Annals of Pure and Applied Mathematics
Vol. 13, No. 2, 2017, 273-292
ISSN: 2279-087X (P), 2279-0888(online)
Published on 16 May 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v13n2a14

Annals of
Pure and Applied Mathematics

Soft Intersection Ideals of Semiring

P. Murugadas **and** ***M.R.Thirumagal***

Department of Mathematics, Annamalai University, Annamalainagar-608002, India
Corresponding author. Email: bodi_muruga@yahoo.com

Received 11 April 2017; accepted 23 April 2017

Abstract. Molodtsov introduced the concept of soft sets, which can be seen as a new Mathematical tool for dealing with uncertainty. In this paper, we initiate the study of soft intersection ideals of semirings by using the soft set theory. The notions of soft intersection semirings, soft intersection left(right, two-sided) ideals of semiring and soft intersection quasi and bi-ideals of semirings are introduced and several related properties are investigated.

Keywords: Soft set, soft intersection semirings, soft intersection left (right, two-sided) ideals of semiring and soft intersection quasi and bi-ideals of semirings.

AMS Mathematics Subject Classification (2010): 06D72, 16Y60, 16D25

1. Introduction

The notion of a semiring was introduced by Vandiver in 1934. Needless to say, semirings found their full place in Mathematics long before years. The applications of semirings to areas such as optimization theory, graph theory, generalized fuzzy computation, automata theory, formal language theory, coding theory and analysis of computer programs have been extensively studied in the literature (cf. [7,8]). It is also well-known that ideals usually play a fundamental role in algebra, especially in the study of rings. Nevertheless, ideals in a semiring S do not in general coincide with the usual ring ideals if S is a ring, and so many results in ring theory have no analogues in semirings using only ideals. Consequently, some more restricted concepts of ideals such as k-ideals [9] and h-ideals [10] have been introduced in the study of the semiring theory. Moreover, the fuzzy set theory initiated by Zadeh [16] has been successfully applied to generalize many basic concepts in algebra. Rosenfeld [14] proposed the concept of group in order to establish the algebraic structure of fuzzy sets. In fact, several researchers have investigated a fuzzy theory in semirings. They introduced the notions of fuzzy semirings, fuzzy (prime) ideals, fuzzy k-ideals, fuzzy h-ideals and L-fuzzy ideals in semirings, and obtained many related results. However, all of these theories have their own difficulties which are pointed out in [13] by Molodtsov who then proposed a completely new approach for modeling vagueness and uncertainty, that is free from the difficulties. This so-called soft set theory has potential applications in many different fields. Maji et al. [11] firstly worked on detailed theoretical study of soft sets. After that, the properties and applications on the soft set theory have been studied by many authors (e.g. [1, 2, 3, 4, 6, 12, 15, 17]). Feng et al. [5] dealt with the algebraic structure of semirings by applying soft set theory and defined the notion of a soft semiring and focused on the algebraic properties of soft semirings. In this paper, we make a new approch to the classical semiring theory via soft sets, with the concept of soft intersection semiring and soft intersection ideals of semirings.

2. Preliminaries

Molodtsov [13] defined the notion of a soft set in the following way: Let U be an initial universe set and E be a set of parameters. The power set of U is denoted by $P(U)$ and A is a subset of U. A pair (F, A) is called a soft set over U, where $F : A \to P(U)$. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [13].

Example 2.1. Let $U = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ be the set of seven cars and E $= \{$ expensive, fuel efficiency, spacious, maintenance free , ecofriendly, high security measure$\}$ are set of parameters. Let (F, P) be a soft set representing the "suitable cars" given by $(F, P) = \{expensive\,cars = \{c_2, c_3, c_5, c_7\}$, fuel efficiency $= \{c_1, c_2, c_3, c_4\}$, spacious $= \{c_4, c_5, c_6, c_7\}$, maintenance free $= \{c_2, c_4, c_6, c_7\}$ ecofriendly $= \{c_1, c_2, c_3,$, high security measure $= \{c_3, c_4, c_6, c_7\}\}$. Suppose that Mr X wants to buy a car consisting the parameter fuel efficiency, spacious, eco friendly, high security measure which forms the subset $P = \{$fuel efficiency, spacious, eco friendly, high security measure$\}$ of the set E. The problem is to select the car which is suitable with the choice parameters set by Mr X.

Definition 2.2. Let $f_A,\ f_B \in S(U)$. Then, f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$ if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 2.3. Let $f_A,\ f_B \in S(U)$. Then, union of f_A and f_B denoted by $f_A \cup f_B$, is defined as $f_A \cup f_B = f_{A \cup B}$, where $f_{A \cup B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

Definition 2.4. Let $f_A,\ f_B \in S(U)$. Then, intersection of f_A and f_B denoted by $f_A \cap f_B$, is defined as $f_A \cap f_B = f_{A \cap B}$, where $f_{A \cap B}(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

Definition 2.5. Let $f_A,\ f_B \in S(U)$. Then, $\wedge$-product of f_A and f_B denoted by $f_A \wedge f_B$, is defined as $f_A \wedge f_B = f_{A \wedge B}$, where $f_{A \wedge B}(x, y) = f_A(x) \cap f_B(y)$ for all $(x, y) \in E \times E$.

Definition 2.6. Let f_A and f_B be soft sets over the common universe U and Ψ be a function from A to B. Then, soft image of f_A under Ψ , denoted by $\Psi(f_A)$, is a soft set over U by

$$(\Psi(f_A))(b)=\begin{cases}\bigcup\{f_A(a)\mid a\in A \text{ and } \Psi(a)=b\}, \text{if } \Psi^{-1}(b)\neq\emptyset,\\ 0, otherwise\end{cases}$$

for all $b\in B.$ And soft pre-image (or soft inverse image) of f_B under Ψ, denoted by $\Psi^{-1}(f_B),$ is a soft set over U by $\Psi^{-1}(f_B)(a)=f_B(\Psi(a))$ for all $a\in A.$

Definition 2.7. Let f_A be a soft set over U and $\alpha\subseteq U.$ Then, upper α-inclusion of $f_A,$ denoted by $U(f_A;\alpha),$ is defined as $U(f_A;\alpha)=\{x\in A\mid f_A(x)\supseteq\alpha\}.$

3. Soft Intersection sum, product and soft characteristic function

In this section, we define soft intersection sum, product and soft characteristic function and study their properties.

Definition 3.1. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection sum f_S+g_S is defined by

$$(f_S+g_S)(x)=\begin{cases}\bigcup\limits_{x=y+z}\{f_S(y)\cap g_S(z)\}, \text{ if there exists } y,z\in S\\ \emptyset, otherwise.\end{cases}$$

for all $x\in S.$

Definition 3.2. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S\circ g_S$ is defined by

$$(f_S\circ g_S)(x)=\begin{cases}\bigcup\limits_{x=yz}\{f_S(y)\cap g_S(z)\}, \text{ if there exists } y,z\in S\\ \emptyset, otherwise.\end{cases}$$

for all $x\in S.$

Example 3.3. Consider the semiring S={0,a,b,c} defined by the following table:

$\oplus$	0	A	b	c
0	0	A	b	c
a	a	B	c	a
b	b	C	a	b
c	c	A	b	c

$\odot$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	b	c
c	0	c	c	c

Let $U=D_3=\{<x,y>: x^3=y^2=e, xy=yx^2\}=\{e,x,x^2,y,yx,yx^2\}$ be the universal set. Let f_S and g_S be soft sets over U such that $f_S(0)=\{e,x,y,yx\},$ $f_S(a)=\{e,x,y^2\},$ $f_S(b)=\{e,y,yx^2\},$ $f_S(c)=\{e,x,x^2,y\}$ and $g_S(0)=\{e,y,y^2\},$ $g_S(a)=\{e,x,yx\},$ $g_S(b)=\{e,yx,yx^2\},$ $g_S(c)=\{e,y,yx\}.$ $(f_S\circ g_S)(a)=(f_S(a)\cap g_S(a))$ $=\{e,x\},$

$(f_S \circ g_S)(b) = (f_S(a) \cap g_S(b)) \cup (f_S(b) \cap g_S(a)) \cup (f_S(b) \cap g_S(b)) = \{e\} \cup \{e\} \cup \{e, yx^2\} = \{e, yx^2\}$,
$(f_S \circ g_S)(c) = (f_S(a) \cap g_S(c)) \cup (f_S(b) \cap g_S(c)) \cup (f_S(c) \cap g_S(a)) \cup (f_S(c) \cap g_S(b)) \cup (f_S(c) \cap g_S(c))$
$= \{e\} \cup \{e, y\} \cup \{e, x\} \cup \{e\} \cup \{e\} \cup \{e, y\}$
$= \{e, x, y\}$.

Theorem 3.4. Let $f_S, g_S, h_S \in S(U)$. Then,
(1) $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.
(2) $f_S \circ g_S \neq f_S \circ g_S$.
(3) $f_S \circ (g_S \cup h_S) = (f_S \circ g_S) \cup (f_S \circ h_S)$ and $(f_S \cup g_S) \circ h_S = (f_S \circ h_S) \cup (g_S \circ h_S)$.
(4) $f_S \circ (g_S \tilde{\cap} h_S) = (f_S \circ g_S) \tilde{\cap} (f_S \circ h_S)$ and $(f_S \tilde{\cap} g_S) \circ h_S = (f_S \circ h_S) \circ (g_S \circ h_S)$.
(5) If $f_S \tilde{\subseteq} g_S$, *then* $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S$ *and* $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S$.
(6) If $t_S, l_S \in S(U)$ such that $t_S \tilde{\subseteq} f_S$ and $l_S \tilde{\subseteq} g_S$, then $l_S \circ g_S \tilde{\subseteq} f_S \circ g_S$.
(7) $(f_S + g_S) + h_S = f_S + (g_S + h_S)$.
(8) $f_S \circ (g_S + h_S) = (f_S \circ g_S) + (f_S \circ h_S)$.

Proof: (1) and (2) follows from Definition 3.1 and Example 3.1.

(3) Let $a \in S$. If a is not expressible as $a = xy$, then $f_S \circ (g_S \tilde{\cap} h_S)(A) = \emptyset$.

Similarly, $((f_S \circ g_S) \tilde{\cap} (f_S \cup h_S))(a) = (f_S \circ g_S)(a) \cup (f_S \cup h_S)(a) = \emptyset \cup \emptyset = \emptyset$.
Now, let there exist $x, y \in S$ such that $a = xy$.

$$(f_S \circ (g_S \tilde{\cap} h_S))(a) = \bigcup_{a=xy} (f_S(x) \cap (g_S \tilde{\cup} h_S)(y))$$
$$= \bigcup_{a=xy} [(f_S(x) \cap g_S(y)) \cup (f_S(x) \cap h_S(y))]$$
$$= \bigcup_{a=xy} [(f_S(x) \cap g_S(y))] \cup [\bigcup_{a=xy} [(f_S(x) \cap h_S(y))]$$
$$= (f_S \circ g_S)(a) \cup (f_S \circ h_S)(a)$$
$$= [(f_S \circ g_S) \tilde{\cup} (f_S \circ h_S)](a).$$

Thus $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cup} (g_S \circ h_S)$.
Similarly, we can prove(4) is also clear.

(5) Let $x \in S$. If x is not expressible as $x = yz$, then
$(f_S \circ h_S)(x) = (g_S \circ h_S)(x) = \emptyset$. Otherwise,

$$(f_S \circ h_S)(x) = \bigcup_{x=yz} (f_S(y) \cap h_S(z))$$

$$\subseteq \bigcup_{x=yz} (g_S(y) \cap h_S(z)) \text{ (since } f_S(y) \subseteq g_S(y)) = (g_S \circ h_S)(x).$$

Similarly, one can show that $h_S \circ f_S \,\tilde{\subseteq}\, h_S \circ g_S$.

(6) can be proved similar to (5).

(7) Let $x \in S$. If x is not expressible as $x = y + z$, then $((f_S + g_S) + h_S)(x) = \varnothing$. Otherwise,

If $x = y + z$, then

$$((f_S + g_S) + h_S)(x) = \bigcup_{x=y+z} \{(f_S + g_S)(y) \cap h_S(z)\}$$

$$= \begin{cases} \bigcup_{x=y+z} \{ \bigcup_{y=y_1+y_2} (f_S(y_1) \cap g_S(y_2)) \cap h_S(z)\} \\ \bigcup_{x=y+z} \{\varnothing \cap h_S(z)\} = \varnothing \; if \; y \neq y_1 + y_2 \end{cases} = \begin{cases} \bigcup_{x=y+z} \bigcup_{y=y_1+y_2} (f_S(y_1) \cap g_S(y_2) \cap h_S(z) \\ \varnothing, otherwise \end{cases}$$

$$= \begin{cases} \bigcup_{x=y_1+y_2+z} (f_S(y_1) \cap g_S(y_2) \cap h_S(z)) \\ \varnothing, otherwise \end{cases} = \begin{cases} \bigcup_{x=y_1+(y_2+z)} (f_S(y_1) \cap (g_S(y_2) \cap h_S(z)) \\ \varnothing, otherwise \end{cases}$$

$$(f_S + g_S) + h_S = f_S + (g_S + h_S).$$

(7) Let $x \in S$. If x is not expressible as $x = yz$, then $(f_S \circ (g_S + h_S))(x) = \varnothing$. Otherwise,

If $x = yz$, then

$$(f_S \circ (g_S + h_S))(x) = \bigcup_{x=yz} \{(f_S(y) \cap (g_S + h_S)(z))\}$$

If $z \neq z_1 + z_2$, then $(g_S + h_S)(z) = \varnothing$ and so $(f_S \circ (g_S + h_S))(x) = \varnothing$. Therefore assume $z = z_1 + z_2$,

$$= \begin{cases} \bigcup_{x=y(z_1+z_2)} \{f_S(y) \cap \{ \bigcup_{z=z_1+z_2} (g_S(z_1) \cap h_S(z_2))\}\} \\ \varnothing, otherwise \end{cases}$$

$$= \begin{cases} \bigcup_{x=y(z_1+z_2)} \{\{f_S(y) \cap g_S(z_1)\} \cup \{f_S(y) \cap (g_S(z_2)\}\} \\ \varnothing, otherwise \end{cases}$$

$$= \begin{cases} \bigcup_{x=yz_1+yz_2)} \{\{(f_S \circ g_S) \cap (f_S \circ h_S)(yz_2)\}\} \\ \varnothing, otherwise \end{cases}$$
$$= ((f_S \circ g_S) + (f_S \circ h_S))(x)$$
$$\Rightarrow f_S \circ (g_S + h_S) = (f_S \circ g_S) + (f_S \circ h_S).$$

Definition 3.5. Let X be a subset of S. We denote by S_X the soft characteristic function of X and is define as

$$S_X(x) = \begin{cases} U, & \text{if } x \in X, \\ \varnothing, & \text{if } x \notin X. \end{cases}$$

It is obvious that the soft characteristic function is a soft set over U, that is, $S_X : S \to P(U)$.

Theorem 3.6. Let X and Y be nonempty subsets of a semiring S. Then, the following properties hold:

(i) if $X \subseteq Y$, then $S_X \tilde{\subseteq} S_Y$.

(ii) $S_X \tilde{\cap} S_Y = S_{X \cap Y}, S_X \tilde{\cup} S_Y = S_{X \cup Y}$.

(iii) $S_X \circ S_Y = S_{XY}$.

(iv) $S_X + S_Y = S_{X+Y}$.

Proof: (i) is straight forward by Definition 3.3.

(ii) Let s be any element of S. Suppose $s \in X \cap Y$. Then, $s \in X$ and $s \in Y$. Thus, we have

$$(S_X \tilde{\cap} S_Y)(s) = S_X(s) \cap S_Y(s) = U \cap U = U = S_{X \cap Y}(s).$$

Suppose $s \notin X \cap Y$. Then, $s \notin X$ or $s \notin Y$. Hence, we have

$$(S_X \tilde{\cap} S_Y)(s) = S_X(s) \cap S_Y(s) = \varnothing = S_{X \cap Y}(s).$$

Let s be any element of S. Suppose $s \in X \cup Y$. Then, $s \in X$ or $s \in Y$. Thus, we have

$$(S_X \tilde{\cup} S_Y)(s) = S_X(s) \cup S_Y(s) = U = S_{X \cup Y}(s).$$

Suppose $s \notin X \cup Y$. Then, $s \notin X$ and $s \notin Y$.
Hence, we have

$$(S_X \tilde{\cup} S_Y)(s) = S_X(s) \cup S_Y(s) = \varnothing = S_{X \cup Y}(s).$$

(iii) Let s be any element of S. Suppose $s \in XY$. Then, $s = xy$ for some $x \in X$ and $y \in Y$. Thus we have

$$(S_X \circ S_Y)(s) = \bigcup_{s=xy} (S_X(x) \cap S_Y(y)) \supseteq S_X(x) \cap S_Y(y) = U.$$

This implies that $(S_X \circ S_Y)(s) = U$. Since $s = xy \in XY$, $S_{XY}(s) = U$. Thus,

$S_X \circ S_Y = S_{XY}$.

In another case, when $s \notin XY$, we have $s \neq xy$ for all $x \in X$ and $y \in Y$. If $s = mn$ for some $m, n \in S$, then we have

$$(S_X \circ S_Y)(s) = \bigcup_{s=mn} (S_X(m) \cap S_Y(n)) = \varnothing = S_{XY}(s).$$

If $s \neq mn$ for all $m, n \in S$, then $(S_X \circ S_Y)(s) = \varnothing = S_{XY}(s)$. Therefore in all the cases, we have $S_X \circ S_Y = S_{XY}$.

4. Soft intersection semiring

Definition 4.1. Let S be a semiring and f_S be a soft set over U. Then, f_S is called a soft intersection semiring of S,if

(1) $f_S(x+y) \supseteq f_S(x) \cap f_S(y)$

(2) $f_S(xy) \supseteq f_S(x) \cap f_S(y)$ for all $x, y \in S$.

Example 4.2. Consider the semiring S = {0,a,b,c} defined by the following table:

$\oplus$	0	a	b	c
0	0	a	b	c
a	a	b	c	a
b	b	c	a	b
c	c	a	b	c

$\odot$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	b	c
c	0	c	c	c

Let $U = D_3 = \{<x, y>: x^3 = y^2 = e, xy = yx^2\} = \{e, x, x^2, y, yx, yx^2\}$ be the universal set. Let f_S and g_S be soft sets over U such that $f_S(0) = \{e, x, y, yx\}$, $f_S(a) = \{e, x, y^2\}$, $f_S(b) = \{e, y, yx^2\}$, $f_S(c) = \{e, x, x^2, y\}$.

Clearly f_S is a SI-semiring over U.

It is easy that if $f\_S = U$ for all $x \in S$, then f_S is a SI-semiring over U. We denote that such a kind of SI-semiring by $\tilde{S}$. It is obvious that $\tilde{S} = S_S$, that is $S_x(x) = U$ for all $x \in S$.

Lemma 4.3. Let f_S be any SI-semiring over U. Then, we have the following:

(1) $\tilde{S} \circ \tilde{S} \mathrel{\tilde{\subseteq}} \tilde{S}$.

(2) $f_S \circ \tilde{S} \mathrel{\tilde{\subseteq}} \tilde{S}$ and $\tilde{S} \circ f_S \mathrel{\tilde{\subseteq}} \tilde{S}$.

(3) $f_S \mathrel{\tilde{\cup}} \tilde{S} = \tilde{S}$ and $\tilde{S} \mathrel{\tilde{\cap}} f_S \mathrel{\tilde{\subseteq}} \tilde{S}$.

(4) $\tilde{S} + \tilde{S} \mathrel{\tilde{\subseteq}} \tilde{S}$.

(5) $f_S + \tilde{S} \mathrel{\tilde{\subseteq}} \tilde{S}$ and $\tilde{S} + f_S \mathrel{\tilde{\subseteq}} \tilde{S}$.

Proof: Obviously (1),(2) and (3) are true.

(4) for any $x \in S$

$(\tilde{S}+\tilde{S})(x) = \bigcup_{x=a+b}(\tilde{S}(a) \cap \tilde{S}(b))$

$= \bigcup_{x=a+b} (U) \subseteq U = \tilde{S}(x)$

(5) $(f_S + \tilde{S})(x) = \bigcup_{x=a+b} (f_S(a) \cap \tilde{S}(b)) = \bigcup_{x=a+b} (f_S(a)) \subseteq \bigcup_{x=a+b} (U) = \bigcup_{x=a+b} \tilde{S}(x)$

$(f_S + \tilde{S})(x) \subseteq \tilde{S}(x) \Rightarrow (f_S + \tilde{S}) \subseteq \tilde{S}$.

Similarly, we can prove $\tilde{S} + f_S \subseteq \tilde{S}$.

Theorem 4.4. Let f_S be a soft set over U. Then, f_S is a SI-semiring over U if and only if

(1) $f_S + f_S \tilde{\subseteq} f_S$.

(2) $f_S \circ f_S \tilde{\subseteq} f_S$.

Proof: Assume that f_S is a SI-semiringover U. Let $a \in S$. If $(f_S + f_S)(a) = \emptyset$, then it is obvious that $(f_S + f_S)(a) \subseteq f_S(a)$, thus $f_S + f_S \subseteq f_S$.

Otherwise, there exist elements $x, y \in S$ such that $a = x + y$. Then, since f_S is a SI-semiring over U, we have:

(1) $(f_S + f_S)(a) = \bigcup_{a=x+y} (f_S(x) \cap f_S(y))$

$\subseteq \bigcup_{a=x+y} f_S(x+y) = \bigcup_{a=x+y} f_S(a) = f_S(a)$.

Thus, $f_S + f_S \tilde{\subseteq} f_S$.

(2) is similar to (1).

Conversely, assume that (1) and (2) are true. Let $x, y \in S$ and $a = x + y$. Then, we have

$f_S(x+y) = f_S(a) \supseteq (f_S + f_S)(a) = \bigcup_{a=x+y} (f_S(x) \cap f_S(y) \supseteq f_S(x) \cap f_S(y)$.

$f_S(x+y) \supseteq f_S(x) \cap f_S(y)$.

Similarly, $f_S(xy) \supseteq f_S(x) \cap f_S(y)$

Hence, f_S is an SI-semiring over U.

Theorem 4.5. Let X be a nonempty subset of a semiring S. Then, X is a subsemiring of S if and only if S_X is a SI-semiring of S.

Proof: Assume that X is a subsemiring of S, that is, $XX \subseteq X$ and $X + X \subseteq X$. Then, we have

$S_X + S_X = S_{X+X} \subseteq S_X$ and $S_X S_X \Rightarrow S_{XX} \subseteq S_X$ (by Theorem 3.2(iii) and Theorem 3.2(iv)) and so S_X is a SI-semiring over U.

Conversely, let $x \in X + X$ and S_X be a SI-semiring of S. Then, by Theorem 4.1, $S_X(x) \supseteq (S_X + S_X)(x) = \bigcup_{x=a+b} (S_X(a) \cap S_X(b)) \Rightarrow S_{X+X}(x) = U$ implying that $S_X(x)$

$=U$, hence $X+X\subseteq X$ and let $x\in X$ and S_X be a SI-semiring of S. Then, $S_X(x)\supseteq(S_X\circ S_X)(x)=S_{XX}(x)=U$ implying that $S_X(x)=U$. Hence $XX\subseteq X$ and so, X is a subsemiring of S.

Proposition 4.6. Let f_S and f_T be a SI-semiring over U. Then, $f_S\wedge f_T$ is a SI-semiring over U.

Proof: Let $(x_1,y_1),(x_2,y_2)\in S\times T$. Then,

$f_{S\wedge T}((x_1,y_1)+(x_2,y_2))=f_{S\wedge T}(x_1+x_2,y_1+y_2)=f_S(x_1+x_2)\cap f_T(y_1+y_2)$

$\supseteq[f_S(x_1)\cap f_S(x_2)]\cap[f_T(y_1)\cap f_T(y_2)]=[f_S(x_1)\cap f_T(y_1)]\cap[f_S(x_2)\cap f_T(y_2)]$

$=f_{S\wedge T}(x_1,y_1)\cap f_{S\wedge T}((x_2,y_2)$

and

$f_{S\wedge T}((x_1,y_1)(x_2,y_2))=f_{S\wedge T}(x_1x_2,y_1y_2)=f_S(x_1x_2)\cap f_T(y_1y_2)$

$\supseteq[f_S(x_1)\cap f_S(x_2)]\cap[f_T(y_1)\cap f_T(y_2)]=[f_S(x_1)\cap f_T(y_1)]\cap[f_S(x_2)\cap f_T(y_2)]$

$=f_{S\wedge T}(x_1,y_1)\cap f_{S\wedge T}((x_2,y_2)$.

Therefore, $f_S\wedge f_T$ is a SI-semiring over U.

Definition 4.7. Let f_S, f_T be a SI-semirings over U. Then, the product of soft intersection semirings f_S and f_T isdefined as $f_S\times f_T=f_{S\times T}$ where $f_{S\times T}(x,y)=f_S(x)\times f_T(y)$ for all $(x,y)\in S\times T$.

Proposition 4.8. If f_S and f_T are SI-semiring over U. Then, so is $f_S\times f_T$ over $U\times U$.

Proof: By Definition 4.2, $f_S\times f_T=f_{S\times T}$ where $f_{S\times T}(x,y)=f_S(x)\times f_T(y)$ for all $(x,y)\in S\times T$.

Then, for all $(x_1,y_1),(x_2,y_2)\in S\times T$, $f_{S\times T}((x_1,y_1)+(x_2,y_2))=f_{S\times T}(x_1+x_2,y_1+y_2)$

$=f_S(x_1+x_2)\times f_T(y_1+y_2)\supseteq[f_S(x_1)\cap f_S(x_2)]\times[f_T(y_1)\cap f_T(y_2)]$

$=[f_S(x_1)\times f_T(y_1)]\cap[f_S(x_2)\times f_T(y_2)]=f_{S\times T}(x_1,y_1)\cap f_{S\times T}((x_2,y_2)$

and

$f_{S\times T}((x_1,y_1)(x_2,y_2))=f_{S\times T}(x_1x_2,y_1y_2)=f_S(x_1x_2)\times f_T(y_1y_2)$

$\supseteq[f_S(x_1)\cap f_S(x_2)]\times[f_T(y_1)\cap f_T(y_2)]=[f_S(x_1)\times f_T(y_1)]\cap[f_S(x_2)\times f_T(y_2)]$

$=f_{S\times T}(x_1,y_1)\cap f_{S\times T}((x_2,y_2))$.

Therefore, $f_S\times f_T=f_{S\times T}$ is a SI-semiring over $U\times U$.

Proposition 4.9. If f_S and h_S are SI-semiring over U. Then, so is $f_S\tilde{\cap}h_S$ over U.

Proof: Let $x,y\in S$, then

$(f_S\tilde{\cap}h_S)(x+y)=f_S(x+y)\cap h_S(x+y)\supseteq(f_S(x)\cap f_S(y))\cap(h_S(x)\cap h_S(y))$

$= (f_S(x) \cap h_S(x)) \cap (f_S(y) \cap h_S(y)) \ = (f_s \tilde{\cap} h_S)(x) \cap (f_S \tilde{\cap} h_S)(y)$

and

$(f_S \tilde{\cap} h_S)(xy) = f_S(xy) \cap h_S(xy) \ \supseteq (f_S(x) \cap f_S(y)) \cap (h_S(x) \cap h_S(y))$

$= (f_S(x) \cap h_S(x)) \cap (f_S(y) \cap h_S(y)) = (f_S \tilde{\cap} h_S)(x) \cap (f_S \tilde{\cap} h_S)(y).$

Therefore, $f_S \cap h_s$ is a SI-semiring over U.

Proposition 4.10. Let f_S be a soft set over U and α be a subset of U such that $\alpha \in Im(f_S)$, where $Im(f_S) = \{\alpha \subseteq U : f_S(x) = \alpha, \textit{for } x \in S\}$. If f_S is a SI-semiring over U, then $U(f_S;\alpha)$ is a subsemiring of S.

Proof: Let $f_S(x) = \alpha$ for some $x \in S$, then $\emptyset \neq U(f_S;\alpha) \subseteq S$. Let $x, y \in U(f_S;\alpha)$, then $f_S(x) \supseteq \alpha$ and $f_S(y) \supseteq \alpha$. We need to show that $xy \in U(f_S;\alpha)$ and $x + y \in U(f_S, \alpha)$. Since f_S is a SI-semiring over U, it follows that

$f_S(xy) \supseteq f_S(x) \cap f_S(y) \ \supseteq \alpha \cap \alpha = \alpha \ \Rightarrow xy \in U(f_S;\alpha).$

$f_S(x+y) \supseteq f_S(x) \cap f_S(y) \ \supseteq \alpha \cap \alpha = \alpha.$

This shows $x + y \in U(f_S;\alpha)$.

Definition 4.11. Let f_S be a SI-semiring over U. Then, the subsemiring $U(f_S;\alpha)$ are called upper α-subsemiring of f_S.

Proposition 4.12. Let f_S be a soft set over U, $U(f_S;\alpha)$ be upper α-subsemiring of f_S for each $\alpha \subseteq U$ and $Im(f_S)$ be an ordered set by inclusion. Then, f_S is a SI-semiring over U.

Proof: Let $x, y \in S$ and $f_S(x) = \alpha_1$ and $f_S(y) = \alpha_2$. Suppose that $\alpha_1 \subseteq \alpha_2$. It is obvious that $x \in U(f_S;\alpha_1)$ and $y \in U(f_S;\alpha_2)$. Since $\alpha_1 \subseteq \alpha_2, x, y \in U(f_S;\alpha_1)$ and since $U(f_S;\alpha)$ is a subsemiring of S for all $\alpha \subseteq U$, it follows that $xy \in U(f_S;\alpha_1), x + y \in U(f_S, \alpha_1)$. Hence, $f_S(x+y) \supseteq \alpha_1 = \alpha_1 \cap \alpha_2 = f_S(x) \cap f_S(y)$ and $f_S(xy) \supseteq \alpha_1 = \alpha_1 \cap \alpha_2 = f_S(x) \cap f_S(y)$. Thus, f_S is a SI-semiring over U.

Proposition 4.13. Let f_S and f_T be soft sets over U and ϕ be a semiring isomorphism from S to T. If f_S is a SI-semiring over U, then so is $\phi(f_S)$.

Proof: Let $t_1, t_2 \in T$. Since ϕ is surjective, then there exists $s_1, s_2 \in S$ such that $\phi(s_1) = t_1$ and $\phi(s_2) = t_2$. Then,

$(\phi(f_S))(t_1t_2) = \bigcup\{f_S(s) : s \in S, \phi(s) = t_1t_2\}$

$= \bigcup\{f_S(s) : s \in S, s = \phi^{-1}(t_1t_2)\} \ = \bigcup\{f_S(s) : s \in S, s = \phi^{-1}(\phi(s_1s_2)) = s_1s_2\}$

$= \bigcup \{ f_S(s_1 s_2) : s_i \in S, \phi(s_i) = t_i, i = 1,2\}$

$\supseteq \bigcup \{ f_S(s_1) \cap f_S(s_2) : s_i \in S, \phi(s_i) = t_i, i = 1,2\}$

$= (\bigcup \{ f_S(s_1) : s_1 \in S, \phi(s_1) = t_1\}) \cap (\bigcup \{ f_S(s_2) : s_2 \in S, \phi(s_2) = t_2\})$

$= (\phi(f_S))(t_1) \cap (\phi(f_S))(t_2)$ and

$(\phi(f_S))(t_1 + t_2) = \bigcup \{ f_S(s) : s \in S, \phi(s) = t_1 + t_2\} = \bigcup \{ f_S(s) : s \in S, s = \phi^{-1}(t_1 + t_2)\}$

$= \bigcup \{ f_S(s) : s \in S, s = \phi^{-1}(\phi(s_1 + s_2)) = s_1 + s_2\}$

$= \bigcup \{ f_S(s_1 + s_2); s_i \in S, \phi(s_i) = t_i, i = 1,2\}$

$\supseteq \bigcup \{ f_s(s_1) \cap f_S(s_2); s_i \in S, \phi(s_i) = t_i, i = 1,2\}$

$= (\bigcup \{ f_S(s_1) : s_1 \in S, \phi(s_1) = t_1\}) \cap (\bigcup \{(f_S(s_2), s_2 \in S, \phi(s_2) = t_2)\})$

$= \phi(f_S)(t_1) \cap (\phi(f_S))(t_2).$

Hence $\phi(f_S)$ is a SI-semiring over U.

Proposition 4.14. Let f_S and f_T be soft sets over U and ϕ be a semiring homomorphism from S to T. If f_T is a SI-semiring over U, then so is $\phi^{-1}(f_T)$.

Proof: Let $s_1, s_2 \in S.$ Then,

$\phi^{-1}(f_T)(s_1 s_2) = f_T(\phi(s_1 s_2)) = f_t(\phi(s_1)\phi(s_2))) \supseteq f_T(\phi(s_1)) \cap f_T(\phi(s_2))$

$\phi^{-1}(f_T)(s_1 s_2) = (\phi^{-1}(f_T))(s_1) \cap (\phi^{-1}(f_T))(s_2)$

Let $s_1, s_2 \in S.$ Then,

$\phi^{-1}(f_T)(s_1 + s_2) = f_T(\phi(s_1 + s_2)) = f_T((\phi(s_1)) + (\phi(s_2))) \supseteq f_T(\phi(s_1)) \cap f_T(\phi(s_2))$

$\phi^{-1}(f_T)(s_1 + s_2) \supseteq (\phi^{-1}(f_T)(s_1) \cap \phi^{-1}(f_T)(s_2)).$

Hence, $\phi^{-1}(f_T)$ is an SI- semiring over U.

5. Soft intersection left (right, two-sided) ideals of semiring

Definition 5.1. A soft set f_S over U is called a soft intersection left (right, two-sided) ideals of S over U if

(1) $f_S(x + y) \supseteq f_S(x) \cap f_S(y)$

(2) $f_S(xy) \supseteq f_S(x)(f_s(xy) \supseteq f_S(y))$

for all $x, y \in S.$ A soft set over U is called a soft intersection two-sided ideal (soft intersection ideal) of S if it is both soft intersection left and soft intersection right ideal of S over U.

Example 5.2. Consider the semiring S={0,x,1} defined by the following table:

$\oplus$	0	x	1
0	0	x	1
x	X	0	x
1	1	x	0

$\odot$	0	x	1
0	0	0	0
x	0	x	x
1	0	x	1

Let f_S be soft set over S such that $f_S(0)=\{0,x,1\}$, $f_S(x)=\{0,x\}$, $f_S(1)=\{x\}$. Then, one can easily show that f_S is a SI-ideal of S over U.

Theorem 5.3. Let f_S be a SI-semiring over U. Then, f_S is a SI-left ideal of S over U if and only if

(1) $f_S + f_S \tilde{\subseteq} f_S$.

(2) $S \circ f_S \tilde{\subseteq} f_S$.

Proof: Assume that f_S is a SI-left ideal of S over U. Let $a \in S$. If $(f_S + f_S)(a) = \emptyset$, then it is obvious that $(f_S + f_S)(a) \subseteq f_S(a)$, thus $f_S + f_S \subseteq f_S$.

Otherwise, there exist elements $x, y \in S$ such that $a = x + y$. Then, since f_S is a SI-left ideal of S over U, we have:

$$(f_S + f_S)(a) = \bigcup_{a=x+y} (f_S(x) \cap f_S(y)) \subseteq \bigcup_{a=x+y} f_S(x+y) = \bigcup_{a=x+y} f_S(a) = f_S(a)$$

Thus, $f_S + f_S \tilde{\subseteq} f_S$.

(2) If $(\tilde{S} \circ f_S)(a) = \emptyset$, then it is obvious that

$(\tilde{S} \circ f_S)(a) \subseteq f_S(a)$, thus $\tilde{S} \circ f_S \tilde{\subseteq} f_S$.

Otherwise, there exist elements $x, y \in S$ such that $a = xy$. Then, since f_S is a SI-left ideal of S over U, we have:

$$(\tilde{S} \circ f_S)(a) = \bigcup_{a=xy} (\tilde{S}(x) \cap f_S(y)) \subseteq \bigcup_{a=xy} (U \cap f_S(xy)) = \bigcup_{a=xy} (U \cap f_S(a)) = f_S(a)$$

Thus, $\tilde{S} \circ f_S \subseteq f_S$.

Conversely, assume that (1) and (2) are true. Let $x, y \in S$ and $a = x + y$. Then,we have:

$$f_S(x+y) = f_S(a) \supseteq (f_S + f_S)(a) = \bigcup_{a=x+y} (f_S(x) \cap f_S(y) \supseteq f_S(x) \cap f_S(y).$$

Similarly, $f_S(xy) \supseteq f_S(y)$

Hence, f_S is an SI-left ideal over U. This competes the proof.

Theorem 5.4. Let f_S be a SI-semiring over U. Then, f_S is a SI-right ideal of S over U if and only if

(1) $f_S + f_S \subseteq f_S$.

(2) $f_S \circ \tilde{S} \tilde{\subseteq} f_S$.

Proof: Similar to the proof of Theorem 5.1.

Corollary 5.5. $\tilde{S}$ is both SI-right and SI-left ideal of S.

Proof: Follows from Lemma 4.1-(1).

Theorem 5.6. Let X be a nonempty subset of a semiring S. Then, X is a left (right, two-sided) ideal of S if and only if S_X is an SI-left (right, two-sided) ideal of S over U.

Proof: We give the proof for the SI-left ideals. Assume that X is a left ideal of S, that is, $X + X \subseteq X$ and $SX \subseteq X$. Then, we have $S_X + S_X = S_{X+X} \subseteq S_X$ and $\tilde{S} \circ S_X = S_S \circ S_X = S_{SX} \tilde{\subseteq} S_X$. Thus, S_X is an SI-left ideal of S over U by Theorem 5.1.

Conversely, let $x \in SX$ and S_X be an SI-left ideal of S over U. Then, $S_X(x) \supseteq (\tilde{S} \circ S_X)(x) = (S_S \circ S_X)(x) = S_{SX}(x) = U$ implying that $S_X(x) = U$, hence $x \in X$. Similarly $S_X(x) \supseteq (S_X + S_X)(x + x)$. Thus, $S_X \subseteq X$ and X is a left ideal of S.

Theorem 5.7. Let f_S and g_S be SI-left (right) ideals of a semiring S. Then $f_S + g_S$ is a SI-left (right) ideal of S.

Proof: Suppose f_S, g_S are SI-left ideals of a semiring S and $x, y \in S$. If $(f_S + g_S)(x) = \varnothing$ or $(f_S + g_S)(y) = \varnothing$ then, $(f_S + g_S)(x) \cap (f_S + g_S)(y) = \varnothing \subseteq (f_S + g_S)(x + y)$. If $(f_S + g_S)(x) \neq \varnothing$ and $(f_S + g_S)(y) \neq \varnothing$ then, $(f_s + g_S)(y) = \bigcup_{y=c+d} \{f_S(c) \cap g_S(d)\}$.

Thus,

$$(f_S + g_S)(x) \cap (f_S + g_S)(y) = \Big(\bigcup_{x=a+b} \{f_S(a) \cap g_S(b)\}\Big) \cap \Big(\bigcup_{y=c+d} \{f_S(c) \cap g_S(d)\}\Big)$$

$$= \bigcup_{x=a+b} \bigcup_{y=c+d} \{(f_S(a) \cap g_S(b)) \cap (f_S(c) \cap g_S(d))\}$$

$$= \bigcup_{x=a+b} \bigcup_{y=c+d} \{(f_S(a) \cap f_S(c)) \cap (g_S(b) \cap g_S(d))\}$$

$$\subseteq \bigcup_{x=a+b} \bigcup_{y=c+d} \{(f_S(a+c) \cap g_S(b+d))\}$$

$$\subseteq (f_S + g_S)(x + y).$$

Again, if $(f_S + g_S)(x) = \varnothing$ then $(f_S + g_S)(x) \subseteq (f_S + g_S)(yx)$. If $(f_S + g_S)(x) \neq \varnothing$, then

$(f_S+g_S)(x)=\bigcup_{x=a+b}\{f_S(a)\cap g_S(b)\}\ \subseteq\{\bigcup_{x=a+b}\{f_S(ya)\cap g_S(yb)\}$
$\subseteq\bigcup_{yx=c+d}\{f_S(c)\cap g_S(d)\}=(f_S+g_S)(yx).$

Hence f_S+g_S is a soft fuzzy left ideal of S.

Theorem 5.8. If f_S, g_S are SI-left (right) ideals of a semiring S, then $f_S\circ g_S$ is a SI-left (right) ideal of S.

Proof: Suppose f_S, g_S are SI-left ideals of a semiring S and $x, y\in S$. If $(f_S\circ g_S)(x)=\varnothing$ or $(f_S\circ g_S)(y)=\varnothing,$

then $(f_S\circ g_S(x))\cap(f_S\circ g_S)(y)=\varnothing\subseteq(f_S\circ g_S)(x+y).$

If $(f_S\circ g_S)(x)\neq\varnothing$ and $(f_S\circ g_S(y))\neq\varnothing,$ then

$(f_S\circ g_S)(x)=\bigcup_{x=ab}\{(f_S(a)\cap g_S(b))\}$

$(f_S\circ g_S)(y)=\bigcup_{y=cd}\{(f_S(c)\cap g_S(d))\}$

$(f_S\circ g_S)(x)\cap(f_S\circ g_S)(y)=[\bigcup_{x=ab}\{((f_S(a)\cap g_S(b))\}]\cap[\bigcup_{y=cd}\{((f_S(c)\cap g_S(d))\}]$

$=\bigcup_{x=ab}\bigcup_{y=cd}[((f_S(a)\cap g_S(b))]\cap[[((f_S(a)\cap g_S(b)]\subseteq\bigcup_{x+y=ef}[((f_S(e)\cap g_S(f))]$

$=(f_S\circ g_S)(x+y).$

Again, if $(f_S\circ g_S)(x)=\varnothing$ then $(f_S\circ g_S)(x)\subseteq(f_S\circ g_S)(yx).$

If $(f_S\circ g_S)\neq\varnothing,$ then

$(f_S\circ g_S)(x)=\bigcup_{x=ab}\{(f_S(a)\cap g_S(b)\}\subseteq\bigcup_{x=ab}\{(f_S(ya)\cap g_S(b)\}$

$\subseteq\bigcup_{yx=cd}\{(f_S(c)\cap g_S(d)\}=(f_S\circ g_S)(yx).$

Hence $f_S\circ g_S$ is a SI- left ideal of S.

Theorem 5.9. Let f_S be a soft set over U. Then, if f_S is a SI-left (right, two sided) ideal of S over U, f_S is a SI-semiring over U.

Proof: We give the proof for SI-ideals. Let f_S be a SI-left ideal of S over *U*. Then, $f_S(x+y)\supseteq f_S(x)\cap f_S(y)$ and $f_S(xy)\supseteq f_S(y)$ for all $x, y\in S$. Thus $f_S(xy)\supseteq f_S(y)\supseteq f_S(x)\cap f_S(y),$ so f_S is a SI-semiring over U.

Theorem 5.10. Let f_S be a SI-right ideal and g_S a soft intersection left ideal of a semiring S. Then $f_S\circ g_S\,\tilde{\subseteq}\, f_S\,\tilde{\cap}\, g_S.$

Proof: Let f_S and g_S be a SI-right ideal of S. Then, since $f_S, g_S \tilde{\subseteq} \tilde{S}$ always holds, we have $f_S \circ g_S \tilde{\subseteq} f_S \circ \tilde{S} \tilde{\subseteq} f_S$ and $f_S \circ g_S \tilde{\subseteq} \tilde{S} \circ g_S \tilde{\subseteq} g_S$. Hence $f_S \circ g_S \tilde{\subseteq} f_S \tilde{\cap} g_S$.

Proposition 5.11. Let f_S be a soft set over U and α be a subset of U such that $\alpha \in Im(f_S)$. If f_S is an SI-left (right) ideal of S over U, the $U(f_S;\alpha)$ is a left (right) ideal of S over U.

6. Soft intersection quasi-ideals of semiring

Definition 6.1. A SI-semiring f_S over U is called a soft intersection quasi-ideal of S over U.

(i) $f_S(x+y) \supseteq f_S(x) \cap f_S(y)$ for all $x, y \in S$

(ii) $(\tilde{S} \circ f_S) \cap (f_S \circ \tilde{S}) \subseteq f_S$.

Theorem 6.2. A soft set f_S of a semiring S is a soft intersection quasi-ideal of S if and only if each nonempty level subset $U(f_S;\alpha)$ of f_S is a quasi-ideal of S.

Proof: Suppose f_S is a fuzzy quasi-ideal of S. Let $a, b \in U(f_S;\alpha)$. Then $f_S(a) \supseteq \alpha$ and $f_S(b) \supseteq \alpha$. As $f_S(a+b) \supseteq f_S(a) \cap f_S(b)$, so $f_S(a+b) \supseteq \alpha$. Hence $a+b \in U(f_S;\alpha)$. Let $x \in U(f_S;\alpha)\tilde{S} \cap \tilde{S}U(f_S;\alpha)$. Then $x = \sum_{i=1}^{m} u_i r_i$ and $x = \sum_{k=1}^{p} s_k v_k$ for some $u_i, v_k \in U(f_S;\alpha)$ and $r_i, s_k \in S$.

Now,

$$f_S(x) \supseteq [(\tilde{S} \circ f_S) \cap (f_S \circ \tilde{S})](x) = (\tilde{S} \circ f_S)(x) \cap (f_S \circ \tilde{S})(x)$$

$$= \bigcup_{x=\sum_{k=1}^{p} s_k v_k} \left[\tilde{S}(s_k) \cap f_S(v_k) \right] \cap \bigcup_{x=\sum_{i=1}^{m} u_i r_i} \left[f_S(u_i) \cap \tilde{S}(r_i) \right] \supseteq \alpha \cap \alpha = \alpha.$$

.So, $f_S(x) \supseteq \alpha$. Thus, $x \in U(f_S;\alpha)$. Hence $U(f_S;\alpha)S \cap SU(f_S;\alpha) \subseteq U(f_S;\alpha)$.

Conversely, assume that each nonempty subset $U(f_S;\alpha)$ of S is a quasi-ideal of S. Let $a, b \in S$ be such that $f_S(a+b) \subset f_S(a) \cap f_S(b)$. Take $\alpha \subseteq U$ such that $f_S(a+b) \subset \alpha \subseteq f_S(a) \cap f_S(b)$. Then $a, b \in U(f_S;\alpha)$ but $a+b \notin U(f_S;\alpha)$, a contradiction. Hence $f_S(a+b) \supseteq f_S(a) \cap f_S(b)$.

Let $x \in S$. If possible let $f_S(x) \subset [(f_S \circ \tilde{S}) \cap (\tilde{S} \circ f_S)](x)$. Take $\alpha \subseteq U$ such that $f_S(x) \subset \alpha \subseteq [(f_S \circ \tilde{S}) \cap (\tilde{S} \circ f_S)](x)$. If $[(f_S \circ \tilde{S}) \cap (\tilde{S} \circ f_S)](x) \supseteq \alpha$, then

$$[(\tilde{S}\circ f_S)\cap(f_S\circ\tilde{S})](x)=\bigcup_{x=\sum_{k=1}^{p}s_kv_k}\left[\tilde{S}(s_k)\cap f_S(v_k)\right]\cap\bigcup_{x=\sum_{i=1}^{m}u_ir_i}\left[f_S(u_i)\cap\tilde{S}(r_i)\right].$$

Hence, $\bigcup_{x=\sum_{k=1}^{p}s_kv_k}[\tilde{S}(s_k)\cap f_s(v_k)]\supseteq\alpha$ and $\bigcup_{x=\sum_{i=1}^{m}s_kv_k}[f_S(u_i)\cap\tilde{S}(r_i)]\supseteq\alpha$.

so, $f_S(u_i)\supseteq\alpha, f_S(v_k)\supseteq\alpha,$ that is, $u_i,v_k\in U(f_s;\alpha)$ for all *i*, *k*. Thus $\sum_{i=1}^{m}u_ir_i\in U(f_S;\alpha)S$ and $\sum_{k=1}^{p}s_kv_k\in SU(f_S;\alpha)$. This implies $x\in U(f_S;\alpha)S\cap SU(f_S;\alpha)\subseteq U(f_S;\alpha,)$ and hence $x\in U(f_S;\alpha)$, that is $f_S(x)\supseteq\alpha$, a contradiction. Hence $(f_S\circ\tilde{S})\cap(\tilde{S}\circ f_S)\subseteq f_S$. Thus f_S is a soft intersection quasi-ideal of S.

Corollary 6.3. Let Q be a nonempty subset of a semiring S. Then Q is a quasi-ideal of S if and only if the characteristic function $\tilde{S}$ of Q is a fuzzy quasi-ideal of S.

Proposition 6.4. The intersection of any two SI-quasi-ideals of a semiring S is a SI-quasi-ideal of S.

Proof: Let f_S, g_S be SI-semiring quasi-ideals of a semiring S and $x,y\in S$. Then

$$(f_S\cap g_S)(x+y)=f_S(x+y)\cap g_S(x+y)\supseteq[f_S(x)\cap f_S(y)]\cap[g_S(x)\cap g_S(y)]$$
$$=[f_S(x)\cap g_S(x)]\cap[f_S(y)\cap g_S(y)]=(f_S\cap g_S)(x)\cap(f_S\cap g_S)(y).$$

Also,

$$((f_S\cap g_S)\circ\tilde{S})\cap(\tilde{S}\circ(f_S\cap g_S))\subseteq(f_S\circ\tilde{S})\cap(\tilde{S}\circ f_S)\subseteq f_S.$$

$$((f_S\cap g_S)\circ\tilde{S})\tilde{S}\circ(f_S\cap g_S))\subseteq(g_S\circ\tilde{S})\cap(\tilde{S}\circ g_S)\subseteq g_S.$$

Thus $((f_S\cap g_S)\circ\tilde{S})\cap(\tilde{S}\circ(f_S\cap g_S))\subseteq f_S\cap g_S$.

Corollary 6.5. Let f_S and g_S be SI- right and SI- left ideals of a semiring S, respectively. Then $f_S\cap g_S$ is a SI- quasi-ideal of S.

7. Soft intersection bi-ideals of semiring

Definition 7.1. A soft set over U is called a SI- bi-ideals of S over U if

(1) $f_S(x+y)\supseteq f_S(x)\cap f_S(y)$

(2) $f_S(xy)\supseteq f_S(x)\cap f_S(y))$

(3) $f_S(xyz)\supseteq f_S(x)\cap f_S(z)$ for all $x,y,z\in S$.

Example 7.2. Consider the semiring S= {0,a,b,c} defined by the following table:

$\oplus$	0	a	b	c
0	0	a	b	c
a	a	b	c	a
b	b	c	a	b
c	c	a	b	c

$\odot$	0	a	B	c
0	0	0	0	0
A	0	a	B	c
B	0	b	B	c
C	0	c	C	c

Define the soft set f_S over $U = Z_4$ such that $f_S(0) = \{\overline{0}, \overline{1}, \overline{2}\}$, $f_s(a) = \{\overline{0}, \overline{1}\}$, $f_S(b) = \{\overline{0}\}$, $f_S(c) = \{\overline{1}, \overline{2}\}$. Clearly f_S is a SI-Bi-ideal of S over U.

Theorem 7.3. Let f_S be a soft set over U. Then, f_S is a SI-bi-ideal of S over U if and only if

(i) $f_S + f_S \tilde{\subseteq} f_S$

(ii) $f_S \circ f_S \tilde{\subseteq} f_S$

(iii) $f_S \circ \tilde{S} \circ f_S \tilde{\subseteq} f_S$.

Proof: Let f_S be a SI- bi-ideal of S and $x \in S$. Then

(1) $(f_S + f_S)(x) = \bigcup_{x=y+z} \{f_S(y) \cap f_S(z)\}$

$\subseteq \bigcup_{x=y+z} f_S(y+z) = f_S(x)$.

Therefore $f_S + f_S \subseteq f_S$.

(2) $f_S \circ f_S \subseteq f_S$ is evident.

(3) Let $s \in S$. If $(f_S \circ \tilde{S} \circ f_S)(s) = \varnothing$, then $f_S \circ \tilde{S} \circ f_S \subseteq f_S$. Otherwise, there exist element $x, y, p, q \in S$ such that $s = xy$ and $x = pq$.

Then, since f_S is an SI-bi-ideal of S over U, we have

$$(f_S \circ \tilde{S} \circ f_S)(s) = [(f_S \circ \tilde{S} \circ f_S)(s)] = \bigcup_{s=pqy} (f_S(p) \cap f_S(y)) \subseteq \bigcup_{s=pqy} f_S(pqy)$$

$$= f_S(xy) = f_S(s)$$

Hence, $f_S \circ \tilde{S} \circ f_S \subseteq f_S$. Here, note that if $x \neq pq$, then $(f_S \circ \tilde{S} \circ f_S \subseteq f_S)(a) = \varnothing$, and so $(f_S \circ \tilde{S} \circ f_S) = \varnothing \subseteq f_S(s)$.

Conversely, assume (1) and (2). By theorem 4.1, f_S is a SI-semiring of S. Let $x, y, z \in S$ and $s = xyz$. Then, since $f_S \circ \tilde{S} \circ f_S \subseteq f_S$, we have

$$f_S(xyz) = f_S(s) \supseteq (f_S \circ \tilde{S} \circ f_S)(s) = [(f_S \circ \tilde{S}) \circ f_S](s)$$

$= \bigcup_{s=(xy)z} [((f_S \circ \tilde{S}(xy) \cap f_S(z))] \supseteq (f_S \circ \tilde{S})(xy) \cap f_S(z) = [\bigcup(f_S(x) \cap \tilde{S}(y))] \cap f_S(z)$

$\supseteq ((f_S(x) \cap \tilde{S})(y)) \cap f_S(z) = (f_S(x) \cap U) \cap f_S(z) = f_S(x) \cap f_S(z).$

Thus, f_S is a SI-bi-ideal of S over U.

Theorem 7.4. Let X be a nonempty subset of a semiring S. Then, X is a bi-ideal of S if and only if S_X is a SI-bi-ideal of S over U.

Proof: Assume that X is a bi-ideal of \$S,\$ that is, $X + X \subseteq X$, $XX \subseteq X$ and $XSX \subseteq X$. Then, we have

$S_X + S_X = S_{X+X} \tilde{\subseteq} S_X$ and

$S_X \circ S_X = S_{XX} \tilde{\subseteq} S_X$ (since xx⊆ X). Thus, S_X is a SI-semiring over U.

Moreover $S_X \circ \tilde{S} \circ S_X = S_X \circ S_s \circ S_X = S_{XSX} \tilde{\subseteq} S_X (since\ XSX \subseteq X)$

This means that S_X is a bi-ideal of S.

Conversely, let S_X be an SI-bi-ideal of S. It means that S_X is a SI-semiring. Let $x \in X + X$. Then, $S_X(x) \supseteq (S_X + S_X)(x) = S_{X+X}(x) = U \Rightarrow x \in X \Rightarrow X + X \subseteq X$ and let $x \in XX$. Then, $S_X(x) \supseteq (S_X \circ S_X)(x) = S_{XX}(x) = U \Rightarrow x \in X \Rightarrow XX \subseteq X$.

Therefore X is a subsemiring of S. Next, let $y \in XSX$. Thus $S_X(y) \supseteq (S_X \circ S \circ S_X)(y) = (S_X \circ S_s \circ S_X)(y) = S_{XSX}(y) = U$ and so $y \in X$. Thus $XSX \subseteq X$ and X is a bi-ideal of S.

Theorem 7.5. Every SI-left (right, two sided) ideal of a semiring S over U is a SI-bi-ideal of S over U.

Proof: Let f_S be a SI-left (right, two sided) ideal of a semiring S over U and $x, y, z \in S$. Then f_S is as SI-semiring by Theorem (5.6.). Moreover,

$f_S(xyz) = f_S((xy)z) \supseteq f_S(z) \supseteq f_S(x) \cap f_S(z)$. Thus f_S is a SI-bi-ideal of S.

Theorem 7.6. Let f_S be any SI-ideal of a semiring S and g_S any SI-bi-ideal of S. $f_S \circ g_S$ and $g_S \circ f_S$ are SI-bi-ideals of S.

Proof: To show that $f_S \circ g_S$ is a SI-bi-ideal of S, first we need to show that $f_S \circ g_S$ is a SI-semiring. Thus

$(f_S \circ g_S) \circ (f_S \circ g_S) = f_S \circ (g_S \circ (f_S \circ g_S))$

$\tilde{\subseteq} f_S \circ (g_S \circ (\circ g_S))\ (since\ f_S \tilde{\subseteq} \tilde{S}) = f_S \circ (g_S \circ (\tilde{S} \circ g_S))$

$\tilde{\subseteq} f_S \circ g_S (since\ g_S \circ (\tilde{S} \circ g_S) \tilde{\subseteq} g_S, (g_S\ being\ SI-bi-ideal)$

Hence by Theorem 4.1, $f_S \circ g_S$ is a SI-semiring over U. Moreover we have

$$(f_S \circ g_S) \circ \tilde{S} \circ (f_S \circ g_S) = f_S \circ (g_S \circ (\tilde{S} \circ f_S) \circ g_S)$$

$$\tilde{\subseteq} f_S \circ (g_S \circ (\tilde{S} \circ g_S))(since\ S \circ f_S \tilde{\subseteq} \tilde{S}) \tilde{\subseteq} f_S \circ g_S.$$

Thus, it follow that $f_S \circ g_S$ is a SI-bi-ideal of S. It can be proved in a similar way that $g_S \circ f_S$ is a SI-bi-ideal of S over U.

Theorem 7.7. Intersection of a non-empty collection of SI- bi-ideal of S over U is also a SI- bi-ideal of S over U.
Proof: The proof follows by routine verifications.

Theorem 7.8. Let $\{f_i : i \in I\}$ be a family of SI- bi-ideal of S over U such that $f_i \subseteq f_j$ or $f_j \subseteq f_i$ for $i, j \in I$. Then $\bigcup_{i \in I} f_i$ is a SI- bi-ideal of S over U.
Proof: The proof follows by routine verifications

Theorem 7.9. In a semiring every SI- quasi ideals are SI- bi-ideals.

8. Conclusion

Through this paper, SI- semiring, SI- left (right, two-sided) ideals of semiring, SI- quasi ideal of semiring and SI- bi- ideal of semirings are studied and properties pertaining to them elicited.

Acknowledgement. The first author is thankful to the University Grants Commission, New Delhi-110021, India, for providing OBC National fellowship under grant no:F.4-1/2016-17/NFO-2015-17-OBC-TAM-29759/(SA-III/Website) 01.04.2016

REFERENCES

1. N.C.Agman and S.Enginoglu, Soft set theory and uni-int decision making, *Eur. J. Oper. Res.*, 207 (2010) 848 - 855.
2. N.C.Agman and S.Enginoglu, Soft matrix theory and its decision making, *Comput. Math. Appl.*, 59 (2010) 3308 - 3314.
3. J.Ahsan, K.Saifullah and M.Farid Khan, Fuzzy semirings, *Fuzzy Sets and Systems*, 60 (1993) 309-320.
4. M.I.Ali, F.Feng, X.Liu, W.K.Min and M.Shabir, On some new operations in soft set theory, *Comput. Math. Appl.*, 57 (2009) 1547-1553.
5. F.Fenga, Y.B.Jun and X.Zhao, Soft semirings, *Comput. and Math. Appl.*, 56 (2008) 2621-628.
6. F.Feng, X.Y.Liu, V.L.-Fotea and Y.B.Jun, Soft sets and soft rough sets, *Inform. Sci.*, 181 (2011) 1125-1137.
7. K.Glazek, *A Guide to the Literature on Semirings and their Applications in Mathematics and Information Sciences*, Kluwer, Dordrecht, 2002.
8. J.S.Golan, *Semirings and Affine Equations over Them: Theory and Applications*, Kluwer, Dordrecht, 2003.
9. M.Henriksen, Ideals in semirings with commutative addition, *Amer. Math. Soc. Notices*, 6 (1958) 321.
10. K.Iizuka, On the Jacobson radical of a semiring, *Tohoku Math. J.*, 11 (2) (1959) 409 - 421.

11. P.K.Maji, A.R.Roy and R.Biswas, An application of soft sets in a decision making problem, *Comput. Math. Appl.*, 44 (2002)1077–1083.
12. P.K.Maji, R.Biswas and A.R.Roy, Soft set theory, *Comput. Math. Appl.*, 45 (2003) 555-562.
13. D.A.Molodtsov, Soft set theory-first results, *Comput. Math. Appl.*, 37 (1999) 19-31.
14. A.Rosenfeld, Fuzzy groups, *J. Math. Anal. Appl.*, 35 (1971) 512 - 517.
15. A.Sezgin and A.O.Atag¨un, On operations of soft sets, *Comput. Math. Appl.*, 61 (2011) 1457-1467.
16. L.A.Zadeh, Fuzzy sets, *Information and Control,* 8 (1995) 338-353.
17. Y.Zou and Z.Xiao, Data analysis approaches of soft sets under incomplete information, *Knowl. Base. Syst.*, 21 (2008) 941-945.