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Adjacent Vertex Distinguishing Total Coloring of Line and Splitting Graph of Some Graphs

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Abstract. In this paper, we prove the existence of the adjacent vertex distinguishing total coloring of line graph of Fan graph F_n, double star $K_{1,n,n}$, Friendship graph F^n and splitting graph of path P_n, cycle C_n, star $K_{1,n}$ and sun let graph S^{2n} in detail.

Keywords: simple graph, adjacent vertex distinguishing total coloring, line graph, splitting graph, path, cycle, star, sun let, fan, double star and friendship graph.

AMS Mathematics Subject Classification (2010): 05C15, 05C38, 05C76

1. Introduction

Throughout this paper, we consider finite, simple, connected and undirected graph $G=(V(G),E(G))$. For every vertex $u,v\in V(G)$, the edge connecting two vertices is denoted by $uv\in E(G)$. For all other standard concepts of graph theory, we see [1,2,5]. For graphs G_1 and G_2, we let $G_1\cup G_2$ denotes their union, that is, $V(G_1\cup G_2)=V(G_1)\cup V(G_2)$ and $E(G_1\cup G_2)=E(G_1)\cup E(G_2)$. If a simple graph G has two adjacent vertices of maximum degree, then $\psi_{avt}(G)\geq\Delta(G)+2$. Otherwise, if a simple graph G does not have two adjacent vertices of maximum degree, then $\psi_{avt}(G)=\Delta(G)+1$. The well-known $AVDTC$ conjecture, made by Zhang et al. [8] says that every simple graph G has $\chi_{avt}(G)\leq\Delta(G)+3$. AVDTC of tensor product of graphs are discussed in litrature [6,7]. (2,1)-total labelling of cactus graphs and adjacent vertex distinguishing edge colouring of cactus graphs are discussed in literature [3,4]. Throughout the paper, we denote the path graph, cycle graph, complete graph, star graph, sun let, fan graph, double star and friendship graph by P_n, C_n, K_n, $K_{1,n}$, S^{2n}, F_n, $K_{1,n}$ and F^n respectively. A proper total coloring of G is an assignment of colors to the vertices and the edges such that no two adjacent vertices and adjacent edges are assigned with the same color, no edge and its end vertices are assigned with the same color and for any pair of adjacent vertices have distinct set of colors.

A total k-coloring of the graph G is adjacent vertex distinguishing, if a mapping $f: V(G) \cup E(G) \to \{1,2,\cdots,k\}$, $k \in \mathbb{Z}^+$ such that any two adjacent or incident elements in $V(G) \cup E(G)$ have different colors. The minimum number of colors required to give an adjacent vertex distinguishing total coloring (abbreviated as AVDTC) to the graph G is denoted by $\chi_{avt}(G)$.

First, we find the adjacent vertex distinguishing total chromatic number of Fan graph, double star graph, Friendship graph and complete graph.

Proposition 1.1. $\chi_{avt}(F_n) = n+1,\ n > 3$.

Proof: Fan graph denoted by F_n, is a graph obtained by joining all vertices of the path P_n to a further vertex called the center vertex v. Thus F_n contains $n+1$ vertices and $2n-1$ edges. Here, we have the vertex and edge set $V(F_n) = \{v, v_1, v_2, \cdots, v_n\}$ and $E[F_n] = \left\{\left(\bigcup_{i=1}^{n} vv_i\right) \cup \left(\bigcup_{i=1}^{n-1} v_i v_{i+1}\right)\right\}$. Now we define $f: V(F_n) \cup E(F_n) \to \{1,2,\cdots,k\}$ given by,

$$For\ 1 \le i \le n,\quad f(v_i) = i\ \ and\ \ f(v) = n+1$$
$$For\ 1 \le i \le n-1,\quad f(v_i v_{i+1}) = i+2 and f(vv_1) = n$$
$$For\ 2 \le i \le n,\quad f(v_i v_n) = i-1$$

$\therefore\ \ \chi_{avt}(F_n) = n+1,\ n > 3$.

Proposition 1.2. $\chi_{avt}(K_{1,n,n})) = n+1,\ \ for\ n > 2$.

Proof: The double star $K_{1,n,n}$ is obtained from the star $K_{1,n}$ by adding a new pendent edge of the existing n pendent vertices. Here v is the apex vertex. The vertex set and edge set of $K_{1,n,n}$ as follows,

$$V[K_{1,n,n}] = \left\{\left(\bigcup_{i=1}^{n} v \cup v_i \cup v_i'\right)\right\}\ and\ E[K_{1,n,n}] = \left\{\left(\bigcup_{i=1}^{n} (vv_i \cup v_i v_i')\right)\right\}$$

Then $|V(K_{1,n,n})| = 2n+1$ and $|E(K_{1,n,n})| = 2n$. We define

$f: V(K_{1,n,n}) \cup E(K_{1,n,n}) \to \{1,2,\cdots,k\}$ given by

$f(v) = n+1,\ f(vv_n) = 1,\ f(v_{n-1}v_{n-1}') = 1 and f(v_n v_n') = 2$.

For $1 \le i \le n,\ \ f(v_i) = i and f(v_i') = n+1,$

For $1 \le i \le n-1,\ \ f(vv_i) = i+1$

For $1 \le i \le n-2,\ \ f(v_i v_i') = i+2$.

$\therefore\ \ \chi_{avt}(K_{1,n,n})) = n+1,\ $ for $n > 2$.

Proposition 1.3. $\chi_{avt}(F^n) = 2n+1,\ n \ge 2$.

Proof: The friendship graph F^n can be constructed by joining n copies of the cycle graph C_3 with a common vertex say v. Thus, the vertex set and edge set are $\{v, v_1, v_2, \cdots, v_{2n}\}$ and $E[F^n] = \left\{\left(\bigcup_{i=1}^{2n} vv_i\right) \cup \left(\bigcup_{i=1}^{n} v_{2i-1}v_{2i}\right)\right\}$. Here $|V(F^n)| = 2n+1$ and $|E(F^n)| = 3n$. Now we define $f : V(F^n) \cup E(F^n) \to \{1,2,\cdots,k\}$ given by for $n \geq 2$,
For $1 \leq i \leq n$,
$f(v_{2i-1}) = 2,\ f(v_{2i}) = 3,\ f(vv_{2i-1}) = 2i-1,\ f(vv_{2i}) = 2i \textit{ and } f(v) = f(v_{2i-1}v_{2i}) = 2n+1.$

$\therefore \quad \chi_{avt}(F^n) = 2n+1,\ n \geq 2$.

Proposition 1.4. *The complete graph K_n admits $AVDTC$ and*

$$\chi_{avt}(K_n) = \begin{cases} n+1, & if\ n \equiv 0\,(mod\ 2) \\ n+2, & if\ n \equiv 1\,(mod\ 2) \end{cases}$$

Proof: Let $x_1, x_2, \cdots x_n$ be the vertices of complete graph K_n. We define $f : V(K_n) \cup E(K_n) \to \{1,2,\cdots,k\}$ given by

Case-1 When $n \equiv 0\,(mod\ 2)$
For $1 \leq i, j \leq n$

$$\begin{aligned} f(x_i) &= i \\ f(x_i x_j) &= i+j,\ if\ i+j = n+1 \end{aligned}$$

For $i + j \neq n+1$

$$f(x_i x_j) \equiv \begin{cases} \dfrac{i+j}{2}, & if\ \ i+j \equiv 0\,(mod\ 2) \\ \left(\left\lceil \dfrac{n+1}{2} \right\rceil (i+j)\right) (mod\ n+1), & if\ \ i+j \equiv 1\,(mod\ 2) \end{cases}$$

Case-2 When $n \equiv 1\,(mod\ 2)$
For $1 \leq i, j \leq n$

$$\begin{aligned} f(x_i) &= i, \\ f(x_i x_j) &= i+j,\ if\ i+j = n+2 \end{aligned}$$

For $i + j \neq n+2$

$$f(x_i x_j) \equiv \begin{cases} \dfrac{i+j}{2}, & if\ \ i+j \equiv 0\,(mod\ 2) \\ \left(\left\lceil \dfrac{n+2}{2} \right\rceil (i+j)\right) (mod\ n+2), & if\ \ i+j \equiv 1\,(mod\ 2) \end{cases}$$

$$\therefore \chi_{avt}(K_n) = \begin{cases} n+1, & if\ \ n \equiv 0\,(mod\ 2) \\ n+2, & if\ \ n \equiv 1\,(mod\ 2) \end{cases}$$

We observe that these graphs are admits the adjacent vertex distinguishing total coloring conjecture. Next, we discuss the line graph of these graphs admits AVDTC conjecture.

2. AVDTC of line graph

A graph G and its line graph $L(G)$ is a graph such that each vertex of $L(G)$ represents an edge of G and two vertices of $L(G)$ are adjacent iff their corresponding edges are adjacent in G. In this section, the adjacent vertex distinguishing total coloring of line graph of Fan graph $L(F_n)$, double star $L(K_{1,n,n})$ and Friendship graph $L(F^n)$ are discussed.

Theorem 2.1. *The line graph of Fan graph $L(F_n)$ admits $AVDTC$ and*

$$\chi_{avt}L(F_n) = n+3, \textit{ for } n > 3$$

Proof: From the proposition (1.1), Let us consider the edges of F_n as $vv_i = x_i$, *for* $1 \le i \le n$ and $v_i v_{i+1} = y_i$, *for* $1 \le i \le n-1$.

$$V[L(F_n)] = \left\{\left(\bigcup_{i=1}^{n} x_i\right) \cup \left(\bigcup_{i=1}^{n-1} y_i\right)\right\}$$

Hence $|V(L(F_n))| = 2n-1$ and $|E(L(F_n))| = \frac{n^2+5n-8}{2}$. Note that $\{x_1, x_2, \cdots, x_n\}$ forms the complete graph K_n with n vertices. Now we define $f : V(L(F_n)) \cup E(L(F_n)) \to \{1,2,\cdots,k\}$ as follows. For this, first we color a clique in $L(F_n)$ using the proposition (1.4). Now the remaining vertices and edges of $L(F_n)$ are colored as follows.

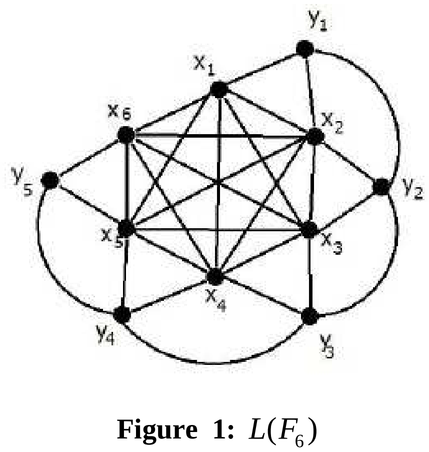

Figure 1: $L(F_6)$

Case-1. When n is even.

For $1 \le i \le n-1$, $f(x_i y_i) = n+2$; $f(x_{i+1} y_i) = n+3$; $f(y_i) = f(x_i x_{i+1})$;

For $1 \le i \le n-2$, $f(y_i y_{i+1}) = i+1$.

Case-2. When n is odd.
For $1 \le i \le n-1$, $f(x_i x_{i+1}) = f(y_i)$; $f(x_{i+1} y_i) = n+3$;
For $1 \le i \le n-2$, $f(y_i y_{i+1}) = i+1$,
For $1 \le i \le \lfloor \frac{n}{2} \rfloor$, $f(x_{2i} y_{2i}) = i$; $f(x_{2i-1} y_{2i-1}) = \lceil \frac{n}{2} \rceil + i$.
Hence Line graph of fan graph admits AVDTC.

$\therefore \quad \chi_{avt}(L(F_n)) = n+3$. Hence the theorem.

Theorem 2.2. *The line graph of double star graph $L(K_{1,n,n})$ admits AVDTC and*

$$\chi_{avt} L(K_{1,n,n}) = n+2, \text{ for } n > 3.$$

Proof: From the proposition (1.2), Let us consider the edges of $K_{1,n,n}$ as $vv_i = x_i$ and $v_i v_i' = y_i$ for $1 \le i \le n$. $V[L(K_{1,n,n})] = \left\{ \bigcup_{i=1}^{n} (x_i \cup y_i) \right\}$ Hence $|V(L(K_{1,n,n}))| = 2n$ and $|E(L(K_{1,n,n}))| = \frac{n^2+n}{2}$. Note that $\{x_1, x_2, \cdots, x_n\}$ forms the complete graph K_n with n vertices. Now we define $f : V(L(K_{1,n,n})) \cup E(L(K_{1,n,n})) \to \{1,2,\cdots,k\}$ as follows. For this, first we color a clique in $L(K_{1,n,n})$ using the proposition (1.4). Now the remaining vertices and edges are colored as follows.

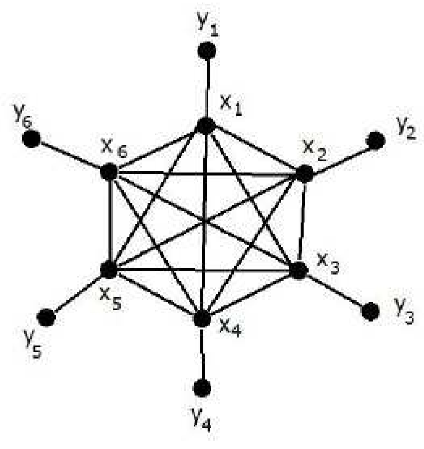

Figure 2: $L(K_{1,6,6})$

Case-1. When n is even.
For $1 \le i \le n$, $f(x_i y_i) = n+2$; $f(y_i) = n+1$.

Case-2. When n is odd.

For $1 \le i \le n$, $f(y_i) = n+2$. For $1 \le i \le \lfloor \frac{n}{2} \rfloor$, $f(x_{2i}y_{2i}) = i$.

For $1 \le i \le \lceil \frac{n}{2} \rceil$, $f(x_{2i-1}y_{2i-1}) = \lceil \frac{n}{2} \rceil + i$.

Hence the Line graph of double star admits AVDTC.

$\therefore \quad \chi_{avt}(L(K_{1,n,n})) = n+2.$ Hence the theorem.

Theorem 2.3. *The line graph of friendship graph* $L(F^n)$ *admits* $AVDTC$ *and*

$$\chi_{avt}L(F^n) = 2n+3, \; for \; n \ge 2$$

Proof: From the proposition (1.3), Let us consider the edges of F^n as $v_{2i-1}v_{2i} = y_i$, $for \; 1 \le i \le 2n$ and $vv_i = x_i$, $for \; 1 \le i \le n$.

$$V[L(F^n)] = \left\{ \left(\bigcup_{i=1}^{n} y_i \right) \cup \left(\bigcup_{i=1}^{2n} x_i \right) \right\}$$

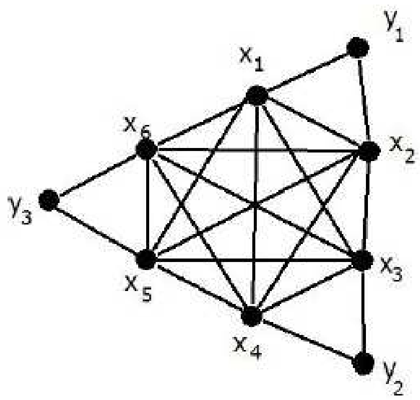

Figure 3: $L(F^3)$

Hence $|V(L(F^n))| = 3n$ and $|E(L(F^n))| = n(2n+1)$. Note that $\{x_1, x_2, \cdots, x_{2n}\}$ forms the complete graph K_{2n} with $2n$ vertices and x_{2i-1} and x_{2i} are adjacent with y_i for $1 \le i \le n$. Now we define $f : V(L(F^n)) \cup E(L(F^n)) \to \{1,2,\cdots,k\}$ as follows. For this, first we color a clique in $L(F^n)$ using the poposition (1.4). The remaining are colored for $1 \le i \le n$, $f(x_{2i-1}y_i) = n+2$; $f(x_{2i}x_i) = n+3$; $f(y_i) = f(x_{2i-1}x_{2i})$. Hence the Line graph of friendship graph admits AVDTC. $\therefore \quad \chi_{avt}(L(F^n)) = 2n+3.$

3. AVDTC of splitting graph

For a graph G the splitting graph of G is obtained by adding a new vertex v'

corresponding to each vertex v of G such that $N(v) = N(v')$. The resultant graph is denoted as $Sp(G)$. In this section, the adjacent vertex distinguishing total coloring of Splitting graph of path P_n, cycle C_n and star $K_{1,n}$ and sun let S^{2n} graphs are discussed.

Theorem 3.1. *The splitting graph of path graph $Sp(P_n)$ admits $AVDTC$ and*

$$\chi_{avt} Sp(P_n) = \begin{cases} 6, & for\ n \geq 4 \\ 5, & for\ n = 3 \end{cases}$$

Proof: Let we denote $\{v_1, v_2, \cdots, v_n\}$ as the vertices of P_n and $\{v'_1, v'_2, \cdots, v'_n\}$ as the new vertices. The vertex set and edge set of $Sp(P_n)$ are

$$V[Sp(P_n)] = \left\{ \left(\bigcup_{i=1}^{n} v_i \cup v'_i \right) \right\} \ and \ \ E[Sp(P_n)] = \left\{ \left(\bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup v_i v'_{i+1} \cup v'_i v_{i+1}) \right) \right\}$$

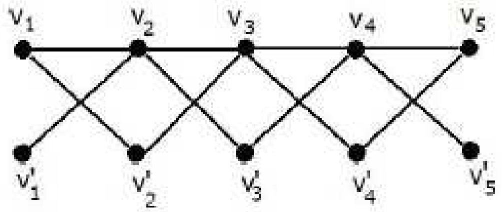

Figure 4: $Sp(P_5)$

Hence $|V(Sp(P_n))| = 2n$ and $|E(Sp(P_n))| = 3(n-1)$. This theorem is trivial for $n = 3$. Now we define $f : V(Sp(P_n)) \cup E(Sp(P_n)) \to \{1, 2, \cdots, k\}$ given by for $n \geq 4$,

$$For\ 1 \leq i \leq n, \quad f(v_i) = f(v'_i) = \begin{cases} \{1\}, if\ i \equiv 1\ (mod\ 2) \\ \{2\}, if\ i \equiv 0\ (mod\ 2) \end{cases}$$

$$For\ 1 \leq i \leq n-1, \quad f(v_i v_{i+1}) = \begin{cases} \{3\}, if\ i \equiv 1\ (mod\ 2) \\ \{4\}, if\ i \equiv 0\ (mod\ 2) \end{cases}$$

$$f(v_i v'_{i+1}) = 5 \ \ and \ \ f(v'_i v_{i+1}) = 6$$

Clearly, the color classes of any two adjacent vertices are different.

$\therefore \quad \chi_{avt}(Sp(P_n)) = 6, \quad n \geq 4.$ Hence the theorem.

Theorem 3.2. *The splitting graph of cycle graph $Sp(C_n)$ admits $AVDTC$ and*

$$\chi_{avt} Sp(C_n) = 6, \ \ n \geq 4.$$

Proof: Let we denote $\{v_1, v_2, \cdots, v_n\}$ as the vertices of C_n and $\{v'_1, v'_2, \cdots, v'_n\}$ as the new vertices. The vertex set and edge set of $Sp(C_n)$ is given by

$$V[Sp(C_n)] = \left\{ \left(\bigcup_{i=1}^{n} v_i \cup v_i' \right) \right\} \; and$$

$$E[Sp(C_n)] = \left\{ \left(\bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup v_i' v_{i+1} \cup v_i v_{i+1}') \right) \cup (v_n v_1 \cup v_n' v_1 \cup v_n v_1') \right\}$$

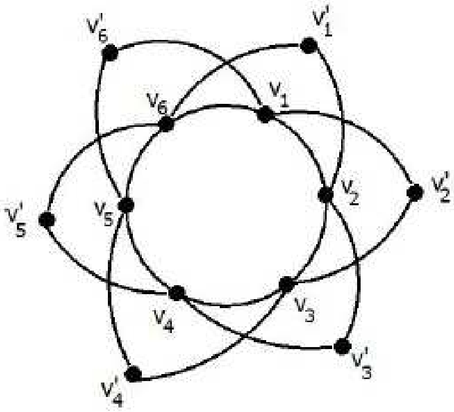

Figure 5: $Sp(C_6)$

Then $|V(Sp(C_n))| = 2n$ and $|E(Sp(C_n))| = 3n$.

Define $f: V(Sp(C_n)) \cup E(Sp(C_n)) \rightarrow \{1, 2, \cdots, k\}$ as follows.

Case-1. When n is even.

$$For \;\; 1 \le i \le n, \quad f(v_i) = f(v_i') = \begin{cases} 1, & if \;\; i \equiv 1 \, (mod \;2) \\ 2, & if \;\; i \equiv 0 \, (mod \;2) \end{cases}$$

$$For \; 1 \le i \le n-1, \quad f(v_i v_{i+1}) = \begin{cases} 3, & if \;\; i \equiv 1 \, (mod \;2) \\ 4, & if \;\; i \equiv 0 \, (mod \;2) \end{cases}$$

$$f(v_i' v_{i+1}) = f(v_n' v_1) = 6, \; f(v_i v_{i+1}') = f(v_n v_1') = 5, \; and f(v_n v_1) = 4.$$

Case-2. When n is odd.

$$For \quad 1 \le i \le n-1, \; f(v_i) = f(v_i') = \begin{cases} 1, & if \;\; i \equiv 1 \, (mod \;2) \\ 2, & if \;\; i \equiv 0 \, (mod \;2) \end{cases}$$

$$For \; 1 \le i \le n-2, \; f(v_i v_{i+1}) = \begin{cases} 3, & if \;\; i \equiv 1 \, (mod \;2) \\ 4, & if \;\; i \equiv 0 \, (mod \;2) \end{cases}$$

$$For \;\; 1 \le i \le n-1, \;\; f(v_i' v_{i+1}) = f(v_n' v_1) = 6, \;\; f(v_i v_{i+1}') = f(v_n v_1') = 5, \; f(v_n) = 4,$$

$$f(v_n v_1) = 2 \; and \;\; f(v_{n-1} v_n) = 1$$

The color classes of any two adjacent vertices are different.

$\therefore \quad \chi_{avt}(Sp(C_n)) = 6$.

Hence the theorem.

Theorem 3.3. *The splitting graph of star graph* $Sp(K_{1,n})$ *admits* $AVDTC$ *and*

$$\chi_{avt} Sp(K_{1,n}) = 2n+1, \ n \geq 2.$$

Proof: Let $\{v, v_1, v_2, \cdots, v_n\}$ be the vertices of star $K_{1,n}$ and $\{v', v_1', v_2', \cdots, v_n'\}$ be the new vertices of $K_{1,n}$. Here v is the apex vertex. The vertex set and edge set of $Sp(K_{1,n})$ is given by

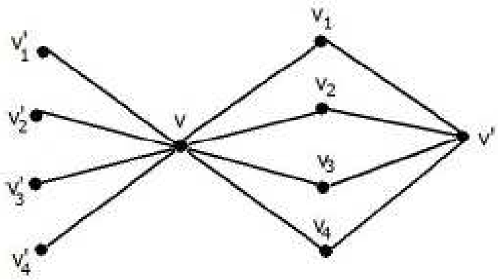

Figure 6: $Sp(K_{1,4})$

$$V[S(K_{1,n})] = \left\{ \left(\bigcup_{i=1}^{n} v_i \cup v_i' \right) \cup v \cup v' \right\} \ and \ E[Sp(K_{1,n})] = \left\{ \bigcup_{i=1}^{n} (vv_i \cup vv_i' \cup v'v_i) \right\}$$

Then $|V(Sp(K_{1,n}))| = 2(n+1)$ and $|E(Sp(K_{1,n}))| = 3n$.

We define $f : V(Sp(K_{1,n})) \cup E(Sp(K_{1,n})) \rightarrow \{1, 2, \cdots, k\}$ given by

For $1 \leq i \leq n$,

$$f(v_i) = f(v_i') = i, \ f(v) = f(v') = n+1, \ f(vv_i') = f(v'v_i) = n+1+i$$

For $1 \leq i \leq n-1$,

$$f(vv_i) = i+1, \ f(vv_n) = 1$$

Clearly, the color classes of any two adjacent vertices are different.

$\therefore \quad \chi_{avt}(Sp(K_{1,n})) = 2n+1, \ n \geq 2.$

Hence the theorem.

Theorem 3.4. *The splitting graph of sun let graph* S^{2n} *admits* $AVDTC$ *and*

$$\chi_{avt} Sp(S^{2n}) = 8, \ n \geq 4.$$

Proof: Let $\{v_1, v_2, \cdots, v_n, v_1', v_2', \cdots v_n'\}$ be the vertices of sun let graph S^{2n}. Here $deg(v_i) = 3$ and $deg(v_i') = 1$ for $i = 1, 2, \cdots, n$. Now consider the new vertices $\{u_1, u_2, \cdots, u_n, u_1', u_2', \cdots, u_n'\}$ of S^{2n}. The vertex set and edge set of $Sp(S^{2n})$ is given by

$$V[Sp(S^{2n})] = \left\{ \left(\bigcup_{i=1}^{n} v_i \cup v_i' \cup u_i \cup u_i' \right) \right\}$$

$$E[Sp(S^{2n})] = \left\{ \bigcup_{i=1}^{n-1} (v_i v_{i+1} \cup v_i u_{i+1} \cup v_{i+1} u_i) \cup \bigcup_{i=1}^{n} (v_i v_i' \cup v_i u_i' \cup u_i v_i') \cup (v_n v_1 \cup v_n u_1 \cup v_1 u_n) \right\}$$

Then $|V(Sp(S^{2n}))| = 4n$ and $|E(Sp(S^{2n}))| = 6n$.

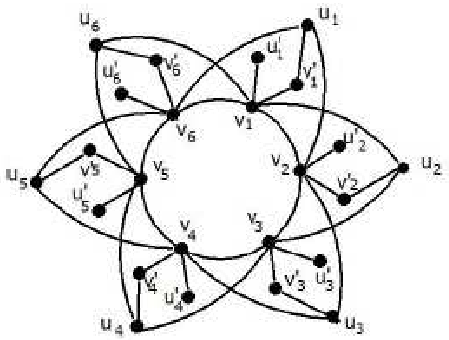

Figure 7: $Sp(S^6)$

We define $f : V(Sp(S^{2n})) \cup E(Sp(S^{2n})) \to \{1,2,\cdots,k\}$ given by

Case-1. If $n \equiv 1 \ (mod\ 2)$.

For $1 \le i \le n-1$

$$f(v_i) = f(u_i) = \begin{cases} 1, & if\ i \equiv 1 \ (mod\ 2) \\ 2, & if\ i \equiv 0 \ (mod\ 2) \end{cases}$$

For $1 \le i \le n-2$

$$f(v_i v_{i+1}) = \begin{cases} 3, & if\ i \equiv 1 \ (mod\ 2) \\ 4, & if\ i \equiv 0 \ (mod\ 2) \end{cases}$$

and $f(v_n) = 4$, $f(v_n v_1) = 2$, $f(v_{n-1} v_n) = 1$

For $1 \le i \le n-1$

$$f(v_i u_{i+1}) = f(v_n u_1) = 5,\ f(v_{i+1} u_i) = f(v_1 u_n) = 6$$

For $1 \le i \le n$

$$f(v_i') = f(u_i') = 3,\ f(v_i v_i') = 7 \ and \ f(u_i v_i') = f(v_i u_i') = 8$$

Case-2. If $n \equiv 0 \ (mod\ 2)$.

For $1 \le i \le n$

$$f(v_i) = f(u_i) = \begin{cases} 1, & if\ i \equiv 1 \ (mod\ 2) \\ 2, & if\ i \equiv 0 \ (mod\ 2) \end{cases}$$

For $1 \le i \le n-1$

$$f(v_i v_{i+1}) = \begin{cases} 3, & if\ i \equiv 1 \ (mod\ 2) \\ 4, & if\ i \equiv 0 \ (mod\ 2) \end{cases} \quad and \quad f(v_n v_1) = 4.$$

For $1 \le i \le n-1, \quad f(v_i u_{i+1}) = f(v_n u_1) = 5,\ f(v_{i+1} u_i) = f(v_1 u_n) = 6$

For $1 \le i \le n, \quad f(v_i') = f(u_i') = 3,\ f(v_i v_i') = 7$ *and* $f(u_i v_i') = f(v_i u_i') = 8$

$\therefore \quad \chi_{avt}(Sp(S^{2n})) = 8,\ \ n \ge 4.$

Therefore the splitting graph of sun let graph admits AVDTC conjecture. Hence the theorem.

4. Conclusion

We found the adjacent vertex distinguishing total chromatic number of line graph of Fan graph, double star, Friendship graph and splitting graph of path, cycle, star and sun let graph. Also, it is proved that the adjacent vertex distinguishing total coloring conjecture is true for these graphs.

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