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Some Cycle Related 4-cordial Graphs

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Abstract. We discuss here, *4*-cordial labeling of some cycle related graphs. We prove that middle graph of the cycle and splitting graph of the cycle are *4*-cordial. In addition to this we prove that double wheel and flower graph are *4*-cordial.

Keywords: Abelian group; *4*-cordial labeling; middle graph; splitting graph; double wheel; flower graph.

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1. Introduction

Throughout this work, by a graph we mean finite, connected, undirected & simple graph $G=(V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling* (*an edge labeling*).

A latest survey on various graph labeling problems can be found in Gallian [1].

Definition 1.2. Let $<A, *>$ be any Abelian group. A graph $G=(V(G), E(G))$ is said to be A-cordial labeling if there is a mapping $f: V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e=uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(a)-v_f(b)| \leq 1$; if for all $a, b \in A$

(ii) $|e_f(a)-e_f(b)| \leq 1$; if for all $a, b \in A$

where, $v_f(a)$=the number of vertices with label a;
$v_f(b)$=the number of vertices with label b;
$e_f(a)$=the number of edges with label a;
$e_f(b)$=the number of edges with label b.

We note that if $A=<Z_k, +_k>$, that is additive group of modulo k then the labeling is known as k-cordial labeling.

Here, we consider $A=< Z_4 ,+_4>$, that is additive group of modulo *4* then the labeling is known as *4*-cordial labeling.

The concept of *A*-cordial labeling was introduced by Hovey [3] and proved the following results.

- All the connected graphs are *3*-cordial.
- All the trees are *3*-cordial.
- All the trees are *4*-cordial.
- Cycles are *k*-cordial for all odd *k*.

Kanani and Rathod [4] proved the following results.

- All the Wheels W_n are *4*-cordial.
- All the fans f_n are *4*-cordial.
- All the friendship graphs F_n are *4*-cordial.
- All the Helms H_n are *4*-cordial.

Rathod and Kanani [5] proved the following results.

- The middle graph $M(P_n)$ of path P_n is *4*-cordial.
- The total graph $T(P_n)$ of path P_n is *4*-cordial.
- The splitting graph $S'(P_n)$ of path is *4*-cordial.
- The square graph P^2_n of path is *4*-cordial.
- The triangular snake TS_n is *4*-cordial.

Rathod and Kanani [6]derived the following *4*-cordial graphs.

- The splitting graph $S'(K_{1,n})$ of star $K_{1,n}$ is *4*-cordial.
- The triangular book graph B_n is *4*-cordial.
- The one point union f_3^n of *n* copies of a fan f_3 is *4*-cordial.

Rathod and Kanani [7] proved the following results.

- The graph $Z\text{-}P_n$ is *4*-cordial for all *n*.
- The braid graph *B(n)* is *4*-cordial for all *n*.
- The triangular ladder TL_n is *4*-cordial for all *n*.
- The irregular quadrilateral snake $IQ(S_n)$ is *4*-cordial for all *n*.

2. Some useful definitions of standard graphs

1. The ***Middle Graph M (G)*** of G is the graph whose vertex set is *V (G) U E(G)* and in which two vertices are adjacent if and only if either they are adjacent edges of *G* or one is a vertex of *G* and the other is an edge incident with it.
2. The ***Splitting Graph S' (G)*** of graph G is obtained by adding to each vertex *v* a new vertex *v'* such that *v'* is adjacent to every vertex which is adjacent to *v* in *G* in other words *N (v) = N (v').*
3. The ***Double Wheel Graph*** DW_n of size *n* can be composed of $2C_n + K_1$ it consists of two cycles of size *n* where vertices of two cycles are all connected to a central vertex.
4. The ***Flower Graph*** Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm.

For any undefined term in graph theory we rely upon Gross and Yellen [3].

3. Main results

Theorem 2.1. The Middle Graph $M(C_n)$ of cycle C_n is 4-cordial.

Proof: Let $G = M(C_n)$ be the middle graph of cycle C_n and $v_1,v_2,\ldots,v_n$ be the vertices of C_n. Let $v'_1,v'_2,\ldots,v'_n$ be the vertices added corresponding to the edges $e_1,e_2,\ldots,e_n$ respectively to obtain $M(C_n)$. Here, $|V(G)| = 2n$ and $|E(G)| = 3n$.

To define 4-cordial labeling $f: V(G) \to Z_4$ we consider the following cases.

<u>**Case 1**</u>: $n \equiv 0 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_n$ by
$3,1,2,0,1,3,0,2;\ 3,1,2,0,1,3,0,2;\ldots,1,3,0,2$ and the vertices $v'_1,v'_2,\ldots,v'_n$ by
$1,0,3,2;1,0,3,2;\ldots,1,0,3,2$ successively.

<u>**Case 2**</u>: $n \equiv 1 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_n$ by
$3,1,2,0,1,3,0,2;\ 3,1,2,0,1,3,0,2;\ldots,1,3,0,2,3$ and the vertices $v'_1,v'_2,\ldots,v'_{n-2}$ by
$1,0,3,2;1,0,3,2;\ldots,1,0,3$ successively and the vertices v'_{n-1},v'_n by $1,2$.

<u>**Case 3**</u>: $n \equiv 2 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_{n-1}$ successively by
$3,1,2,0,1,3,0,2;\ 3,1,2,0,1,3,0,2;\ldots,1,3,0,2,3$ and the vertex v_n by 2.
Label the vertices $v'_1,v'_2,\ldots,v'_n$ by $1,0,3,2;1,0,3,2;\ldots,1,0,3,2,1,0$ successively.

<u>**Case 4**</u>: $n \equiv 3 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_n$ by
$3,1,2,0,1,3,0,2;\ 3,1,2,0,1,3,0,2;\ldots,1,3,0,2,3,1,2$. Label the vertices $v'_1,v'_2,\ldots,v'_{n-1}$ successively by $1,0,3,2;1,0,3,2;\ldots,1,0,3,2,1,0$ and the vertex v'_n by 0.

<u>**Case 5**</u>: $n \equiv 4 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_{n-2}$ successively by
$3,1,2,0,1,3,0,2;\ 3,1,2,0,1,3,0,2;\ldots,1,3,0,2,3,1$ and the vertices v_{n-1},v_n by $3,0$.
Label the vertices $v'_1,v'_2,\ldots,v'_{n-2}$ successively by $1,0,3,2;1,0,3,2;\ldots,1,0,3,2,1,0$ and the vertices v'_{n-1},v'_n by $2,2$.

<u>**Case 6**</u>: $n \equiv 5 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_n$ successively by
$3,1,2,0,1,3,0,2;\ 3,1,2,0,1,3,0,2;\ldots,1,3,0,2,3,1,2,0,1$. Label the vertices $v'_1,v'_2,\ldots,v'_{n-1}$ successively by $1,0,3,2;1,0,3,2;\ldots,1,0,3,2,1,0,3,2$ and the vertex v'_n by 0.

<u>**Case 7**</u>: $n \equiv 6 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_{n-2}$ successively by
$3,1,2,0,1,3,0,2;\ 3,1,2,0,1,3,0,2;\ldots,1,3,0,2,3,1,2,0$ and the vertices v_{n-1},v_n by $3,1$.
Label the vertices $v'_1,v'_2,\ldots,v'_{n-2}$ successively by $1,0,3,2;1,0,3,2;\ldots,1,0,3,2,1,0,3,2$ and the vertices v'_{n-1},v'_n by $0,2$.

<u>**Case 8**</u>: $n \equiv 7 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_{n-1}$ successively by

3,1,2,0,1,3,0,2; 3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2,0,1,3 and the vertex v_n by *1*.
Label the vertices $v'_1, v'_2, \ldots, v'_{n-3}$ successively by *1,0,3,2;1,0,3,2;…,1,0,3,2,1,0 ,3,2* and the vertices v'_{n-2}, v'_{n-1}, v'_n by *2,0,3*.

Table 1 show that above defined labeling pattern satisfies the vertex conditions and edge conditions for *4*-cordial labeling. Hence, the Middle Graph $M(C_n)$ of cycle C_n is *4*-cordial.

Let $n = 8a + b,\ a,b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$
2,6	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) + 1 = e_f(1) = e_f(2) + 1 = e_f(3)$
3	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) + 1 = e_f(2) = e_f(3) + 1$
5	$v_f(0) = v_f(1) = v_f(2) + 1 = v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2) = e_f(3)$
7	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) + 1 = e_f(2) + 1 = e_f(3) + 1$

Table 1:

Illustration 2.2. The Middle graph $M(C_5)$ and its *4*-cordial labeling is shown in *Figure 1*.

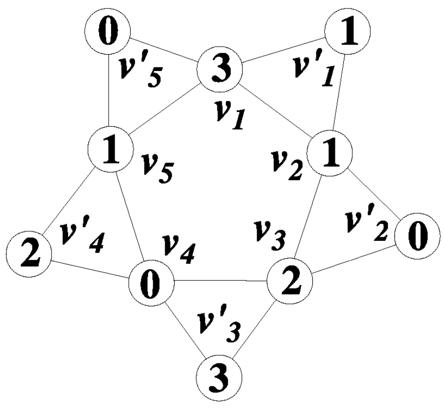

Figure 1: 4-cordial labeling of middle graph $M(C_5)$ of cycle C_5 graph.

Theorem 2.3. The splitting graph $S'(C_n)$ of cycle C_n is *4*-cordial.

Proof: Let $G = S'(C_n)$ be the splitting graph of cycle C_n and $v_1, v_2, \ldots, v_n$ be the vertices of C_n. Let G be the graph obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G. Here, $|V(G)| = 2n$ and $|E(G)| = 3n$.
To define *4*-cordial labeling $f: V(G) \rightarrow Z_4$ we consider the following cases.

<u>Case 1</u>: $n \equiv 0 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_n$ by

3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2 and the vertices $v'_1, v'_2, \ldots, v'_n$ by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2* successively.

Case 2: $n \equiv 1 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-1}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2* and the vertex v_n by 2. Label the vertices $v'_1, v'_2, \ldots, v'_{n-2}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0* and the vertices v'_{n-1}, v'_n by *1,3*.

Case 3: $n \equiv 2 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-2}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2* and the vertices v_{n-1}, v_n by *0,2*.

Label the vertices $v'_1, v'_2, \ldots, v'_{n-3}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0* and the vertices v'_{n-2}, v'_{n-1}, v'_n by *1,2,3*.

Case 4: $n \equiv 3 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_n$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2*. Label the vertices $v'_1, v'_2, \ldots, v'_{n-2}$ successively by*3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3* and the vertices v'_{n-1} , v'_n by *0,1*.

Case 5: $n \equiv 4 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-1}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2* and the vertex v_n by *3*.

Label the vertices $v'_1, v'_2, \ldots, v'_{n-4}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2* and the vertices $v'_{n-3}, v'_{n-2}, v'_{n-1}, v'_n$ by *0,1,0,2*.

Case 6: $n \equiv 5 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-1}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2,0* and the vertex v_n by 2. Label the vertices $v'_1, v'_2, \ldots, v'_{n-2}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2* and the vertices v'_{n-1} ,v'_n by *3,0*.

Case 7: $n \equiv 6 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-1}$ successively by *3,1,2,0,1,3,0,2; 3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2,0,1* and the vertex v_n by *2*.

Label the vertices $v'_1, v'_2, \ldots, v'_{n-2}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2,0* and the vertices v'_{n-1}, v'_n by *0,3*.

Case 8: $n \equiv 7 \pmod 8$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-1}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2,0,1,3* and the vertex v_n by *2*.

Label the vertices $v'_1, v'_2, \ldots, v'_{n-3}$ successively by *3,1,2,0,1,3,0,2;3,1,2,0,1,3,0,2;…,1,3,0,2,3,1,2,0* and the vertices v'_{n-2}, v'_{n-1}, v'_n by *0,1,3*.

Table 2 show that above defined labeling pattern satisfies the vertex conditions and edge conditions for *4*-cordial labeling. Hence, the Splitting Graph $S'(C_n)$ of cycle C_n is *4*-cordial.

Let $n = 8a + b$, $a,b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1	$v_f(0) + 1 = v_f(1) = v_f(2) + 1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2)+1 = e_f(3)$
2	$v_f(0) = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1)+1 = e_f(2) = e_f(3)+1$
3	$v_f(0) = v_f(1) +1= v_f(2) +1= v_f(3) + 1$	$e_f(0) + 1 = e_f(1) = e_f(2)+1 = e_f(3)$
5	$v_f(0) +1= v_f(1) +1= v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2)+1 = e_f(3)$
6	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)+1$	$e_f(0) = e_f(1) + 1 = e_f(2) = e_f(3) + 1$
7	$v_f(0) + 1 = v_f(1) = v_f(2)+1 = v_f(3)$	$e_f(0)+1 = e_f(1) + 1 = e_f(2)= e_f(3) + 1$

Table 2:

Illustration 2.4. The Splitting graph $S'(C_8)$ and its *4*-cordial labeling is shown in Figure 2.

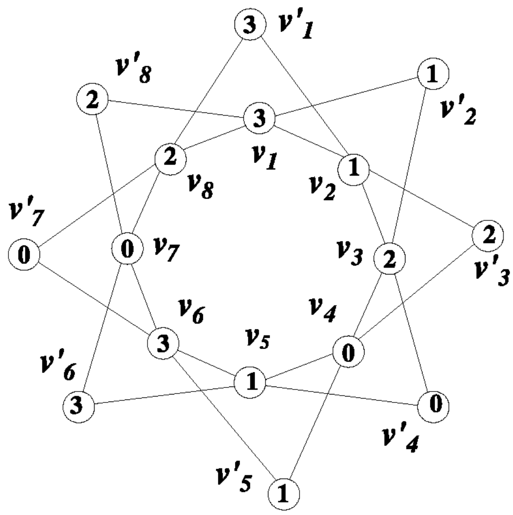

Figure 2: 4-cordial labeling of splitting graph $S'(C_8)$ of cycle C_8 graph.

Theorem 2.5. The double wheel graph DW_n is *4*-cordial.

Proof: Let G = DW_n be the double wheel graph. Let $v_1,v_2,\ldots,v_n$ be the *n* vertices of first wheel and $v'_1,v'_2,\ldots,v'_n$ be the vertices of other wheel. Let *v* be the apex vertex, which is common for both the wheels. Here, $|V(G)| = 2n+1$ and $|E(G)| = 4n$.

To define *4*-cordial labeling $f: V(G) \rightarrow Z_4$ we consider the following cases.

Label the apex vertex *v* by *0*.

<u>**Case 1**</u>: $n \equiv 0 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_n$ by

1,3,2,0, 3,1,2,0;1,3,2,0,3,1,2,0;…,3,1,2,0 and the vertices $v'_1,v'_2,\ldots,v'_n$ by

1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;…,3,1,0,2 successively.

Case 2: $n \equiv 1 \ (mod8)$.

In this case label the vertices $v_1, v_2, \ldots, v_n$ successively by $1,3,2,0, 3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0,1$. Label the vertices $v'_1, v'_2, \ldots, v'_{n-3}$ successively by $1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;\ldots,3,1$ and the vertices v'_{n-2}, v'_{n-1}, v'_n by $2,2,3$.

Case 3: $n \equiv 2 \ (mod8)$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-5}$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3$ and the vertices $v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}, v_n$ by $2,2,1,2,0$. Label the vertices $v'_1, v'_2, \ldots, v'_n$ successively by $1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;\ldots,3,1,0,2,1,3$.

Case 4: $n \equiv 3 \ (mod8)$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-1}$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0,1,3$ and the vertex v_n by 3. Label the vertices $v'_1, v'_2, \ldots, v'_{n-2}$ successively by $1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;\ldots,3,1,0,2,1$ and the vertices v'_{n-1}, v'_n by $2,0$.

Case 5: $n \equiv 4 \ (mod8)$.

In this case label the vertices $v_1, v_2, \ldots, v_n$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0,1,3,2,0$ and the vertices $v'_1, v'_2, \ldots, v'_n$ successively by $1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;\ldots,3,1,0,2,1,3,0,2$.

Case 6: $n \equiv 5 \ (mod8)$.

In this case label the vertices $v_1, v_2, \ldots, v_n$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0,1,3,2,0,3$. Label the vertices $v'_1, v'_2, \ldots, v'_{n-3}$ successively by $1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;\ldots,3,1,0,2,1,3$ and the vertices v'_{n-2}, v'_{n-1}, v'_n by $2,2,1$.

Case 7: $n \equiv 6 \ (mod8)$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-2}$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0,1,3,2,0$ and the vertices v_{n-1}, v_n by $1,2$. Label the vertices $v'_1, v'_2, \ldots, v'_{n-2}$ successively by $1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;\ldots,3,1,0,2,1,3,0,2$ and the vertices v'_{n-1}, v'_n by $2,3$.

Case 8: $n \equiv 7 \ (mod8)$.

In this case label the vertices $v_1, v_2, \ldots, v_{n-3}$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0,1,3,2,0$ and the vertices $v_{n-2}, v_{n-1} . v_n$ by $2,1,3$. Label the vertices $v'_1, v'_2, \ldots, v'_{n-2}$ successively by $1,3,0,2,3,1,0,2;1,3,0,2,3,1,0,2;\ldots,3,1,0,2,1,3,0,2,3$ and the vertices v'_{n-1}, v'_n by $0,1$.

Table 3 show that above defined labeling pattern satisfies the vertex conditions and edge conditions for *4*-cordial labeling. Hence, the Double wheel Graph DW_n is *4*-cordial.

Let $n = 8a + b$, $a,b \in N \cup \{0\}$.

b	Vertex conditions	Edge conditions
0,4	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
1,5	$v_f(0) + 1 = v_f(1) = v_f(2) = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
2,6	$v_f(0)+1 = v_f(1)+1 = v_f(2) = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
3,7	$v_f(0) = v_f(1) = v_f(2)+1= v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$

Table 3:

Illustration 2.6. The double wheel graph DW_6 and its 4-cordial labeling is shown in Figure 3.

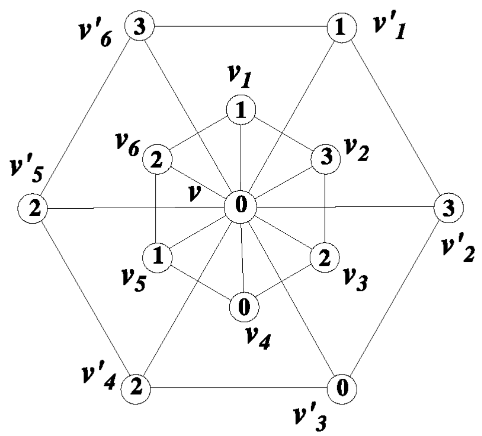

Figure 3: 4-cordial labeling of double wheel graph DW_6.

Theorem 2.7. The flower graph FL_n is 4-cordial.

Proof: Let G = FL_n be the Flower graph obtained from the helm H_n. Let v be the apex vertex and $v_1,v_2,\ldots,v_n$ be the n vertices of cycle and $v'_1,v'_2,\ldots,v'_n$ be the pendant vertices. The flower graph is obtained from helm H_n by joining each pendant vertex to the apex vertex of helm. Here, $|V(G)| = 2n+1$ and $|E(G)| = 4n$.

To define 4-cordial labeling $f: V(G) \rightarrow Z_4$ we consider the following cases.

Label the apex vertex v by 0.

<u>**Case 1**</u>: $n \equiv 0 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_n$ by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0$ and the vertices $v'_1,v'_2,\ldots,v'_n$ by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,2,3$ successively.

<u>**Case 2**</u>: $n \equiv 1 \pmod 8$.

In this case label the vertices $v_1,v_2,\ldots,v_n$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;\ldots,3,1,2,0,1$. Label the vertices $v'_1,v'_2,\ldots,v'_{n-1}$ successively by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,2,3$ and the vertex v'_n by 3.

Case 3: $n\equiv 2\ (mod8)$.

In this case label the vertices $v_1,v_{2,},\ldots,v_{n-2}$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;,\ldots,3,1,2,0$ and the vertices v_{n-1} ,v_n by $3,1$.
Label the vertices $v'_1,v'_{2,},\ldots,v'_{n-2}$ successively by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,2,3$ and the vertices v'_{n-1} ,v'_n by $2,0$.

Case 4: $n\equiv 3\ (mod8)$.

In this case label the vertices $v_1,v_{2,},\ldots,v_n$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;,\ldots,3,1,2,0,1,3,2$. Label the vertices $v'_1,v'_{2,},\ldots,v'_{n-1}$ successively by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,2,3,1,0$ and the vertex v'_n by 2.

Case 5: $n\equiv 4\ (mod8)$.

In this case label the vertices $v_1,v_{2,},\ldots,v_{n-1}$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;,\ldots,3,1,2,0,1,3,2$ and the vertex v_n by 2. Label the vertices $v'_1,v'_{2,},\ldots,v'_n$ successively by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,2,3,1,0,3,2$.

Case 6: $n\equiv 5\ (mod8)$.

In this case label the vertices $v_1,v_{2,},\ldots,v_n$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;,\ldots,3,1,2,0,1,3,2,0,3$. Label the vertices $v'_1,v'_{2,},\ldots,v'_n$ successively by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,2,3,1,0,3,2,1$.

Case 7: $n\equiv 6\ (mod8)$.

In this case label the vertices $v_1,v_{2,},\ldots,v_n$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;,\ldots,3,1,2,0,1,3,2,0,3,1$.
Label the vertices $v'_1,v'_{2,},\ldots,v'_{n-2}$ successively by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,3,2,1,0,2,3$ and the vertices v'_{n-1},v'_n by $2,0$.

Case 8: $n\equiv 7\ (mod8)$.

In this case label the vertices $v_1,v_{2,},\ldots,v_{n-3}$ successively by $1,3,2,0,3,1,2,0;1,3,2,0,3,1,2,0;,\ldots,3,1,2,0,1,3,2,0$ and the vertices v_{n-2} , v_{n-1} .v_n by $2,1,3$.
Label the vertices $v'_1,v'_{2,},\ldots,v'_{n-1}$ successively by $1,0,3,2,1,0,2,3;1,0,3,2,1,0,2,3;\ldots,1,0,3,2,1,0,2,3,1,0$ and the vertex v'_n by 2.

Table *4* show that above defined labeling pattern satisfies the vertex conditions and edge conditions for *4*-cordial labeling. Hence, the Flower Graph FL_n is *4*-cordial.

Let $n = 8a + b,\ a,b \in N\ U\ \{0\}$.

b	Vertex conditions	Edge conditions
$0,2,6$	$v_f(0) = v_f(1)+1 = v_f(2)+1 = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
$1,5$	$v_f(0) = v_f(1) = v_f(2)+1 = v_f(3)$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
4	$v_f(0)+1 = v_f(1)+1 = v_f(2) = v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$
$3,7$	$v_f(0) = v_f(1) = v_f(2)= v_f(3)+1$	$e_f(0) = e_f(1) = e_f(2) = e_f(3)$

Table 4 :

Illustration 2.8. The Flower graph FL_5 and its *4*-cordial labeling is shown in Figure 4.

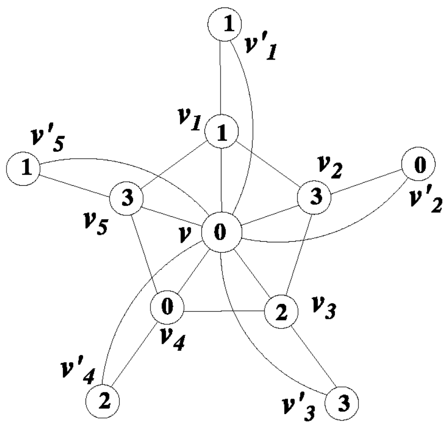

Figure 4: 4-cordial labeling of flower graph FL_5.

3. Concluding remarks

To investigate similar results for other graph families and for other *k*-cordial labeling is an open area of research.

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