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3-Equitable and Total Magic Cordial Labeling for the Extended Duplicate Graph of Splitting Graph of Path

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Abstract. In this paper, we prove that the extended duplicate graph of splitting graph of path admits 3-equitable and total magic cordial labeling.

Keywords: Graph labeling, duplicate graph, splitting graph, 3-equitable, total magic cordial

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

All graphs in this paper are simple, finite and undirected. The vertex set and edge set of a graph is denoted by V(G) and E(G) respectively. If the vertices of the graph are assigned values subject to certain conditions then it is known as graph labeling. A useful survey to know about the numerous labeling methods is by Gallian [5]. Sampathkumar has introduced the concept of duplicate graph [4]. A new labeling called total Magic cordial labeling and 3-equitable labeling introduced by Cahit [3], Girija and Elumalai have proved that the edge magic total labeling of the cycle C_n with p_3 chords [8]. Thirusangu, Selvam and Ulaganathan have proved that the total magic cordial labeling for the extended duplicate graph of twig graphs [2]. Sutha, Thirusangu and Bala have proved that the some graph labelings on middle graph of extended duplicate graph of a path [6]. Thirusangu, Ulaganathan and Vijaya kumar have proved that the some cordial labeling of duplicate graph of ladder graph [7]. Mary et al. have proved that the Labelings on jahangir graph and extended duplicate graph of Jahangir graph [10]. Graph labelings have many applications within mathematics as well as to several areas of computer science and communication networks. Kalantari, Khosrovshahi and Mitchell [1,9] tried to find applications of magic labeling in optimisation theory, especially for the travelling salesmen problem. In this paper, we prove that the existence of extended duplicate graph of splitting graph of path admits 3-equitable and total magic cordial labeling.

2. Preliminaries

In this section, we give the basic definitions relevant to this paper. Let G(V,E) be a finite, simple and undirected graph with p vertices and q edges.

Definition 2.1. (Splitting graph) For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph Spl(G) thus obtained is called splitting graph of G.

Illustration 1. (The splitting graph of path Spl(P_6))

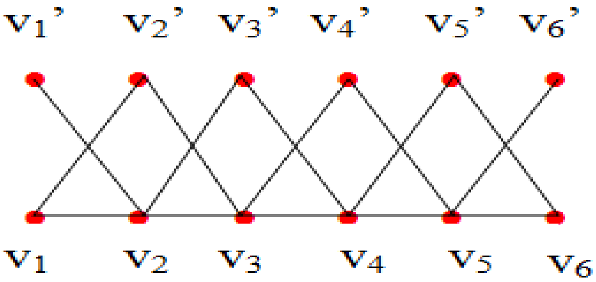


Figure 1:

Definition 2.2. (Duplicate graph) Let G (V,E) be a simple graph and the duplicate graph of G is DG = (V_1, E_1) ,where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in$ V, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge *ab* is in E if and only if both ab' and $a'b$ are edges in E_1.

Definition 2.3. (Extended duplicate graph of splitting graph of path) Let DG = (V_1,E_1) be a duplicate graph of splitting graph of path G(V,E). Extended duplicate graph of splitting graph of path is obtained by adding the edge v_2v_2' to the duplicate graph. It is denoted by EDG Spl(P_m). Clearly it has 4m vertices and 6m-5 edges, where $m \geq 2$ is the number of length.

Illustration 2. Extended duplicate graph of splitting graph of a path

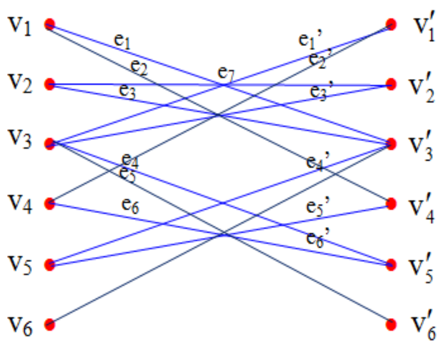


Figure 2: ***EDG(Spl(P_3))***

Definition 2.5. (3-equitable labeling) A function $f : V \rightarrow \{0,1,2\}$ such that each edge uv receives the label $|f(u) - f(v)|$ is said to be 3-equitable labeling if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq 2$.

Definition 2.6. (**Total magic cordial labeling)** A graph G(V,E) is said to admit total total magic cordial labeling if $f : V \cup E \rightarrow \{0, 1\}$ such that

$(i)\{f(x) + f(y) + f(xy)\}$ (mod 2) is constant for all edges $xy \in$ E.

$(ii)|f(0) - f(1)| \leq 1$, where

a) $f(0)$ denotes the sum of the number of the vertices labeled with ‘ 0’ and the number of edges labeled with ’ 0’ .

b) $f(1)$ denotes the sum of the number of the vertices labeled with ‘1’ and the number of edges labeled with ‘1’ .

3. Main results

Algorithm 3.1.

Procedure [3-Equitable labeling for EDG(Spl(P_m)), m≥ 2]

V ← {v_1, v_2, … ,v_{2m}, v'_1, v'_2, … ,v'_{2m}}

E ← {e_1, e_2, … ,e_{3m-3}, e_{3m-2}, e'_1,e'_2,...,e'_{3m-3}}

$v_1 \leftarrow 1$, $v_2 \leftarrow 1$, $v'_1 \leftarrow 2$, $v'_2 \leftarrow 0$

for i=0 to [(m-2)/3] **do**

$v_{3+6i} \leftarrow 0$
$v_{4+6i} \leftarrow 0$
$v'_{3+6i} \leftarrow 1$
$v'_{4+6i} \leftarrow 2$

end for

for i=0 to [(m-3)/3] **do**

$v_{5+6i} \leftarrow 2$
$v_{6+6i} \leftarrow 0$
$v'_{5+6i} \leftarrow 2$
$v'_{6+6i} \leftarrow 1$

end for

for i=0 to [(m-4)/3] **do**

$v_{7+6i} \leftarrow 1$
$v_{8+6i} \leftarrow 1$
$v'_{7+6i} \leftarrow 2$
$v'_{8+6i} \leftarrow 0$

end for
end procedure

Theorem 3.1. The extended duplicate graph of splitting path graph $Spl(P_m)$, $m \geq 2$ admits 3-equitable labeling.

Proof: Let $Spl(P_m)$, $m \geq 2$ be a splitting path graph . In order to label the vertices, define a function $f : V \rightarrow \{0,1,2\}$ as given in algorithm 3.1.

The vertices v_1, v_2, v'_1 and v'_2 receive label '1' , '1' , '2' and '0' respectively **;**

for $0 \leq i \leq [(m-2)/3]$, the vertices v_{3+6i} receive label '0' ; the vertices v_{4+6i} receive label '0' ; the vertices v'_{3+6i} receive label '1' ; the vertices v'_{4+6i} receive label '2' **;**

for $0 \leq i \leq [(m-3)/3]$, the vertices v_{5+6i} receive label '2' ; the vertices v_{6+6i} receive label '0' ; the vertices v'_{5+6i} receive label '2' and the vertices v'_{6+6i} receive label '1' **;**

for $0 \leq i \leq [(m-4)/3]$, the vertices v_{7+6i} receive label '1' ; the vertices v_{8+6i} receive label '1' ; the vertices v'_{7+6i} receive label '2' ; the vertices v'_{8+6i} receive label '0' ;

Thus, the entire 4m vertices are labeled.

To obtain the labels for edges , we define the induced function $f^* : E \rightarrow \{0,1,2\}$ such that $f^*(v_i v_j) = | f(v_i) - f(v_j) |$ where $v_i , v_j \in V$

The induced function yields the label '0' for the edges e_1, e_3, e'_3 **;** the label '1' for the edges e_2, e_{3m-2} **;** the label '2' for the edges e'_1 and e'_2 **;**

for $0 \leq i \leq [(m-3)/3]$ and $0 \leq j \leq 1$, the edges $e_{4+9i+2j}$ receive label '2' ; the edges e_{5+9i} receive label '1'; the edges e'_{4+9i+j} receive label '1' ; the edges e'_{6+9i} receive label '0' **;**

for $0 \leq i \leq [(m-4)/3]$ and $0 \leq j \leq 1$, the edges e_{7+9i} receive label '0' ; the edges e_{8+9i+j} receive label '0' ; the edges e'_{7+9i+j} receive label '1' ; the edges e'_{9+9i} receive label '0' ;

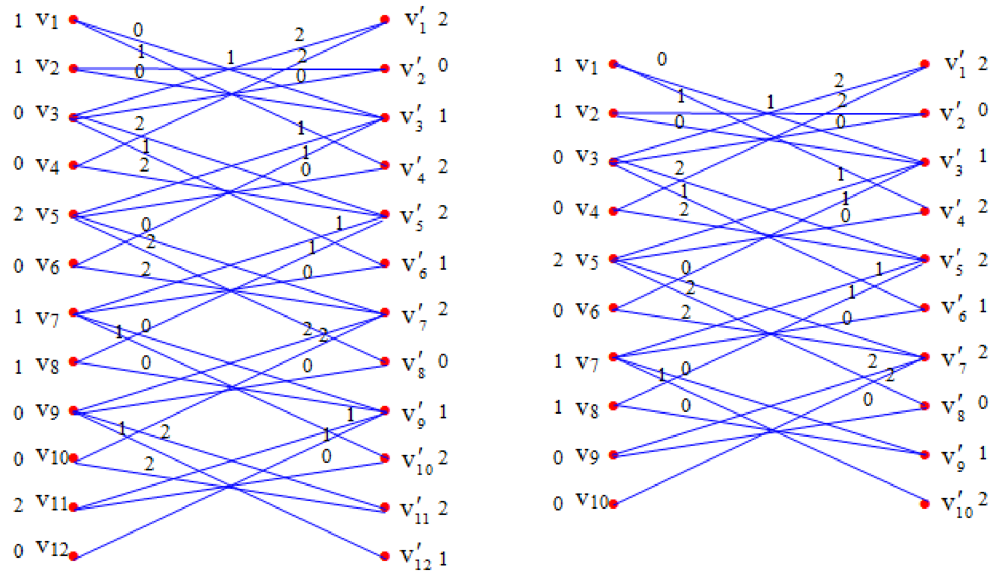
for $0 \leq i \leq [(m-5)/3]$ and $0 \leq j \leq 1$, the edges $e_{10+9i+2j}$ receive label '0' ; the edges e_{11+9i} receive label '1' ; the edges $e'_{10+9i+j}$ receive label '2' ; the edges e'_{12+9i} receive label '0';

case (i) If (m-2)mod 3 = 0, then 2m-1 edges receive label '0', 2m-2 edges receive label '1' and 2m-2 edges receive label '2';

case (ii) If (m-2)mod 3 $\neq$ 0,then 2m-1 edges receive label '1', 2m-2 edges receive label '2' and 2m-2 edges receive label '0';

Thus in both the cases all the 6m-5 edges are labeled.Hence the extended duplicate graph of splitting path graph $Spl(P_m)$, $m \geq 2$ is 3-equitable labeling.

Illustration 3. 3-equitable labeling for edg of spilitting graph of path.



(a) ***EDG(Spl(P₆))*** **(b)** ***EDG(Spl(P₅))***

Figure 3:

3.2. Total magic cordial labeling

Algorithm 3.2.

Procedure[Total magic cordial labeling for($Spl(P_m)$), $m \geq 2$]

$V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v'_1, v'_2, \ldots, v'_{2m}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{3m-3}, e_{3m-2}, e'_1, e'_2, \ldots, e'_{3m-3}\}$

$v_1 \leftarrow 1$, $v_2 \leftarrow 1$, $e_{3m-2} \leftarrow 1$

$v'_1 \leftarrow 0$, $v'_2 \leftarrow 0$

for i=0 to [(m-2)/2] **do**

$v_{3+4i} \leftarrow 1$

$v_{4+4i} \leftarrow 0$

$v'_{3+4i} \leftarrow 1$

$v'_{4+4i} \leftarrow 0$

end for

for i = 0 to [(m-3)/2] **do**

$v_{5+4i} \leftarrow 1$

$v_{6+4i} \leftarrow 1$

$v'_{5+4i} \leftarrow 0$

$v'_{6+4i} \leftarrow 0$

end for

for i = 0 to [(m-2)/2] **do**

e_{1+6i} ← 0
e_{2+6i} ← 1
e_{3+6i} ← 0
e'_{1+6i} ← 1
e'_{2+6i} ← 0
e'_{3+6i} ← 1
end for
for i = 0 to [(m-3)/2] **do**
e_{4+6i} ← 1
e_{5+6i} ← 1
e_{6+6i} ← 0
e'_{4+6i} ← 0
e'_{5+6i} ← 0
e'_{6+6i} ← 1
end for
end procedure

Theorem 3.2. The extended duplicate graph of splitting path graph Spl(P_m), m ≥ 2 admits total magic cordial labeling.

Proof: Let *Spl(P_m)*, m ≥ 2 be a splitting path graph . In order to label the vertices and edges, define a function *f* : V U E→ {0,1} as given in algorithm 3.2.

The vertices v_1, v_2, v'_1, v'_2 and e_{3m-2} receive label '1' , '1' , '0' ,'0'and '1'respectively **;**

for 0 ≤ i ≤ [(m-2)/2] , the vertices v_{3+4i} receive label '1' ; the vertices v_{4+4i} receive label '0' ; the vertices v'_{3+4i} receive label '1' and the vertices v'_{4+4i} receive label '0'**;**

for 0 ≤ i ≤ [(m-3)/2] , the vertices v_{5+4i} receive label '1' ; the vertices v_{6+4i} receive label '1' ; the vertices v'_{5+4i} receive label '0' ; the vertices v'_{6+4i} receive label '0' ;

for 0 ≤ i ≤ [(m-2)/2] , the edges e_{1+6i} receive label '0' ; the edges e_{2+6i} receive label '1' ; the edges e_{3+6i} receive label '0' ; the edges e'_{1+6i} receive label '1' ; the edges e'_{2+6i} receive label '0' ; the edges e'_{3+6i} receive label '1' ;

for 0 ≤ i ≤ [(m-3/2] , the edges e_{4+6i} receive label '1' ; the edges e_{5+6i} receive label '1' ; the edges e_{6+6i} receive label '0' ; the edges e'_{4+6i} receive label '0' ; the edges e'_{5+6i} receive label '0' ; the edges e'_{6+6i} receive label '1' ;

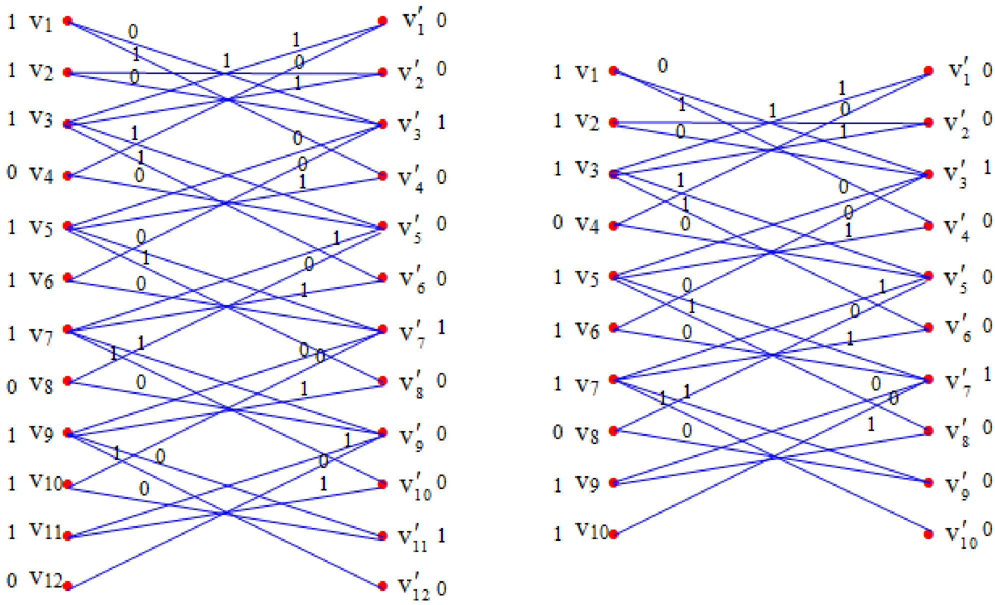
Thus all the 6m-5 edges are labeled.

The induced function f * : V U E→ {0,1} is defined as f * (v_i v_j) = {*f* (v_i) + *f* (v_j) + *f* (v_i v_j)} (mod 2) ; $v_i v_j$ ∈ E

Thus the induced function yields the total magic cordial constant '0'.

Hence the extended duplicate graph of splitting path graph S*plP*$_m$), m ≥ 2 admits total magic cordial labeling.

illustration 4. Total magic cordial labeling for edg of spilitting graph of path



(a) ***EDG(Spl(P$_6$))*** **(b)** ***EDG(Spl(P$_5$))***

Figure 4:

4. Conclusion

In this paper, we presented algorithms and prove that the extended duplicate graph of splitting graph path Spl(P$_m$), m ≥ 2 admits 3-equitable and total magic cordial labeling.

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