is called a (right (left)) k-ideal of a semiring S if $x+y, y\in I$ implies $x\in I.$

Definition 2.5. A mapping $\mu: X\times Q\to[0,1]$, where X, Q are arbitrary non-empty sets, is called a Q-fuzzy set of X. An upper level set of a Q-fuzzy set μ denoted by $U(\mu;t)$ is defined as $U(\mu;t)=\{x\in X \mid \mu(x,q)\geq t, \forall q\in Q\}$ and a lower level set of a Q-fuzzy set μ denoted by $L(\mu;t)$ is defined as $L(\mu;t)=\{x\in X \mid \mu(x,q)\leq t, \forall q\in Q\},$ for all $t\in[0,1].$

Definition 2.6. A fuzzy set μ of a semiring S is said to be a fuzzy semiring if for all $x, y \in S$,
1) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}$
2) $\mu(xy) \geq min\{\mu(x), \mu(y)\}$.

Definition 2.7. A fuzzy set μ is called a fuzzy left ideal(right ideal) of semiring S if for all $x, y \in S$
1) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}$
2) $\mu(xy) \geq \mu(y)(\mu(xy) \geq \mu(x))$.

Definition 2.8. A fuzzy left ideal μ is called a fuzzy left k-ideal of a semiring S if for all $x, y, z \in S, x+y=z$ implies $\mu(x) \geq min\{\mu(y), \mu(z)\}$.

Definition 2.9. An intuitionistic fuzzy set defined on non-empty sets X is an object of the form $A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle \mid x \in X\}$, where the function $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the non-membership (namely $\lambda_A(x)$) for each element $x \in X$, to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for each $x \in X$. Obviously, every intuitionistic Q-fuzzy set μ we can have $A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle \mid x \in X\}$.
For any intuitionistic fuzzy set A,B of X, define
$(A \cap B)(x) = (min\{\mu_A(x), \mu_B(x)\}, max\{\lambda_A(x), \lambda_B(x)\})$ and
$(A \cup B)(x) = (max\{\mu_A(x), \mu_B(x)\}, min\{\lambda_A(x), \lambda_B(x)\})$

Definition 2.10. An intuitionistic Q-fuzzy set defined on non-empty sets X and Q is an object of the form $A = \{\langle x, q, \mu_A(x,q), \lambda_A(x,q)\rangle \mid x \in X, q \in Q\}$, where the function $\mu_A : X \times Q \to [0,1]$ and $\lambda_A : X \times Q \to [0,1]$ denote the degree of membership (namely $\mu_A(x,q)$) and the non-membership (namely $\lambda_A(x,q)$) for each element $x \in X, q \in Q$ to the set A, respectively, and $0 \leq \mu_A(x,q) + \lambda_A(x,q) \leq 1$ for each $x \in X$, $q \in Q$. Obviously, every Q-fuzzy set μ we can have $A = \{\langle x, q, \mu_A(x,q), \lambda_A(x,q)\rangle \mid x \in X, q \in Q\}$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic Q-fuzzy set $A = \{\langle x, q, \mu_A(x,q), \lambda_A(x,q)\rangle \mid x \in S, q \in Q\}$. Obviously for an IQFS $A = (\mu_A, \lambda_A)$ in X, when $\lambda(x,q) = 1 - \mu(x,q)$, for every $x \in X, q \in Q$, the IQFS A is a QFS.

Example 2.11. Consider the semiring $S = (Z_6, \oplus, \odot)$ and Q be any non empty set. Let $A = \{0, 2, 4\} \subseteq S$. Define $\mu_A : S \times Q \to [0,1]$ and $\lambda_A : S \times Q \to [0,1]$ as

$$\mu_A(x,q)=\begin{cases}0.7, \text{if } x\in A, q\in Q,\\ 0.3, otherwise\end{cases}$$

and

$$\lambda_A(x,q)=\begin{cases}0.2, \text{if } x\in A, q\in Q,\\ 0.6, otherwise.\end{cases}.$$

Clearly $A=(\mu_A,\lambda_A)$ is an Intuitionistic Q - fuzzy set.

Definition 2.12. Consider a fuzzy set μ of a semiring S with the following condition

1) $\mu(x+y,q)\geq min\{\mu(x,q),\mu(y,q)\}$
2) $\mu(xy,q)\geq min\{\mu(x,q),\mu(y,q)\}$
3) $\mu(xy,q)\geq \mu(y,q)$
4) $(\mu(xy,q)\geq \mu(x,q))$ for all $x,y\in S, q\in Q$.

Then μ is said to be a Q-fuzzy semiring if it satisfies (1) and (2) Q-fuzzy left ideal if it satisfies (1) and (3) and Q-fuzzy right ideal if it satisfies (1) and (4).

Throughout this paper $(Q_1,.)$ is a semigroup.

3. Q_1- fuzzy ideals of semi-ring

Definition 3.1. Let $(G_S,.)$ be a semigroup. A subsemigroup A of G_S is said to be a right (left) k-subsemigroup of G_S if $r_1.r_2\in A$ and $r_2\in A (r_1\in A)$ implies $r_1\in A$ $(r_2\in A)$.

In the following example we show that a subsemigroup of a semigroup need not be a right(left) k-subsemigroup of a semigroup.

Example 3.2. Consider the semigroup $(Z_6,\odot)$. $I=\{0,1,3,5\}$ is clearly a subsemigroup of $(Z_6,\odot)$. But I is not a right (left) k-subsemigroup of $(Z_6,\odot)$. If $r_1=2, r_2=3$ then $r_1.r_2=0\in I$ and $r_2=3\in I$ but $r_1=2\notin I$. If $I=\{1,3,5\}$, then I is a right (left) k-subsemigroup of $(Z_6,\odot)$.

Definition 3.3. Let $(Q_1,.)$ be a semigroup. A Q_1-fuzzy set μ of a semi-ring S is said to be a Q_1-fuzzy semi-ring if

1) $\mu(x+y,q)\geq min\{\mu(x,q),\mu(y,q)\}$
2) $\mu(xy,q)\geq min\{\mu(x,q),\mu(y,q)\}$
3) $\mu(x,q_1.q_2)\geq min\{\mu(x,q_1),\mu(x,q_2)\}\ \forall x,y\in S, q,q_1,q_2\in Q_1$.

Example 3.4. Consider the semi-ring $S=(Z_6,\oplus,\odot)$ and $Q_1=(Z_4,\odot)$.

Let $A=\{0,2,4\}\subseteq S$ and $M=\{0,1\}\subseteq Q_1$. Define $\mu: S\times Q_1\rightarrow[0,1]$ as

$$\mu(x,q)=\begin{cases}1, \text{if } x\in A, and\ q\in M,\\ 0, otherwise\end{cases}$$

Clearly it is a Q_1-fuzzy semi-ring of S.

Remark 3.5. The following example shows that every Q_1-fuzzy semi-ring is Q-fuzzy semi-ring of S, but the converse need not be true.

Example 3.6. Consider the semi-ring $S=(Z_6, \oplus, \odot)$ and $Q=(Z_4, \odot)$. Let $A=\{0, 2,4\}\subseteq S$ and $M=\{2,3\}\subseteq Q$.

Define $\mu: S\times Q\to[0,1]$ as

$$\mu(x,q)=\begin{cases}1, \text{if } x\in A, and\ q\in M,\\ 0, otherwise\end{cases}.$$

Since $0=\mu(4,0)=\mu(4,2\times2)\ngeq\mu(4,2)\wedge\mu(4,2)=1$. It does not satisfy condition (3) for Q_1-fuzzy set μ of a semi-ringS. Clearly μ is a Q-fuzzy semi-ring but not Q_1-fuzzy semi-ring. Therefore Q-fuzzy semi-ring does not imply Q_1-fuzzy semi-ring S.

Definition 3.7. An upper level set of a Q_1-fuzzy set μ is denoted by,

1) If $q\in Q_1$ is fixed then $U_q(\mu;t)=\{x\in S\mid \mu(x,q)\geq t\}$,

2) If $x\in S$ is fixed then $\bar{U}_x(\mu;t)=\{q\in Q_1\mid \mu(x,q)\geq t\}$

and a lower level set of a Q_1-fuzzy set μ is denoted by,

1) If $q\in Q_1$ is fixed then $L_q(\mu;t)=\{x\in S\mid \mu(x,q)\leq t\}$,

2) If $x\in S$ is fixed then $\bar{L}_x(\mu;t)=\{q\in Q_1\mid \mu(x,q)\leq t\}$.

Lemma 3.8. Let μ be a Q_1-fuzzy set of a semiring S. Then μ is a Q_1-fuzzy semiring of S iff $U_q(\mu;t)$ is a semiring of S and $\bar{U}_x(\mu;t)$ is a subsemigroup of Q_1 for $q\in Q_1$, $x\in S$ and for all $t\in[0,1]$ whenever nonempty.

Proof: Suppose μ is a Q_1-fuzzy semiring of S and $U_q(\mu;t)$ and $\bar{U}_x(\mu;t)$ are non-empty for $t\in[0,1]$. Let $x,y\in U_q(\mu;t)$. Then $\mu(x,q)\geq t$, $\mu(y,q)\geq t$. Since $\mu(x+y,q)\geq\mu(x,q)\wedge\mu(y,q)\geq t$, implies that $x+y\in U_q(\mu;t)$. Again since $\mu(x\ y, q)\geq\mu(x,q)\wedge\mu(y,q)\geq t$, implies that $xy\in U_q(\mu;t)$. Therefore $U_q(\mu;t)$ is a semiring of S. Let $q_1,q_2\in\bar{U}_x(\mu;t)$ implies that $\mu(x,q_1)\geq t, \mu(x,q_2)\geq t$. Now $\mu(x,q_1.q_2)\geq\mu(x,q_1)\wedge\mu(x,q_2)\geq t$. This implies that $q_1.q_2\in\bar{U}_x(\mu;t)$. Therefore $\bar{U}_x(\mu;t)$ is a subsemigroup of Q_1.

Conversely, assume that each non-empty set $U_q(\mu;t)$ is a semiring of S and $\bar{U}_x(\mu;t)$ is a subsemigroup of Q_1 for all $t\in[0,1]$. If there exists $x,y\in S$ and $q\in Q_1$ such that $\mu(x+y,q)<\mu(x,q)\wedge\mu(y,q)$. Let $t\in[0,1]$ such that $\mu(x+y,q)<t \leq\mu(x,q)\wedge\mu(y,q)$. This shows that, for $x,y\in U_q(\mu;t)$, $x+y\notin U_q(\mu;t)$. This is a contradiction to the fact that $U_q(\mu;t)$ is a semiring of S and hence $\mu(x+y,q)\geq \mu(x,q)\wedge\mu(y,q)\ \forall x,y\in S$. Suppose there exists $x,y\in S$ and $q\in Q_1$ such that $\mu(xy,q)<\mu(x,q)\wedge\mu(y,q)$. Let $t\in[0,1]$ such that $\mu(xy,q)<t\leq\mu(x,q)\wedge\mu(y,q)$. This in case shows that $xy\notin U_q(\mu;t)$ for $x,y\in U_q(\mu;t)$. This is a contradiction to the fact that $U_q(\mu;t)$ is a semiring of S and hence $\mu(xy,q)\geq\mu(x,q)\wedge\mu(y,q)$ $\forall x,y\in S$. Similarly if there exists $x\in S$ and $q_1,q_2\in Q_1$ such that $\mu(x,q_1.q_2)< \mu(x,q_1)\wedge\mu(x,q_2)$. Let $t\in[0,1]$ such that $\mu(x,q_1.q_2)<t\leq\mu(x,q_1)\wedge\mu(x,q_2)$. This in turn shows for $q_1,q_2\in\bar{U}_x(\mu;t)$, $q_1.q_2\notin\bar{U}_x(\mu;t)$ a contradiction to our assumption that $\bar{U}_x(\mu;t)$ is a subsemigroup of Q_1. Hence $\mu(x,q_1.q_2)\geq\mu(x,q_1)\wedge\mu(x,q_2)$ $\forall x\in S, q_1,q_2\in Q_1$. Therefore μ is a Q_1 - fuzzy semiring of S.

Definition 3.9. A Q_1-fuzzy set μ of a semiring S is said to be a Q_1- fuzzy right (left) ideal if

1) $\mu(x+y,q)\geq\mu(x,q)\wedge\mu(y,q)$
2) $\mu(xy,q)\geq\mu(x,q)[\mu(xy,q)\geq\mu(y,q)]$
3) $\mu(x,q_1.q_2)\geq\mu(x,q_1)(\mu(x,q_2))\forall x,y\in S,q,q_1,q_2\in Q_1$.

Example 3.10. Consider the semiring $S=(Z_6,\oplus,\odot)$ and $Q_1=(Z_4,\odot)$. Let $A=\{0,2,4\}\subseteq S$ and $M=\{0,1\}\subseteq Q_1$. Let $\mu:S\times Q_1\to[0,1]$ be defined as in Example [3.4]. Clearly μ is a Q_1- fuzzy right (left) ideal of a semiring S.

Remark 3.11. Every Q_1-fuzzy right(left) ideal of a semiring is Q-fuzzy right (left) ideal of a semiring S, but the converse need not be true.

Example 3.12. Consider the semiring $S=(Z_6,\oplus,\odot)$ and $Q=(Z_4,\odot)$. Let $A=\{0,2,4\}\subseteq S$ and $M=\{2,3\}\subseteq Q$, define $\mu:S\times Q\to[0,1]$ by

$$\mu(x,q)=\begin{cases}1,\text{if } x\in A, q\in M,\\ 0, otherwise.\end{cases}$$

Clearly $\mu(x,q)$ is a Q-fuzzy right (left) ideal of a semiring S, but not Q_1-fuzzy right (left) ideal of a semiring S, since $\mu(0,1)=0\ngeq\mu(0,2)=1$.

Theorem 3.13. Let μ is a Q_1-fuzzy set of a semiring S. Then μ is a Q_1-fuzzy right (left) ideal of S iff $U_q(\mu;t)$ is a right (left) ideal of S and $\bar{U}_x(\mu;t)$ is a right (left) of Q_1 for $q \in Q_1$, $x \in S$ and for all $t \in [0,1]$ whenever nonempty.

Proof: Suppose μ is a Q_1-fuzzy right ideal of S and $U_q(\mu;t), \bar{U}_x(\mu;t)$ are non-empty for $t \in [0,1]$. Let $x, y \in U_q(\mu;t)$. Then $\mu(x,q) \geq t,\ \mu(y,q) \geq t.$ Since $\mu(x+y,q) \geq \mu(x,q) \wedge \mu(y,q) \geq t,$ we have $x+y \in U_q(\mu;t)$. For $x \in U_q(\mu;t), y \in S,\ \mu(x\ y, q) \geq \mu(x,q) \geq t,$ which yields $xy \in U_q(\mu;t)$ for $x \in U_q(\mu;t)$. Therefore $U_q(\mu;t)$ is a right ideal of S. Let $q_1 \in \bar{U}_x(\mu;t)$ and $q_2 \in Q_1$ then $\mu(x,q_1) \geq t.$ Indeed $\mu(x,q_1.q_2) \geq \mu(x,q_1) \geq t.$ This implies that $q_1.q_2 \in \bar{U}_x(\mu;t)$. Therefore $\bar{U}_x(\mu;t)$ is a right of Q_1.

Conversely, assume that each non-empty set $U_q(\mu;t)$ is a right ideal of S and $\bar{U}_x(\mu;t)$ is a right ideal of Q_1. If there exists $x, y \in S$ and $q \in Q_1$ such that $\mu(x+y,q) < \mu(x,q) \wedge \mu(y,q)$. Let $t \in [0,1]$ such that $\mu(x+y,q) < t \leq \mu(x,q) \wedge \mu(y,q)$. Then for $x, y \in U_q(\mu;t)$, we have $x+y \notin U_q(\mu;t)$. This is a contradiction to the hypothesis that $U_q(\mu;t)$ is a right ideal of S and hence $\mu(x+y,q) \geq \mu(x,q) \wedge \mu(y,q)\ \forall\, x,y \in S$. Again if there exists $x, y \in S$ and $q \in Q_1$ such that $\mu(xy,q) < \mu(x,q)$. Let $t \in [0,1]$ such that $\mu(xy,q) < t \leq \mu(x,q)$. This shows that for $x \in U_q(\mu;t), xy \notin U_q(\mu;t)$ contradiction to our hypothesis that $U_q(\mu;t)$ is a right ideal of S and hence $\mu(xy,q) \geq \mu(x,q) \forall x, y \in S$ and $q \in Q_1$. Similarly, if $\mu(x,q_1.q_2) < \mu(x,q_1)$ for $x \in S$ and $q_1, q_2 \in Q_1$. Let $t \in [0,1]$ such that $\mu(x,q_1.q_2) < t \leq \mu(x,q_1)$. Then for $q_1 \in \bar{U}_x(\mu;t),\ q_1.q_2 \notin \bar{U}_x(\mu;t)$, a contradiction to our assumption that $\bar{U}_x(\mu;t)$, is a right ideal of Q_1. Hence $\mu(x,q_1.q_2) \geq \mu(x,q_1)$ $\forall\, x \in S, q_1, q_2 \in Q_1$. Therefore μ is a Q_1-fuzzy right ideal of S.

Definition 3.14. A Q_1-fuzzy right (left) ideal μ of a semiring S is called Q_1 - fuzzy right (left) k - ideal if for all $x, y \in S$ and $q, q_1, q_2 \in Q_1$

1) $\mu(x,q) \geq \mu(x+y,q) \wedge \mu(y,q)$
2) $\mu(x,q_1) \geq \mu(x,q_1.q_2) \wedge \mu(x,q_2)$ $[\mu(x,q_2) \geq \mu(x,q_1.q_2) \wedge \mu(x,q_1)]$.

Example 3.15. Consider the semiring $S = (Z_6, \oplus, \odot)$ and $Q_1 = (Z_4, \odot)$. Let $A = \{0,2,4\} \subseteq S$ and $M = \{0,1\} \subseteq Q_1$. Let $\mu : S \times Q_1 \to [0,1]$ be defined as in Example [3.4]. Clearly μ is a Q_1 - fuzzy right (left) k - ideal of a semiring S.

Remark 3.16. Every Q_1-fuzzy right (left) k-ideal of a semiring is Q-fuzzy right (left) k-ideal of a semiring S, but the converse need not be true.

Example: 3.17. Consider the semiring $S=(Z_6,\oplus,\odot)$ and $Q=(Z_4,\odot)$. Let $A=\{0, 2,4\}\subseteq S$ and $M=\{2,3\}\subseteq Q$, define $\mu:S\times Q\to[0,1]$ by

$$\mu(x,q)=\begin{cases}1,\text{if } x\in A, q\in M,\\ 0, otherwise.\end{cases}$$

Clearly $\mu(x,q)$ is a Q-fuzzy right (left) k-ideal of a semiring S, but not Q_1-fuzzy right(left) k-ideal of a semiring S, since $\mu(0,1)=0\ngeq\mu(0,2)\wedge\mu(0,2)=1$.

Lemma 3.18. Let μ be a Q_1-fuzzy set of a semiring S . Then μ is a Q_1-fuzzy right (left) k-ideal of S iff $U_q(\mu;t)$ is a right (left) k-ideal of S and $\bar{U}_x(\mu;t)$ is a right (left) k-subsemigroup of Q_1 for $q\in Q_1$ and $x\in S$ and for all $t\in[0,1]$ whenever nonempty.

Proof: By Lemma [3.8] $U_q(\mu;t)$ is a right ideal. Let μ be a Q_1-fuzzy right k-ideal of a semiring S. If there exists $x,y\in S, q\in Q_1$ such that $x+y, y\in U_q(\mu;t)$ then $\mu(x+y,q)\geq t$ and $\mu(y,q)\geq t$. Since μ is a Q_1 fuzzy right k-ideal $\mu(x,q)\geq\mu(x+y,q)\wedge\mu(y,q)\geq t$. Thus for $x+y,y\in U_q(\mu;t)$ and $q\in Q_1$ we have $x\in U_q(\mu;t)$. Hence $U_q(\mu;t)$ is a right k-ideal of S.

Similarly if there exists $q_1.q_2, q_2\in\bar{U}_x(\mu;t)$ then $\mu(x,q_1.q_2)\geq t$ and $\mu(x,q_2)\geq t$. Since μ is a Q_1 fuzzy right k-ideal $\mu(x,q_1)\geq\mu(x,q_1.q_2)\wedge\mu(x,q_2)\geq t$ and so $\mu(x,q_1)\geq t$. Therefore $q_1\in\bar{U}_x(\mu;t)$, implying that $\bar{U}_x(\mu;t)$ is a $k-$subsemigroup of Q_1.

Conversely assume that, $U_q(\mu;t)$ is a right k-ideal of S and $\bar{U}_x(\mu;t)$ is a k-subsemigroup of Q_1 for $q\in Q_1$, $x\in S$ and for all $t\in[0,1]$ whenever nonempty. If there exists $x,y\in S$, $q\in Q_1$ such that $\mu(x,q)<\mu(x+y,q)\wedge\mu(y,q)$. Let $t\in[0,1]$ such that $\mu(x,q)<t\leq\mu(x+y,q)\wedge\mu(y,q)$. This shows $x+y,y\in U_q(\mu;t)$ but $x\notin U_q(\mu;t)$. This is a contradiction to the fact that $U_q(\mu;t)$ is a right k-ideal of S. Similarly, if $q_2, q_1.q_2\in\bar{U}_x(\mu;t)$ and $\mu(x,q_1)<\mu(x,q_1.q_2)\wedge\mu(x,q_2)$. Let $t\in[0,1]$ such that $\mu(x,q_1)<t\leq\mu(x,q_1.q_2)\wedge\mu(x,q_2)$. This means $q_2,q_1.q_2\in\bar{U}_x(\mu;t)$ but $q_1\notin\bar{U}_x(\mu;t)$. This is a contradiction to the fact that $\bar{U}_x(\mu;t)$ is a k-subsemigroup of Q_1. Therefore μ is a Q_1-fuzzy right k-ideal of the semiring S.

Definition 1.19. An intuitionistic Q_1-fuzzy set of a semiring S is an object of the form $A=\{\langle x,q,\mu_A(x,q),\lambda_A(x,q)\rangle \mid x\in S, q\in Q_1$ with $\mu(x,q)+\lambda(x,q)\leq 1\}$ is called an intuitionistic Q_1 - fuzzy semiring of S if

1) $\mu_A(x+y,q)\geq \mu_A(x,q)\wedge \mu_A(y,q)$
2) $\mu_A(xy,q)\geq \mu_A(x,q)\wedge \mu_A(y,q)$
3) $\mu_A(x,q_1.q_2)\geq \mu_A(x,q_1)\wedge \mu_A(x,q_2)$
4) $\lambda_A(x+y,q)\leq \lambda_A(x,q)\vee \lambda_A(y,q)$
5) $\lambda_A(xy,q)\leq \lambda_A(x,q)\vee \lambda_A(y,q)$
6) $\lambda_A(x,q_1.q_2)\leq \lambda_A(x,q_1)\vee \lambda_A(x,q_2) \forall x,y\in S, q,q_1,q_2\in Q.$

Example 3.20. Consider the semiring $S=(Z_6,\oplus,\odot)$ and $Q_1=(Z_4,\odot)$. Let $A=\{0, 2,4\}\subseteq S$ and $M=\{0,1\}\subseteq Q_1$. Define $\mu_A: S\times Q_1\to[0,1]$ and $\lambda_A: S\times Q_1\to[0,1]$ as

$$\mu_A(x,q)=\begin{cases}0.7, \text{if } x\in A, q\in M,\\ 0.3, otherwise\end{cases} \quad \text{and } \lambda_A(x,q)=\begin{cases}0.2, \text{if } x\in A, q\in M,\\ 0.6, otherwise.\end{cases}$$

Clearly it is an intuitionistic Q_1-fuzzy semiring of S.

Remark 3.21. Every intuitionistic Q_1-fuzzy semiring is an intuitionistic Q-fuzzy semiring of S but the converse need not be true.

Example 3.22. Consider the semiring $S=(Z_6,\oplus,\odot)$ and $Q=(Z_4,\odot)$. Let $A=\{0, 2,4\}\subseteq S$ and $M=\{2,3\}\subseteq Q$.
Define $\mu: S\times Q\to[0,1]$ and $\lambda_A: S\times Q_1\to[0,1]$ as

$$\mu_A(x,q)=\begin{cases}0.8, \text{if } x\in A, q\in M,\\ 0.3, otherwise\end{cases} \quad \text{and } \lambda_A(x,q)=\begin{cases}0.1, \text{if } x\in A, q\in M,\\ 0.6, otherwise\end{cases}.$$

Since $\mu_A(2,0)=0.3\ngeq \mu(2,2)\wedge\mu_A(2,2)=0.8$ and $\lambda_A(2,0)=0.6\nleq \lambda_A(2,2)\vee \lambda_A(2,2)=0.1$. It does not satisfies conditions (3 and 6) for intuitionistic Q_1-fuzzy semiring A of a semiring S. Therefore intuitionistic Q-fuzzy semiring does not implies intuitionistic Q_1-fuzzy semiring S.

Definition 3.23. Let $A=(\mu_A,\lambda_A)$ be an intuitionistic Q_1-fuzzy set of a semiring S and let $s,t\in[0,1]$. Then the set $S_{A(q)}^{(s,t)}=\{x\in S\mid \mu_A(x,q)\geq s, \gamma_A(x,q)\leq t, q\in Q_1\}$ and $S_{A(x)}^{(s,t)}=\{q\in Q_1\mid \mu_A(x,q)\geq s, \gamma_A(x,q)\leq t, x\in S\}$ is called a (s,t)- level set of $A=(\mu_A,\lambda_A)$. The set $\{(s,t)\in Im(\mu_A)\times Im(\lambda_A)\mid s+t\leq 1\}$ is called image of

$A=(\mu_A,\lambda_A)$.Clearly, $S^{(s,t)}_{A(q)}=U_q(\mu_A;s)\cap L_q(\lambda;t)$ and $S^{(s,t)}_{A(x)}=\bar{U}_x(\mu_A;s)\cap \bar{L}_x(\lambda;t)$, where $U_q(\mu_A;s)[\bar{U}_x(\mu_A;s)]$ and $L_q(\lambda;t)[\bar{L}_x(\lambda;t)]$ are upper and lower level subsets of μ_A and λ_A respectively.

Example 3.24. In Example 3.22, $S^{(.8,.1)}_{A(q)}=\{0,2,4\}$ and $S^{(.8,.1)}_{A(x)}=\{2,3\}$.

Definition 3.25. An intuitionistic Q_1-fuzzy set of a semiring S is an object of the form $A=\{\langle x,q,\mu_A(x,q),\lambda_A(x,q)\rangle \mid x\in S, q\in Q_1\}$ is called an intuitionistic Q_1 - fuzzy right(left) ideal of S if

1) $\mu_A(x+y,q)\geq \mu_A(x,q)\wedge \mu_A(y,q)$
2) $\mu_A(xy,q)\geq \mu_A(x,q)[\mu_A(xy,q)\geq \mu_A(y,q)]$
3) $\mu_A(x,q_1.q_2)\geq \mu_A(x,q_1)(\mu_A(x,q_2))$
4) $\lambda_A(x+y,q)\leq \lambda_A(x,q)\vee \lambda_A(y,q)$
5) $\lambda_A(xy,q)\leq \lambda_A(x,q)[\lambda_A(xy,q)\leq \lambda_A(y,q)]$
6) $\lambda_A(x,q_1.q_2)\leq \lambda_A(x,q_1)(\lambda_A(x,q_2))\forall x,y\in S, q,q_1,q_2\in Q_1$.

Example 3.26. Consider the set of all positive integer $S=Z_0^+$.Clearly it is a semiring under usual multiplication and consider the semigroup $Q_1=(Z_4,\odot)$. Define

$$\mu_{(x,q)}=\begin{cases}0.7,\text{if } x\in\langle 2\rangle, q\in Q_1,\\ 0.2, otherwise\end{cases}. \qquad \lambda(x,q)=\begin{cases}0.3,\text{if } x\in\langle 2\rangle, q\in Q_1,\\ 0.8, otherwise\end{cases}.$$

Then $A=(\mu,\lambda)$ is an intuitionistic Q_1 fuzzy right ideal of S.

Definition 3.27. An intuitionistic Q_1-fuzzy right (left) ideal A in S is said to be an intuitionistic Q_1-fuzzy right (left) k-ideal of S if

1) $\mu_A(x,q)\geq \mu_A(x+y,q)\wedge \mu_A(y,q)$
2) $\mu_A(x,q_1)\geq \mu_A(x,q_1.q_2)\wedge \mu_A(x,q_2)$ $(\mu_A(x,q_2)\geq \mu_A(x,q_1.q_2)\wedge \mu_A(x,q_1))$
3) $\lambda_A(x,q)\leq \lambda_A(x+y,q)\vee \lambda_A(y,q)$
4) $\lambda_A(x,q_1)\leq \lambda_A(x,q_1.q_2)\vee \lambda_A(x,q_2)$ $(\lambda_A(x,q_2)\leq \lambda_A(x,q_1.q_2)\vee \lambda_A(x,q_1))$

for all $x,y\in S$ and $q,q_1,q_2\in Q_1$.

Example 3.28. Consider the semiring $S=(Z_6,\oplus,\odot)$ and $Q_1=(Z_4,\odot)$. Let $A=\{0, 2,4\}\subseteq S$ and $M=\{0,1\}\subseteq Q_1$ defined as in Example [3.20]. Clearly it is an Q_1-fuzzy right(left) k-ideal of a semiring S.

Remark 3.29. Every Q_1-fuzzy right(left) k-ideal of a semiringis a Q-fuzzy right (left) k-ideal of a semiring S but the converse need not be true. We illustrate this through the following example.

Example 3.30. Consider the semiring $S=(Z_6,\oplus,\odot)$ and $Q=(Z_4,\odot)$. Let $A=\{0,2,4\}\subseteq S$ and $M=\{2,3\}\subseteq Q$. Define $\mu:S\times Q\to[0,1], \lambda:S\times Q\to[0,1]$ as

$$\mu_A(x,q)=\begin{cases}1, \text{if } x\in A, q\in M,\\ 0, otherwise\end{cases} \quad \text{and} \quad \lambda_A(x,q)=\begin{cases}0, \text{if } x\in A, q\in M,\\ 1, otherwise\end{cases}.$$

Clearly A is an intuitionistic Q-fuzzy right(left) k-ideal of a semiring S, but not an intuitionistic Q_1-fuzzy right(left) k-ideal of a semiring S. Since $\mu_A(0,1)=0\not\geq \mu_A(0,2)\wedge\mu_A(0,2)=1$ and $\lambda_A(0,1)=0\not\leq\lambda_A(0,2)\vee\lambda_A(0,2)=1$.

Theorem 3.31. An intuitionistic Q_1-fuzzy set $A=\{<x,q,\mu_A(x,q), \lambda_A(x,q)>|\, x\in S, q\in Q_1\}$ in S is an intuitionistic Q_1-fuzzy semiring of S iff any level set $S_{A(q)}^{(s,t)}$ is a semiring of S and $S_{A(x)}^{(s,t)}$ is a subsemigroup of Q_1 respectively for all $s,t\in[0,1]$ with $s+t\leq 1$ whenever nonempty.

Proof: Let A be an intuitionistic Q_1-fuzzy semiring of S and $S_{A(q)}^{(s,t)}$ and $S_{A(x)}^{(s,t)}$ are non-empty for all $s,t\in[0,1]$ with $s+t\leq 1$. Let $x,y\in S_{A(q)}^{(s,t)}$. Then $\mu_A(x,q)\geq s$, $\mu_A(y,q)\geq s$, $\lambda_A(x,q)\leq t$ and $\lambda_A(y,q)\leq t$. Since $\mu_A(x+y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)\geq s$ and $\lambda_A(x+y,q)\leq\lambda_A(x,q)\vee\lambda_A(y,q)\leq t$, $x+y\in S_{A(q)}^{(s,t)}$. Again since $\mu_A(xy,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)\geq s$ and $\lambda_A(xy,q)\leq\lambda_A(x,q)\vee\lambda_A(y,q)\leq t$, $xy\in S_{A(q)}^{(s,t)}$. Therefore $S_{A(q)}^{(s,t)}$ is a semiring of S. Let $q_1,q_2\in S_{A(x)}^{(s,t)}$ implies that $\mu_A(x,q_1)\geq s, \mu_A(x,q_2)\geq s$, $\lambda_A(x,q_1)\leq t$ and $\lambda_A(x,q_2)\leq t$. Since $\mu_A(x,q_1.q_2)\geq\mu_A(x,q_1)\wedge\mu_A(x,q_2)\geq s$ and $\lambda_A(x,q_1.q_2)\leq\lambda_A(x,q_1)\vee\lambda_A(x,q_2)\leq t$ $\forall x\in S$, $q_1,q_2\in S_{A(x)}^{(s,t)}$. This implies that $q_1.q_2\in S_{A(x)}^{(s,t)}$. Therefore $S_{A(x)}^{(s,t)}$ is a subsemigroup of Q_1.

Conversely, assume that each non-empty set $S_{A(q)}^{(s,t)}$ is a semiring of S and $S_{A(x)}^{(s,t)}$ is a subsemigroup of Q_1. If there exists $x,y\in S$ and $q\in Q_1$ such that $\mu_A(x+y,q)<\mu_A(x,q)\wedge\mu_A(y,q)$ and $\lambda_A(x+y,q)>\lambda_A(x,q)\wedge\lambda_A(y,q)$. Let $s,t\in[0,1]$ such that $\mu(x+y,q)<s\leq\mu(x,q)\wedge\mu(y,q)$ and $\lambda_A(x+y,q)>t\geq\lambda_A(x,q)\vee\lambda_A(y,q)$. That is for all $x,y\in S_{A(q)}^{(s,t)}$ we have $x+y\notin S_{A(q)}^{(s,t)}$. This is a contradiction to the hypothesis that $S_{A(q)}^{(s,t)}$ is a semiring of S and hence $\mu_A(x+y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$ $\forall x,y\in S$. Suppose there exists $x,y\in S$ and $q\in Q_1$ such that $\mu_A(xy,q)<$

$\mu_A(x,q)\wedge\mu_A(y,q)$ and $\lambda_A(xy,q)>\lambda_A(x,q)\vee\lambda_A(y,q)$. Let $s,t\in[0,1]$ such that $\mu_A(xy,q)<s\le\mu_A(x,q)\wedge\mu_A(y,q)$ and $\lambda_A(xy,q)>t\ge\lambda_A(x,q)\vee\lambda_A(y,q)$. This in case shows that $xy\notin S^{(s,t)}_{A(q)}$ for all $x,y\in S^{(s,t)}_{A(q)}$. This is a contradiction to the fact that $S^{(s,t)}_{A(q)}$ is a semiring of S and hence $\mu_A(xy,q)\ge\mu_A(x,q)\wedge\mu_A(y,q)$ and $\lambda_A(xy,q)$ $\le\lambda_A(x,q)\vee\lambda_A(y,q)$ $\forall x,y\in S$.

Similarly if there exists $x\in S$ and $q_1,q_2\in Q_1$ such that $\mu_A(x,q_1.q_2)<$ $\mu_A(x,q_1)\wedge\mu_A(x,q_2)$ and $\lambda_A(x,q_1.q_2)>\lambda_A(x,q_1)\vee\lambda_A(x,q_2)$. Let $s,t\in[0,1]$ such that $\mu_A(x,q_1.q_2)<s\le\mu_A(x,q_1)\wedge\mu_A(x,q_2)$ and $\lambda_A(x,q_1.q_2)>t\ge\lambda_A(x,q_1)$ $\vee\lambda_A(x,$ $q_2)$. This in turn shows for $q_1,q_2\in S^{(s,t)}_{A(x)}$, $q_1.q_2\notin S^{(s,t)}_{A(x)}$. This is a contradiction to our assumption that $S^{(s,t)}_{A(x)}$ is a subsemigroup of Q_1. Hence $\mu_A(x,q_1.q_2)\ge\mu_A(x,q_1)\wedge$ $\mu_A(x,q_2)$ and $\lambda_A(x,q_1.q_2)\le\lambda_A(x,q_1)\vee\lambda_A(x,q_2)$ $\forall x\in S, q_1,q_2\in Q_1$. Therefore A is an intuitionistic Q_1 - fuzzy semiring of S.

Theorem 3.32. Let I be a non-empty subset of a semiring S and J be a non-empty subset of a semigroup Q_1. Then an intuitionistic Q_1-fuzzy set $A=(\mu,\lambda)$ defined by

$$\mu(x,q)=\begin{cases}s_2,\text{if } x\in I, q\in J,\\ s_1, otherwise\end{cases}\quad\text{and}\quad \lambda(x,q)=\begin{cases}t_2,\text{if } x\in I, q\in J,\\ t_1, otherwise,\end{cases}$$

where $0\le s_1<s_2\le1$, $0\le t_2<t_1\le1$ and $s_i+t_i\le1$ for each $i=1,2$ is an intuitionistic Q_1 fuzzy right (left) ideal of S if and only if I is a right ideal of S and J is a right ideal of Q_1.

Proof: Let I be a right ideal in S and J be a right ideal in Q_1. Let $x,y\in I$ and $q\in J$, then $x+y\in I$. Therefore $\mu(x+y,q)=s_2$, $\lambda(x+y,q)=t_2$, $\mu(x+y,q)=s_2=$ $\mu(x,q)\wedge\mu(y,q)$ and $\lambda(x+y,q)=t_2=\lambda(x,q)\wedge\lambda(y,q)$. If $x\, or\, y\notin I$, $q\in J$ then $\mu(x,q)\wedge\mu(y,q)=s_1\le\mu(x+y,q)$ and $\lambda(x,q)\wedge\lambda(y,q)=t_1\ge\lambda(x+y,q)$.

Let $x\in I$ *and* $q\in J$, $\mu(xy,q)=s_2=\mu(x,q)$. If $x\notin I\, and\, q\in J$ then $s_1=\mu(x,q)\ ,q)$ and $t_1=\lambda(x,q)\ge\lambda(xy,q)$. Again for $q_1\in J$ and $x\in I$, then $\mu(x,q_1)=s_2$ and $\mu(x,q_1q_2)=s_2$. Therefore $\mu(x,q_1q_2)=\mu(x,q_1)$ and $\lambda(x,q_1q_2)=\lambda(x,q_1)$ for $q_1\notin J$ and for any x, $\mu(x,q_1)=s_1\le\mu(x,q_1q_2)$ for any q_2 and $\lambda(x,q_1)=t_1\ge\lambda(x,q_1q_2)$.

Corollary 3.33. Let I be a non-empty subset of a semiring S and J be a non-empty subset of a semigroup Q_1. Then I is a right (left) ideal of S and J is a right (left) ideal of Q_1 if and only if the intuitionistic Q_1-fuzzy set $A=(\chi_I,1-\chi_I)$ defined by

$$\chi_I(x,q)=\begin{cases}1,\text{if } x\in I, q\in J,\\ 0, otherwise\end{cases}$$

is an intuitionistic Q_1-fuzzy right(left) ideal of S.

Theorem 3.34. An intuitionistic Q_1-fuzzy set $A=(\mu,\lambda)$ in S is an intuitionistic Q_1-fuzzy right (left) ideal of S if and only if the Q_1 fuzzy subsets μ and λ^c are Q_1 fuzzy right(left) ideals of S.

Proof: If $A=(\mu,\lambda)$ is an intuitionistic Q_1 fuzzy right ideal of S, then clearly μ is a Q_1 fuzzy right ideal of S. For all $x,y\in S, q\in Q_1$,

$\lambda^c(x+y,q)=1-\lambda(x+y,q)\geq 1-max\{\lambda(x,q),\lambda(y,q)\}$

$=min\{1-\lambda(x,q),1-\lambda(y,q)\}$ $=min\{\lambda^c(x,q),\lambda^c(y,q)\}$

and

$\lambda^c(xy,q)=1-\lambda(xy,q)\geq 1-\lambda(x,q)=\lambda^c(x,q).$

Also for $q_1,q_2\in Q_1$

$\lambda^c(x,q_1q_2)=1-\lambda(x,q_1q_2)\geq 1-\lambda(x,q_1)=\lambda^c(x,q_1).$

Thus λ^c is a Q_1-fuzzy right ideal of S.

Conversely assume that μ and λ^c are Q_1-fuzzy right ideals of S, then conditions 1,2 and 3 of Definition [3.25], are satisfied. Now for $x,y\in S$ and $q\in Q_1$

$1-\lambda(x+y,q)=\lambda^c(x+y,q)\geq min\{\lambda^c(x,q),\lambda^c(y,q)\}$

$=min\{1-\lambda(x,q),1-\lambda(y,q)\}$ $=1-max\{\lambda(x,q),\lambda(y,q)\}$

which implies $-\lambda(x+y,q)\geq -max\{\lambda(x,q),\lambda(y,q)\}$

implies $\lambda(x+y,q)\leq max\{\lambda(x,q),\lambda(y,q)\}$

and $1-\lambda(xy,q)=\lambda^c(xy,q)\geq\lambda^c(x,q)=1-\lambda(x,q)$

This implies $-\lambda(xy,q)\geq -\lambda(x,q)$ *implies* $\lambda(xy,q)\leq\lambda(x,q).$

Also for $q_1,q_2\in Q_1$, $1-\lambda(x,q_1q_2)=\lambda^c(x,q_1q_2)\geq\lambda^c(x,q_1)=1-\lambda(x,q_1)$

This implies $-\lambda(x,q_1q_2)\geq -\lambda(x,q_1)$ implies $\lambda(x,q_1q_2)\leq\lambda(x,q_1).$

Therefore $A=(\mu,\lambda)$ is an intuitionistic Q_1 fuzzy right ideal of S.

Corollary 3.35. Let $A=(\mu,\lambda)$ be an intuitionistic Q_1 fuzzy set in S. Then A is an intuitionistic Q_1 fuzzy right (left) ideal of S if and only if intuitionistic Q_1-fuzzy set $A_1=(\mu,\mu^c)$ and intuitionistic Q_1 fuzzy set $A_2=(\lambda^c,\lambda)$ are intuitionistic Q_1-fuzzy right (left) ideals of S.

Theorem 3.36. An intuitionistic Q_1-fuzzy set $A=\{<x,q,\mu_A(x,q),\ \lambda_A(x,q)>|\ x\in S,$ $q\in Q_1\}$ in S is an intuitionistic Q_1-fuzzy right (left) ideal in S iff level sets $S_{A(q)}^{(s,t)}$ is a right (left) ideal of S and $S^{(s,t)}_{A(x)}$ is a right (left) ideal of Q_1 respectively for all $s,t \in [0,1]$ with $s+t \leq 1$ whenever nonempty.

Proof: Suppose A is an intuitionistic Q_1-fuzzy right ideal of S and $S^{(s,t)}_{A(q)}$, $S^{(s,t)}_{A(x)}$ are non-empty for $s,t \in [0,1]$, with $s+t \leq 1$. Let $x,y \in S^{(s,t)}_{A(q)}$. Then $\mu_A(x,q) \geq s$, $\mu_A(y,q) \geq s$, $\lambda_A(x,q) \leq t$, $\lambda_A(y,q) \leq t$. Since $\mu_A(x+y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q) \geq s$ and $\lambda_A(x+y,q) \leq \lambda_A(x,q) \vee \lambda_A(y,q) \leq t$. We have $x+y \in S^{(s,t)}_{A(q)}$. Further $\mu_A(xy,q) \geq \mu_A(x,q) \geq s$ and $\lambda_A(xy,q) \leq \lambda_A(x,q) \leq t$. This yields $xy \in S^{(s,t)}_{A(q)}$ for $x,y \in S^{(s,t)}_{A(q)}$. Therefore $S^{(s,t)}_{A(q)}$ is a right ideal of S .

Let $q_1 \in S^{(s,t)}_{A(x)}$ and $q_2 \in Q_1$ then $\mu_A(x,q_1) \geq s$ and $\lambda_A(x,q_1) \leq t$. Indeed $\mu_A(x,q_1.q_2) \geq \mu_A(x,q_1) \geq s$ and $\lambda_A(x,q_1.q_2) \leq \lambda_A(x,q_1) \leq t$. This implies that $q_1.q_2 \in S^{(s,t)}_{A(x)}$. Therefore $S^{(s,t)}_{A(x)}$ is a right ideal of Q_1.

Conversely, assume that each non-empty set $S^{(s,t)}_{A(q)}$ is a right ideal of S and $S^{(s,t)}_{A(x)}$ is a right ideal of Q_1. If there exists $x,y \in S$ and $q \in Q_1$ such that $\mu_A(x+y,q) < \mu_A(x,q) \wedge \mu_A(y,q)$ and $\lambda_A(x+y,q) > \lambda_A(x,q) \wedge \lambda_A(y,q)$. Let $s,t \in [0,1]$ such that $\mu(x+y,q) < s \leq \mu(x,q) \wedge \mu(y,q)$ and $\lambda_A(x+y,q) > t \geq \lambda_A(x,q) \vee \lambda_A(y,q)$. That is for $x,y \in S^{(s,t)}_{A(q)}$ we have $x+y \notin S^{(s,t)}_{A(q)}$. This is a contradiction to the hypothesis that $S^{(s,t)}_{A(q)}$ is a right ideal of S and hence $\mu_A(x+y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$ and $\lambda_A(x+y,q) \leq \lambda_A(x,q) \vee \lambda_A(y,q)$ $\forall x,y \in S$. Again if there exists $x \in S$ and $q \in Q_1$ such that $\mu_A(xy,q) < \mu_A(x,q)$ and $\lambda_A(xy,q) > \lambda_A(x,q)$. Let $s,t \in [0,1]$ such that $\mu_A(xy,q) < s \leq \mu_A(x,q)$ and $\lambda_A(xy,q) > t \geq \lambda_A(x,q)$. This shows that for $x \in S^{(s,t)}_{A(x)}, xy \notin S^{(s,t)}_{A(q)}$ contradiction to our hypothesis that $S^{(s,t)}_{A(q)}$ is a right ideal of S and hence $\mu(xy,q) \geq \mu(x,q)$ $\forall x,y \in S$ and $q \in Q_1$. Similarly, if $\mu_A(x,q_1.q_2) < \mu_A(x,q_1)$ for $x \in S$ and $q_1,q_2 \in Q_1$. Let $s,t \in [0,1]$ such that $\mu_A(x,q_1.q_2) < s \leq \mu_A(x,q_1)$ and $\lambda_A(x,q_1.q_2) > t \geq \lambda_A(x,q_1)$. Then for $q_1 \in S^{(s,t)}_{A(x)}$, $q_1.q_2 \notin S^{(s,t)}_{A(x)}$ a contradiction to our assumption that $S^{(s,t)}_{A(x)}$ is a right ideal of Q_1. Hence $\mu(x,q_1.q_2) \geq \mu(x,q_1)$ and $\lambda_A(x,q_1) \leq \lambda_A(x,q_1.q_2)$ $\forall x \in S, q_1,q_2 \in Q_1$. Therefore A is an intuitionistic Q_1-fuzzy right ideal of S.

Theorem 3.37. An intuitionistic Q_1-fuzzy set $A = \{< x,q,\mu_A(x,q), \lambda_A(x,q) > | x \in S, q \in Q_1\}$ in S is an intuitionistic Q_1-fuzzy right (left) k - ideal in S iff $S^{(s,t)}_{A(q)}$ is a right(left) k - ideal of S and $S^{(s,t)}_{A(x)}$ is a k-subsemigroup of Q_1 respectively for all $s,t \in [0,1]$ with $s+t \le 1$ whenever nonempty.

Proof: Let A be an intuitionistic Q_1-fuzzy right k-ideal of a semiring S. By Theorem 3.2 $S^{(s,t)}_{A(q)}$ is a right ideal of S. If there exists $x, y \in S, q \in Q_1$ such that $x+y,\ y \in S^{(s,t)}_{A(q)}$ then $\mu_A(x+y,q) \ge s$ and $\mu_A(y,q) \ge s,\ \lambda_A(x+y,q) \le t$ and $\lambda_A(y,q) \le t.$ Since A is an intuitionistic Q_1 fuzzy right k-ideal in S, $\mu_A(x,q) \ge \mu_A(x+y,q) \wedge \mu_A(y,q) \ge s$ and $\lambda_A(x,q) \le \lambda_A(x+y,q) \vee \lambda_A(y,q) \le t.$ Thus for $x+y, y \in S^{(s,t)}_{A(q)}$ and $q \in Q_1$ we have $x \in S^{(s,t)}_{A(q)}$. Hence $S^{(s,t)}_{A(q)}$ is a right k-ideal of S.

For $x \in S$ and $q_1, q_2 \in Q_1$ if $q_1.q_2, q_2 \in S^{(s,t)}_{A(x)}$ then $\mu_A(x, q_1.q_2) \ge s,\ \mu_A(x,q_2) \ge s,$ $\lambda_A(x,q_1.q_2) \le t$ and $\lambda_A(x,q_2) \le t.$ Thus $\mu_A(x,q_1) \ge \mu_A(x,q_1.q_2) \wedge \mu_A(x,q_2) \ge s$ and so $\mu_A(x,q_1) \ge s.$ Also $\lambda_A(x,q_1) \le \lambda_A(x,q_1.q_2) \vee \lambda_A(x,q_2) \le t$. Therefore $q_1 \in S^{(s,t)}_{A(x)}$ which implies that $S^{(s,t)}_{A(x)}$ is a $k-$subsemigroup of Q_1.

Conversely assume that $S^{(s,t)}_{A(q)}$ is a right k-ideal of S and $S^{(s,t)}_{A(x)}$ is a k-subsemigroup of Q_1 for $q \in Q_1,\ x \in S$ and for all $s,t \in [0,1]$ such that $s+t \le 1$ whenever nonempty. If there exists $x, y \in S,\ q \in Q_1$ such that $\mu_A(x,q) < \mu_A(x+y,q) \wedge \mu_A(y,q)$ and $\lambda_A(x,q) > \lambda_A(x+y,q) \vee \lambda_A(y,q).$ Let $s,t \in [0,1]$ such that $\mu_A(x,q) < s \le \mu_A(x+y,q) \wedge \mu_A(y,q)$ and $\lambda_A(x,q) > t \ge \lambda_A(x+y,q) \vee \lambda_A(y,q).$ This shows $x+y, y \in S^{(s,t)}_{A(q)}$ but $x \notin S^{(s,t)}_{A(q)}.$ This is a contradiction to the fact that $S^{(s,t)}_{A(q)}$ is a right k-ideal of $S.$ Similarly, for $q_2, q_1.q_2 \in S^{(s,t)}_{A(x)},$ if $\mu_A(x,q_1) < \mu_A(x,q_1.q_2) \wedge \mu_A(x,q_2)$ and $\lambda_A(x,q_1) > \lambda_A(x,q_1.q_2) \vee \lambda_A(x,q_2).$ Let $s,t \in [0,1]$ such that $\mu_A(x,q_1) < s \le \mu_A(x, q_1.q_2) \wedge \mu_A(x,q_2)$ and $\lambda_A(x,q_1) > t \le \lambda_A(x,q_1.q_2) \vee \lambda_A(x,q_2).$ This means $q_2,\ q_1.q_2 \in S^{(s,t)}_{A(x)}$ but $q_1 \notin S^{(s,t)}_{A(x)}.$ This is a contradiction to the fact that $S^{(s,t)}_{A(x)}$ is a k-subsemigroup of Q_1.Therefore $\mu_A(x,q_1) \ge \mu_A(x,q_1.q_2) \wedge \mu_A(x,q_2)$ and $\lambda_A(x,q_1) \le \lambda_A(x, q_1.q_2) \vee \lambda_A(x,q_2).$ Hence A is an intuitionistic Q_1-fuzzy right k-ideal of the semiring S.

4. Conclusion

In this paper k-subsemigroup of a semigroup G_s has been introduced in real case and we provide a structure for Q-fuzzy set and Q-intuitionistic fuzzy set. This article is a threshold for a new algebraic structure and a lot of can be done using this new structure.

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REFERENCES

1. K.T.Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87-96.
2. P.Bhattacharya and N.P. Mukherjee, Fuzzy groups, *Inform. Sci.*, 36 (1985) 267-282.
3. TK Dutta and BK Biswas , Fuzzy k-ideals of semirings, *Bull Calcutta Math. Soc.*, 87 (1995) 91-96.
2. M.Henriksen, Ideals in semirings with commutative addition, *Am. Math. Soc.*, (6) (1958) 321.
3. J.Ahsen, J.N.Mordeson and Mohammad Shabi, Fuzzy k-ideals of semiring, *Fuzzy Semirings with Applications*, 278 (1988) 53-82.
4. S.Kar and S.Purkait, Characterization of some k-regularity of semirings in terms of fuzzy ideals of semiring, *Journal of Intelligent and Fuzzy System,* 27(6) (2014) 3089-3101.
5. Kim, On intuitionistic Q-fuzzy semiprime ideals in semigroups, *Adv. in Fuzzy Mathematics,* 1(1) (2006) 15-21.
6. S.Lekkoksung, Q-fuzzy interior ideals in semigroups, *Int. J. contemp. Math. Sci.,* 7 (2012) 357-361.
7. S.Lekkoksung, On Intuitionistic Q-fuzzy k-ideals of semiring, *Int. J. Contemp. Math. Science*, 7(8) (2012) 389-393.
8. W.J.Liu, Fuzzy invarient subgroups and fuzzy ideals, *Fuzzy Sets and Systems*, 8 (1982) 133-139.
9. D.S.Malik and J.N.Mordeson, Extensions of fuzzy subring and fuzzy ideals, *Fuzzy Sets and Systems*, 45 (1992) 245-251.
10. A. Muhammad and W.A. Dudek, Intuitionistic fuzzy left k-ideals of semirings, *Soft Computing*, 12 (2008) 881-890.
11. P.M. Pu and Y.M.Liu, Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence, *Mathematical Analysis and Applications*, 76(2) (1980) 571-599.
12. A. Rosenfeld, Fuzzy groups, *J. Math. Anal. Appl.,* 35 (1971) 512-517.
13. P.Sivaramakrishna Das, Fuzzy groups and level subgroups, *Journal of Mathematical Analysis and Applications,* 84 (1989) 264-269.
14. A.Solairaju and R.Nagarajan, A new structure and constructions of Q-fuzzygroup, *Advance in Fuzzy Mathematics*, 4 (2009) 23-29.
15. X.P.Wang, Z.W.Mo and W.J.Liu, Fuzzy ideals generated by fuzzy point in semigroups, *Sichuan Shifan Daxue Xuebao*, 15(4) (1992) 17-24.
16. L.A. Zadeh, Fuzzy sets, *Information and Control,* 8 (1995) 338-353.