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Different types of Domination in Intuitionistic Fuzzy Graph

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Abstract. In this paper, we introduced a definition of edge domination set using strong edges and edge independent sets of intuitionistic fuzzy graphs. We determine the domination number $\gamma(G)$ and edge domination number $\gamma^{'}(G)$ for several classes of intuitionistic fuzzy graphs and the relation between them are discussed. Also we introduce a regular dominating set and regular independent set in intuitionistic fuzzy graphs with suitable illustrations.

Keywords: Edge dominating set in IFG, edge domination number, edge independent set, regular domination set

AMS Mathematics Subject Classification (2010): 05C72, 03E72, 03F55

1. Introduction

The study of dominating sets in graph was introduced by Ore and Berge in 1962. The edge domination was introduced by Arumugam and Velammal [1]. The edge domination problem has many applications in resource allocation, network routing and encoding theory problems [1, 3]. Somasundaram and Somasundaram [17] introduced the concept of domination in fuzzy graphs and obtain several bounds for the domination number. Domination in fuzzy graphs using strong edges was discussed by Nagoorgani and Chandrasekaran [8]. Parvathi and Thamizhendhi [12] introduced dominating set, domination number, independent set, total dominating and total domination number in intuitionistic fuzzy graphs. Research work of several investigators [4, 5, 6, 9, 13, 15, 16] have motivated us to develop the different types of domination in intuitionistic fuzzy graphs.

This paper is organized as follows. Section 2 contains preliminaries and in section 3, an edge domination number and edge independent number of an IFG is defined using strong edges. Section 4 deals with properties of edge domination in IFGs and the relation between domination number $\gamma(G)$ and edge domination number $\gamma^{'}(G)$. Finally, we introduced regular dominating set and regular independent set in IFG in section 5.

2. Preliminaries

Definition 2.1. [10] *An intuitionistic fuzzy graph (IFG) is of the form* $G = \langle V, E \rangle$ *said to be a minmax IFG if*

(i) $V = \{v_1, v_2, ..., v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\nu_1 : V \to [0,1]$, denote the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$, for every $v_i \in V, (i = 1,2,..., n)$,

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\nu_2 : V \times V \to [0,1]$, are such that

$$\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)],\ \nu_2(v_i, v_j) \le \max[\nu_1(v_i), \nu_1(v_j)],$$

denotes the degree of membership and non-membership of the edge $(v_i, v_j) \in E$ respectively, where $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E.$

For each intuitionistic fuzzy graph G, the degree of hesitance(hesitation degree) of the vertex $v_i \in V$ in G is $\pi_1(v_i) = 1 - \mu_1(v_i) - \nu_1(v_i)$ and the degree of hesitance (hesitation degree) of an edge $e_{ij} = (v_i, v_j) \in E$ in G is $\pi_2(e_{ij}) = 1 - \mu_2(e_{ij}) - \nu_2(e_{ij})$.

Definition 2.2. [11] *An IFG,* $G = (V, E)$ *is said to be complete IFG I*
$\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$ for every $v_i, v_j \in V$.

Definition 2.3. [11] *An IFG,* $G = (V, E)$ *is said to be strong IFG if*
$\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ and $\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$ for every $(v_i, v_j) \in E.$

Definition 2.4. [11] *The complement of an IFG,* $G = (V, E)$ *is an IFG,* $\overline{G} = (\overline{V}, \overline{E})$, *where*

1. $\overline{V} = V$,
2. $\overline{\mu_1(v_i)} = \mu_1(v_i)$ and $\overline{\nu_1(v_i)} = \nu_1(v_i)$, for all $i = 1,2,\ldots, n,$
3. $\overline{\mu_2(v_i, v_j)} = \min(\mu_1(v_i), \mu_1(v_j)) - \mu_2(v_i, v_j)$ and

$\overline{\nu_2(v_i, v_j)} = \max(\nu_1(v_i), \nu_1(v_j)) - \nu_2(v_i, v_j)$ for all $i, j = 1,2,\ldots, n.$

Definition 2.5. [7] *The neighbourhood degree of a vertex is defined as*
$d_N(v) = (d_{N\mu}(v), d_{N\nu}(v))$ where

$$d_{N\mu}(v) = \sum_{w \in N(v)} \mu_1(w) \ \text{and}\ d_{N\nu}(v) = \sum_{w \in N(v)} \nu_1(w).$$

Definition 2.6. [8] *Let* $G = (V, E)$ *be an IFG. Then the degree of a vertex* v_i *is defined by* $d_G(v_i) = (d_\mu(v_i), d_\nu(v_i)) = (K_1, K_2)$
where $K_1 = d_\mu(v_i) = \sum_{v_i \neq v_j} \mu_2(v_i, v_j)$ and $K_2 = d_\nu(v_i) = \sum_{v_i \neq v_j} \nu_2(v_i, v_j)$

Definition 2.7. [8] *An intuitionistic fuzzy graph* $G = (V, E)$ *is said to be a* (K_1, K_2) *-regular if* $d_G(v_i) = (K_1, K_2)$ *for all* $v_i \in V$ *and also* G *is said to be a regular intuitionistic fuzzy graph of degree* (K_1, K_2).

Definition 2.8. [12] *An intuitionistic fuzzy graph* $G = (V, E)$ *is said to be a bipartite if the vertex set* V *can be partitioned into two non empty sets* V_1 *and* V_2 *such that*

1. $\mu_2(v_i, v_j) = 0$ and $\nu_2(v_i, v_j) = 0$ if $(v_i, v_j) \in V_1$ or $(v_i, v_j) \in V_2$.
2. $\mu_2(v_i, v_j) > 0$, $\nu_2(v_i, v_j) < 0$ if $v_i \in V_1$ or $v_j \in V_2$, for some i and j, (or)
   $\mu_2(v_i, v_j) = 0$, $\nu_2(v_i, v_j) < 0$ if $v_i \in V_1$ or $v_j \in V_2$, for some i and j, (or)
   $\mu_2(v_i, v_j) > 0$, $\nu_2(v_i, v_j) = 0$ if $v_i \in V_1$ or $v_j \in V_2$, for some i and j.

Definition 2.9. [12] *A bipartite intuitionistic fuzzy graph* $G = (V, E)$ *is said to be complete if* $\mu_2(v_i, v_j) = \min(\mu_1(v_i), \mu_1(v_j))$ *and* $\nu_2(v_i, v_j) = \max(\nu_1(v_i), \nu_1(v_j))$ *for all* $v_i \in V_1$ *and* $v_j \in V_2$ *and is denoted by* K_{V_1, V_2}.

Definition 2.10. [12] *If* $v_i, v_j \in V \subseteq G$, *the* μ*-strength of connectedness between* v_i *and* v_j *is* $\mu_2^\infty(v_i, v_j) = sup\{\mu_2^k(v_i, v_j) \mid k = 1, 2, \ldots n\}$ *and* ν*-strength of connectedness between* v_i *and* v_j *is* $\nu_2^\infty(v_i, v_j) = inf\{\nu_2^k(v_i, v_j) \mid k = 1, 2, \ldots n\}$.

If u, v are connected by means of paths of length k then $\mu_2^k(u, v)$ is defined as $sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \mu_2(v_2, v_3) \ldots \wedge \mu_2(v_{k-1}, v) \mid (u, v_1, v_2 \ldots v_{k-1}, v \in V)\}$ and $\nu_2^k(u, v)$ is defined as

$$inf\{\nu_2(u, v_1) \vee \nu_2(v_1, v_2) \vee \nu_2(v_2, v_3) \ldots \vee \nu_2(v_{k-1}, v) \mid (u, v_1, v_2 \ldots v_{k-1}, v \in V)\}.$$

Definition 2.11. [12] *An edge* (u, v) *is said to be a strong edge if* $\mu_2(u, v) \geq \mu_2^\infty(u, v)$ *and* $\nu_2(u, v) \geq \nu_2^\infty(u, v)$.

Definition 2.12. [12] *Let* $G = (V, E)$ *be an IFG on* V. *Let* $u, v \in V$, *we say that* u *dominates* v *in* G *if there exists a strong edge between them.*

Definition 2.13. [12] *A subset* S *of* V *is called dominating set in* G *if for every* $v \in V - S$, *there exists* $u \in S$ *such that* u *dominates* v.

Definition 2.14. [12] *A dominating set* S *of an IFG is said to be minimal dominating set if no proper subset of* S *is a dominating set.*

Definition 2.15. [12] *Minimum cardinality among all minimal dominating set is called vertex domination number of* G *and is denoted by* $\gamma(G)$.

Definition 2.16. [12] *Let* $G=(V,E)$ *be an IFG, then the vertex cardinality of* V *is defined by* $|V|=\sum_{v_i\in V}\left(\frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}\right)$ *for all* $v_i\in V$.

Definition 2.17. [12] *Let* $G=(V,E)$ *be an IFG, then the edge cardinality of* E *is defined by* $|E|=\sum_{v_i,v_j\in E}\left(\frac{1+\mu_2(v_i,v_j)-\gamma_2(v_i,v_j)}{2}\right)$ *for all* $(v_i,v_j)\in E$.

3. Edge domination in intuitionistic fuzzy graphs

The concept of edge dominaton in graphs was introduced by Arumugam and Velammal in 1988 and further edge domination and independence in fuzzy graphs is studied by Nagoorgani and Prasanna devi.

We refer to [12] for the terminology of dominaton in intuitionistic fuzzy graphs.

Definition 3.1. *Let* $G=(V,E)$ *be an IFG. Let* e_i *and* e_j *be two adjacent edges of* G. *We say that* e_i *dominates* e_j *if* e_i *is a strong edge in* G.

Definition 3.2. *A subset* S *of* E *is called a edge dominating set in* G *if for every* $e_j\in E-S$, *there exists* $e_i\in S$ *such that* e_i *dominates* e_j.

Definition 3.3. *An edge dominating set* S *of an IFG is said to be minimal edge dominating set if no proper subset of* S *is an edge dominating set.*

Definition 3.4. *Minimum cardinality among all minimal edge dominating set is called edge domination number of* G *and is denoted by* $\gamma'(G)$.

Definition 3.5. *The strong neighbourhood of an edge* e_i *in an IFG* G *is*
$N_s(e_i)=\{e_j\in E(G)\,|\,e_j$ *is strong edge and adjacent to* e_i *in* G $\}$

Example 3.1. *Consider an IFG,* $G=(V,E)$, *such that* $V=\{v_1,v_2,v_3,v_4,v_5,v_6\}$ *and* $E=\{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_3,v_5),(v_4,v_5),(v_4,v_6),(v_6,v_1)\}$.

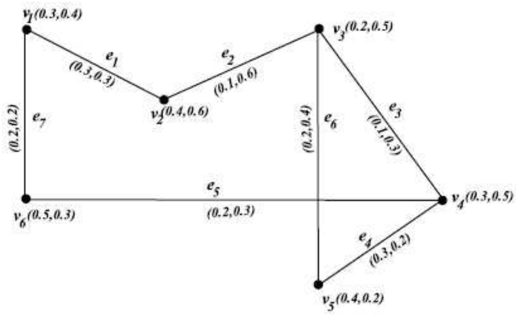

Figure 1:

Here $\{e_1,e_4\},\{e_1,e_2,e_5\}$, $\{e_2,e_5\}$, $\{e_2,e_4,e_7\}$, $\{e_6,e_7\}$ are minimal edge dominating sets of G and $\gamma^{'}(G)=0.7$.

Definition 3.6. *Let $E^{'}$ be a subset of edge set E. Then the node cover of $E^{'}$ is defined as the set of all vertices incident to each edge in $E^{'}$.*

Definition 3.7. *Two edges e_i and e_j are said to be edge independent in an IFG $G=(V,E)$ if $e_i \notin N_s(e_j)$ and $e_j \notin N_s(e_i)$.*

Definition 3.8. *A subset S of E is said to be an intuitionistic fuzzy edge independent set of an IFG G if any two edges in S are edge independent.*

Definition 3.9. *An edge independent set S of G in an IFG is said to be maximal independent if for every edge $e \in E-S$, the set $S \cup \{e\}$ is not independent.*

Definition 3.10. *The minimum cardinality among all maximal edge independent set is called edge independence number of G and it is denoted by $\beta^{'}(G)$.*

Definition 3.11. *An edge in IFG G is called an isolated edge if it is not adjacent to any strong edge in G.*

Example 3.2. *Consider an IFG, $G=(V,E)$, such that $V=\{v_1,v_2,v_3,v_4,v_5\}$ and* $E=\{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_2,v_4),(v_4,v_5),(v_4,v_1),(v_5,v_1)\}$.

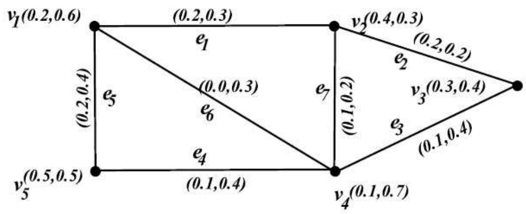

Figure 2:

Here $\{e_1,e_3\}$, $\{e_1,e_4\}$, $\{e_2,e_4\}$, $\{e_2,e_5\}$, $\{e_3,e_5\}$, $\{e_5,e_7\}$, $\{e_6,e_2\}$ are maximal edge independent sets of G and $\beta^{'}(G)=0.75$.

Definition 3.12. *If all the edges are strong edge in an IFG then it is called strengthened IFG.*

Theorem 3.1. *Node cover of an edge dominating set of an IFG G is a dominating set of G.*

Proof: Let $V^{'}$ be the node cover of an edge dominating set D of G. We have to prove that $V^{'}$ is a dominating set of G.

Suppose $V^{'}$ is not a dominating set. Then there exists atleast one vertex u which is not dominating by the elements of $V^{'}$ i.e. $V^{'}$ does not contain any vertex from $N_s(u)$.

Then D does not contain any strong edge which dominates (u, v_i) for all $v_i \in N_s(u)$.

Hence D is not an edge dominating set of G. This is a contradiction. Therefore node cover of an edge dominating set of an IFG is a dominating set of G.

Theorem 3.2. *Let G be an IFG without isolated edges and there exists no edge $e_i \in E$ such that $N_s(e_i) \subseteq D$. If D is a minimal edge dominating set, then $S - D$ is an edge dominating set, where S is the set of all strong edge in G.*

Proof: Let D be a minimal edge dominating set of an IFG G. Suppose $S - D$ is not an edge dominating set. Then there exists atleast one edge $e_i \in D$ which is not dominated by $S - D$. Since G has no isolated edges and there is no edge $e_i \in E$ such that $N_s(e_i) \subseteq D$, e_i is adjacent to atleast one strong edge e_j in S. Since $S - D$ is not an edge dominating set of G, $e_j \notin S - D$. Hence $e_j \in D$.

Therefore $D - \{e_i\}$ is an edge dominating set which is contradiction to the fact that D is minimal edge dominating set.

Thus, every edge in $E - S$ is dominated by an edge in $S - D$.

Therefore $S - D$ is an edge dominating set.

Theorem 3.3. *An edge independent set of an IFG having only strong edges is a maximal edge independent set if and only if it is edge independent and edge dominating set.*

Proof: Let S be an edge independent set of an IFG having only strong edges.

Suppose S is a maximal edge independent set of G. Then for every $e_i \in E - S$, the set $S \cup \{e_i\}$ is not an edge independent set. ie., for every $e_i \in E - S$, there is an edge e_j such that $e_j \in N_s(e_i)$. Thus, S is an edge dominating set of G and also it is a edge independent set of G.

Conversely, Suppose S is both edge independent and edge dominating set of G. We have to prove that S is maximal edge independent set having only strong edges.

Since S is an edge dominating set of G, it has only strong edges. Assume that S is not a maximal edge independent set. Then there exists an edge $e_i \notin S$ such that $S \cup \{e_i\}$ is a edge independent set, there is no edge in S belonging to $N_s(e_i)$ and therefore e_i is not dominated by S.

Hence S cannot be an edge dominating set of G, which is a contradiction. Hence S is a maximal edge independent set of G having only strong edges.

Theorem 3.4. *Every maximal edge independent set in an IFG G having only strong edges is a minimal edge dominating set of G.*

Proof: Let S be a maximal edge independent set having only strong edges of an IFG G.

By Theorem 3.3, S is an edge dominating set of G.

Suppose S is not a minimal edge dominating set of G. Then there exists an edge $e_i \in S$ such that $S-\{e_i\}$ is a edge dominating set. Then atleast one edge e_j in $S-\{e_i\}$ is in $N_s(e_i)$. This contradicts the fact that S is an edge independent set of G. Hence S is a minimal edge dominating set of G.

Theorem 3.5. *Node cover of a maximal edge independent set of an IFG having only strong edges is a dominating set of* G.

Proof: Let S be a maximal edge independent set of an IFG G having only strong edges. Let $V^{'}$ be the node cover of S. By Theorem 3.4, Every maximal edge independent set having only strong edges is a minimal dominating set of G. Then $V^{'}$ is a node cover of a edge dominating set of an IFG G. By Theorem 3.1, node cover of an edge dominating set of an IFG G is dominating set of G. Hence $V^{'}$ is a dominating set of G.

Theorem 3.6. *Every complete IFG is strengthened IFG.*

Proof: Let $G=(V,E)$ be a complete IFG.

By definition of complete IFG

$\mu_2(u,v)=min(\mu_1(u),\mu_1(v))$ and $\nu_2(u,v)=max(\nu_1(u),\nu_1(v))$, for all $u,v\in V$

Suppose G has atleast one non strong (u,v) edge then

$$\mu_2(uv)<\mu_2^{\infty}(u,v) \text{ and } \nu_2(uv)<\nu_2^{\infty}(u,v)$$

which implies that

$\mu_2(u,v)<min(\mu_1(u),\mu_1(v))$ and $\nu_2(u,v)<max(\nu_1(u),\nu_1(v))$, for some $u,v\in V$.

This contradicts our assumption that G is complete IFG. Thus, every edge in complete IFG is a strong edge.

Hence every complete IFG is strengthened IFG.

Corollary 3.7. *Every strong IFG is a strengthened IFG*

Corollary 3.8. *Every self complementary IFG is a strengthened IFG*

Corollary 3.9. *Every complete bipartite IFG* $K_{n,n}$ *is a strengthened IFG*

Theorem 3.10. *Let* $G=(V,E)$ *is an IFG and if* G *is both regular and edge regular IFG then* (μ_2,ν_2) *is a constant function*

Proof: Assume that G is both regular and edge regular IFG

By definition of regular IFG,

$$d_G(v_i)=(d_\mu(v_i),d_\nu(v_i))=(K_1,K_2) \qquad \forall\, v_i\in V$$

By edge regular IFG,

$$d_G(v_iv_j)=(d_\mu(v_iv_j),d_\nu(v_iv_j))=(l_1,l_2) \qquad \forall\,(v_i,v_j)\in E$$

where $d_\mu(v_iv_j)=d_\mu(v_i)+d_\mu(v_j)-2\mu_2(v_iv_j) \qquad \forall(v_i,v_j)\in E$

$$l_1=K_1+K_1-2\mu_2(v_iv_j)$$

$$\mu_2(v_iv_j) = K_1 - \frac{l_1}{2} \qquad \forall (v_i, v_j) \in E$$

and $$d_\nu(v_iv_j) = d_\nu(v_i) + d_\nu(v_j) - 2\nu_2(v_iv_j) \qquad \forall (v_i, v_j) \in E$$

$$l_2 = K_2 + K_2 - 2\nu_2(v_iv_j)$$

$$\nu_2(v_iv_j) = K_2 - \frac{l_1}{2} \qquad \forall (v_i, v_j) \in E$$

Hence (μ_2, ν_2) is a constant function.

Note 1. *Let* $G = (V, E)$ *be a complete IFG with* (μ_2, ν_2) *as a constant function and if* G *is* (K_1, K_2) *regular IFG then* $\mu_2(v_iv_j) = \frac{K_1}{n-1}$ *and* $\nu_2(v_iv_j) = \frac{K_2}{n-1}$ $\forall (v_i, v_j) \in E$.

Theorem 3.11. *Let* $G = (V, E)$ *be a self complementary IFG. Then* G *is a* (K_1, K_2) *edge regular IFG if and only if* $\overline{G}$ *is also* (K_1, K_2) *edge regular IFG*

Proof: Since $G = (V, E)$ is a self complementary IFG.

$$\mu_2(v_i, v_j) = (min(\mu_1(v_i), \mu_1(v_j)))/2 \quad \text{and} \quad \nu_2(v_i, v_j) = (max(\nu_1(v_i), \nu_1(v_j)))/2$$
$$\forall v_i, v_j \in V$$

By the definition of complement,

$$\overline{\mu_2(v_i, v_j)} = min(\mu_1(v_i), \mu_1(v_j)) - \mu_2(v_i, v_j)$$

and $\overline{\nu_2(v_i, v_j)} = max(\nu_1(v_i), \nu_1(v_j)) - \nu_2(v_i, v_j) \qquad \forall v_i, v_j \in V$

Therefore $\overline{\mu_2(v_i, v_j)} = (min(\mu_1(v_i), \mu_1(v_j)))/2$

and $\overline{\nu_2(v_i, v_j)} = (max(\nu_1(v_i), \nu_1(v_j)))/2 \qquad \forall v_i, v_j \in V$

Hence $\overline{\mu_2(v_i, v_j)} = \mu_2(v_i, v_j)$ and $\overline{\nu_2(v_i, v_j)} = \nu_2(v_i, v_j)$

Now $\overline{d_G(v_iv_j)} = (\overline{d_\mu(v_iv_j)}, \overline{d_\nu(v_iv_j)})$

$$\overline{d_\mu(v_iv_j)} = \sum_{\substack{v_iv_k \in E \\ k \neq j}} \overline{\mu_2(v_iv_k)} + \sum_{\substack{v_kv_j \in E \\ k \neq i}} \overline{\mu_2(v_kv_j)}$$

$$= \sum_{\substack{v_iv_k \in E \\ k \neq j}} \mu_2(v_iv_k) + \sum_{\substack{v_kv_j \in E \\ k \neq i}} \mu_2(v_kv_j)$$

$$\overline{d_\mu(v_iv_j)} = d_\mu(v_iv_j)$$

Similarly $\overline{d_\nu(v_iv_j)} = d_\nu(v_iv_j)$

Therefore $\overline{d_G(v_iv_j)} = d_G(v_iv_j) \qquad \forall (v_i, v_j) \in E$

Hence G is (K_1, K_2) edge regular IFG if and only if $\overline{G}$ is also (K_1, K_2) edge regular IFG.

4. Relation between domination number $\gamma(G)$ and edge domination number $\gamma'(G)$ of an IFG

Notation 1. *Let $P(K_n)$ be the number of edges in minimum edge dominating set of K_n.*

Lemma 4.1. $P(K_n) = \frac{n}{2}$, *when n is even.*

Proof: It can be easily verified that $P(K_2) = 1$ and $P(K_4) = 2$.

When $n = 6$, consider K_6. Let $E(K_6) = \{e_1, e_2, ...e_{15}\}$ and if $e_1 \in P(K_6)$ then $N(v_1)$ and $N(v_2)$ constitute 9 edges and since $E(K_6) = 15$ the remaining 6 edges are in induced graph of $G[v_3, v_4, v_5, v_6]$ which is clearly K_4. But $P(K_4) = 2$.

$P(K_6) = 1 + P(K_4) \quad = 1 + 1 + P(K_2) \quad = 1 + 1 + 1$ (3 times)

When $n = 8$, consider K_8. Let $E(K_8) = \{e_1, e_2, ...e_{28}\}$ and if $e_1 \in P(K_8)$ then $N(v_1)$ and $N(v_2)$ constitute 13 edges and since $E(K_8) = 28$ the remaining 15 edges are in induced graph of $G[v_3, v_4, v_5, v_6, v_7, v_8]$ which is clearly K_6. But $P(K_6) = 3$.

$P(K_8) = 1 + P(K_6) \quad = 1 + 1 + P(K_4) \quad = 1 + 1 + 1 + 1$ (4 times)

Similarly, consider K_{2n}. Let $E(K_{2n}) = \{e_1, e_2, ...e_{2n^2-n}\}$ and if $e_i \in P(K_{2n})$ then $N(v_i)$ and $N(v_j)$ constitute $4n-3$ edges and since $E(K_{2n}) = n(2n-1)$ the remaining $2n^2 - 5n + 3$ edges are in induced graph of $G[v_1, v_2, ..., v_{2n-2}]$ which is clearly K_{2n-2}.

$P(K_{2n}) = 1 + P(K_{2n-2}) \quad = 1 + [1 + P(K_{2n-4})]$

$= 1 + [1 + 1 + P(K_{2n-6})] \quad = 1 + [1 + 1 + ... + 1 ((n-1)\, times)]$

$P(K_{2n}) = n$

Hence $P(K_n) = \frac{n}{2}$, when n is even.

Lemma 4.2. $P(K_n) = \frac{n-1}{2}$, *when n is odd.*

Proof: It can be easily verified that $P(K_1) = 0$ and $P(K_3) = 1$.

When $n = 5$, consider K_5. Let $E(K_5) = \{e_1, e_2, ...e_{10}\}$ and if $e_1 \in P(K_5)$ then $N(v_1)$ and $N(v_2)$ constitute 7 edges and since $E(K_5) = 10$ the remaining 3 edges are in induced graph of $G[v_3, v_4, v_5]$ which is clearly K_3. But $P(K_3) = 1$.

$P(K_5) = 1 + P(K_3) \quad = 1 + 1 + P(K_1) \quad = 1 + 1 (2\, times)$

When $n = 7$, consider K_7. Let $E(K_7) = \{e_1, e_2, ...e_{21}\}$ and if $e_1 \in P(K_7)$ then $N(v_1)$ and $N(v_2)$ constitute 11 edges and since $E(K_7) = 21$ the remaining 10 edges are in induced graph of $G[v_3, v_4, v_5, v_6, v_7]$ which is clearly K_5. But $P(K_5) = 2$.

$P(K_7) = 1 + P(K_5) \qquad = 1 + 1 + P(K_3) \qquad = 1 + 1 + 1$ (3 times)

Similarly, consider K_{2n-1}. Let $E(K_{2n-1}) = \{e_1, e_2, ... e_{2n^2-3n+1}\}$ and if $e_i \in P(K_{2n-1})$ then $N(v_i)$ and $N(v_j)$ constitute $4n-5$ edges and since $E(K_{2n-1}) = 2n^2 - 3n + 1$ the remaining $2n^2 - 7n + 6$ edges are in induced graph of $G[v_1, v_2, ..., v_{2n-3}]$ which is clearly K_{2n-3}.

$P(K_{2n-1}) = 1 + P(K_{2n-3})$

$= 1 + [1 + 1 + P(K_{2n-7})] \qquad = 1 + [1 + 1 + ... + 1((n-2)\, times)]$

$P(K_{2n-1}) = n - 1$

Hence $P(K_n) = \dfrac{n-1}{2}$, when n is odd.

Lemma 4.3. *If $G = (V, E)$ is a complete IFG then $P(K_{m+n}) = P(K_m) + P(K_n)$.*

Proof:

Case (i) When m and n is even

$$P(K_{m+n}) = \frac{m+n}{2} = \frac{m}{2} + \frac{n}{2} = P(K_m) + P(K_n).$$

Case (ii) When m is odd and n is even

$$P(K_{m+n}) = \frac{m+n-1}{2} = \frac{m-1}{2} + \frac{n}{2} = P(K_m) + P(K_n).$$

Remark 4.1. *The above result is not true where m and n is both odd.*

Lemma 4.4. *For any complete IFG K_n, $P(K_n) = \dfrac{2n-1+(-1)^n}{4}$.*

Proof: To prove $P(K_n) = \dfrac{2n-1+(-1)^n}{4}$, we use the method of induction.

Case (i) when n is odd

For $n = 1$, $P(K_1) = 0 \quad \because K_1$ is a single vertex.

Assume that the result is true for all $n = 2m+1$, we have to prove for $n = 2m+3$.

$$P(K_{2m+1}) = \frac{4m+2-1+(-1)^{2m+1}}{4} = m$$

$$P(K_{2m+3}) = P(K_{2m+1}) + P(K_2) \quad (\text{By Lemma 4.3})$$

$$P(K_{2(m+1)+1}) = m+1.$$

Hence $P(K_{2m+3})$ is true.

Case (ii) when n is even

For $n=2$, $P(K_2)=1$ $\because$ K_2 is an edge.
Assume that the result is true for $n=2m$, we have to prove for $n=2m+2$.

$$P(K_{2m}) = \frac{4m-1+(-1)^{2m}}{4} = m$$

$$P(K_{2m+2}) = P(K_{2m}) + P(K_2) \quad (\text{By Lemma 4.3})$$

$$P(K_{2(m+1)}) = \frac{4m-1+(-1)^{2m}}{4} + \frac{4-1+(-1)^{2}}{4} = m+1.$$

Hence $P(K_{2m+2})$ is true.
Hence the proof.

Lemma 4.5. *If* $P(K_n) = \dfrac{2n-1+(-1)^n}{4}$ *denote the number of edges in minimal edge dominating set of complete graph* K_n *then*

1. $P(K_{n+1}) = P(K_n)$, when n is even
2. $P(K_{n+1}) = P(K_n)+1$, when n is odd

Proof:
(i) When n is even

$$P(K_{n+1}) = \frac{2(n+1)-1+(-1)^{n+1}}{4} = \frac{2(n)-1+(-1)^n}{4}$$

$$= P(K_n)$$

Hence $P(K_{n+1}) = P(K_n)$, when n is even.
(ii) When n is odd

$$P(K_n)+1 = \frac{2n-1+(-1)^n}{4}+1 = \frac{2n-1+(-1)^n+4}{4}$$

$$= \frac{2n-1+(-1)^n+2+2}{4} = \frac{2(n+1)-1-1+2}{4}$$

$$= \frac{2(n+1)-1+1}{4} = \frac{2(n+1)-1+(-1)^{n+1}}{4}$$

$$= P(K_{n+1})$$

Hence $P(K_{n+1}) = P(K_n)+1$, when n is odd.

Theorem 4.6. *For a complete IFG* G *with* $n \geq 2$ *vertices*

$$\gamma'(G) = \begin{cases} \dfrac{n}{2}\gamma(G), & n \text{ is even} \\ \dfrac{n-1}{2}\gamma(G), & n \text{ is odd} \end{cases}$$

where $\gamma(G)$ and $\gamma'(G)$ is the minimum cardinality of the vertex and edge dominating set.

Proof: Let $G=(V,E)$ be a complete IFG with $n \geq 2$. By Theorem 3.6, G is strengthened IFG. If we choose an edge (u,v) then it will dominate all the edges incident to u and v. The set of edges (u_i, v_j) where no two of them has a vertex in common forms a minimal edge dominating set. Minimum cardinality of edge dominating set is $\gamma^{'}(G)$. Since G is complete IFG, minimum cardinality of vertex dominating set will be only one vertex and it is $\gamma(G)$.

If n is even, then $P(K_n)$ contains $\frac{n}{2}$ edges. (By Lemma 4.1) $\gamma^{'}(G)$ is the sum of minimum cardinality of $\frac{n}{2}$ edges and $\gamma(G)$ is a minimum cardinality of a vertex. Since G is complete IFG, the edges associated with $\gamma(G)$ will have minimum cardinality.

Hence the sum of minimum cardinality of edge dominating set $\gamma^{'}(G) \leq \frac{n}{2}\gamma(G)$.

Similarly, if n is odd, then $P(K_n)$ contains $\frac{n-1}{2}$ edges (By Lemma 4.2) and $\gamma^{'}(G) \leq \frac{n-1}{2}\gamma(G)$.

Hence the result.

Theorem 4.7. *If $G=(V,E)$ is a complete IFG then $\gamma(G) < \gamma^{'}(G)$ if $n > 3$*

Proof: Assume that $G=(V,E)$ is a complete IFG. Then every edge in G is strong edge. $\gamma(G)$ is the minimum cardinality of a vertex.

By Lemma 4.4, $P(K_n)$ contains $(\frac{2n-1+(-1)^n}{4})$ edges.

$\gamma^{'}(G)$ is the sum of minimum cardinality of $(\frac{2n-1+(-1)^n}{4})$ edges.

If $n > 3$ then $\gamma^{'}(G)$ contains more than one edge.

Hence $\gamma(G) < \gamma^{'}(G)$.

Note 2. *Let $G=(V,E)$ is a complete IFG with (μ_1, ν_1) as constant function and $n = 3$ then $\gamma(G) = \gamma^{'}(G)$.*

Note 3. *If G is edge regular IFG with (μ_1, ν_1) as constant function the $\overline{G}$ is edge regular IFG.*

Remark 4.2. *If $\gamma(G)$ and $\gamma^{'}(G)$ is domination number and edge domination number of an IFG with $n > 3$ then*

1. $\frac{2}{n}\gamma^{'}(G) \leq \gamma(G) < \gamma^{'}(G)$, n is even
2. $\frac{2}{n-1}\gamma^{'}(G) \leq \gamma(G) < \gamma^{'}(G)$, n is odd.

Theorem 4.8. *Let $G = (V, E)$ be a self complementary IFG then $\gamma^{'}(G) = \gamma^{'}(\overline{G})$.*

Proof: Since G is self complementary IFG.
Every edge in self complementary IFG is a strong edge and $\overline{\mu_2(v_i, v_j)} = \mu_2(v_i, v_j)$ and $\overline{\nu_2(v_i, v_j)} = \nu_2(v_i, v_j)$.
Also G is isomorphic to $\overline{G}$.
Therefore minimum cardinality of edge dominating set of G and $\overline{G}$ remains same.
Hence $\gamma^{'}(G) = \gamma^{'}(\overline{G})$.

Theorem 4.9. *Let $G = (V, E)$ be a complete bipartite IFG $K_{n,n}$ and if G is both regular and edge regular IFG then $\gamma^{'}(G) = (\frac{2n-2}{n})\gamma(G)$*

Proof: Given $G = (V, E)$ be a complete bipartite IFG $K_{n,n}$ then every edge in G is a strong edge. Assume that G is both regular and edge regular IFG. By Theorem 3.10, (μ_2, ν_2) is a constant function then the vertex domination number $\gamma(G) = n\mu_2(u, v)$ and the edge donmination number $\gamma^{'}(G) = (2n-2)\mu_2(u, v)$.

Hence $\gamma^{'}(G) = (\frac{2n-2}{n})\gamma(G)$.

5 Regular domination in intuitionistic fuzzy graphs

Definition 5.1. *Let $G = (V, E)$ be an IFG. A set S subset of V is called regular intuitionistic fuzzy dominating set if*

1. every vertex in $V - S$ is adjacent to some vertex in S
2. all the vertices in S has the same degree

Example 5.1. *Consider an IFG, $G = (V, E)$, such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_6), (v_1, v_4), (v_4, v_5), (v_5, v_6)\}$.*

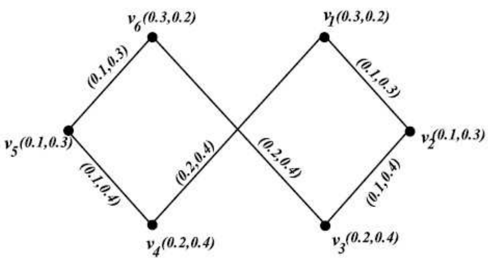

Figure 3:

Definition 5.2. *The minimum cardinality of regular intuitionistic fuzzy dominating set is called regular intuitionistic fuzzy dominating number and denoted by* $\gamma_{rif}(G) = 0.8$

Definition 5.3. *A set* S *subset of* V *is called minimal regular intuitionistic fuzzy dominating set if*

1. Any subset of S is not a intuitionistic fuzzy dominating set
2. all the vertices of S have the same degree

Definition 5.4. *An independent set* S *of an IFG* $G = (V, E)$ *is said to be regular independent IFG if all the vertices of* S *has the same degree*

Definition 5.5. *A set* S *is maximal regular independent set if for every vertex* $v \in V - S$ *the set* $S \cup \{v\}$ *is not regular independent set*

Example 5.2. *Consider an IFG,* $G = (V, E)$*, such that* $V = \{v_1, v_2, v_3, v_4\}$ *and*
$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$.

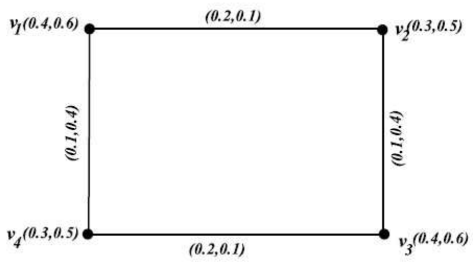

Figure 4:

Here $\{v_1, v_3\}$, $\{v_2, v_4\}$ is a regular independent set.

Theorem 5.1. *A regular independent set is a regular maximal independent set of an IFG if and only if it is regular independent and regular dominating set.*

Proof: Let S be a regular maximal independent set in an IFG, then for every $u \in V - S$, the set $S \cup \{u\}$ is not an independent set. ie., for every $u \in V - S$, there is a vertex $v \in S$ such that u is adjacent to v. Thus, S is dominating set of G and also regular independent set of G. Hence S is regular independent and regular dominating set.

Conversely, Suppose S is both regular independent and regular dominating set of G. We have to prove that S is regular mximal independent set. Assume that S is not a maximal independent set. Then there exists a vertex $u \notin S$ such that $S \cup \{u\}$ is an independent set, there is no vertex in S adjacent to u and therefore u is not dominated by S.

Hence S cannot be a dominating set of G, which is contradiction. Hence S is maximal independent. Thus S is regular maximal independent set.

Theorem 5.2. *Every regular maximal independent set in an IFG* G *is a regular minimal dominating set of* G.

Proof: Let S be a regular maximal independent set in an IFG. By Theorem 5.1, S is regular dominating set of G. Suppose S is not a minimal dominating set of G. Then there exists atleast one vertex $v \in S$ such that $S-\{v\}$ is dominating set. Then atleast one vertex in $S-\{v\}$ is adjacent to v. This contradicts the fact that S is regular independent set of G. Hence S is regular minimal dominating set of G.

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