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Certain Properties on Fuzzy R_0 Topological Spaces in Quasi-coincidence Sense

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Abstract. In this paper, we introduce two notions of R_0 property in fuzzy topological spaces by using quasi-coincidence sense and we show that all these notions satisfy good extension property. Also hereditary, productive and projective properties are satisfied by these notions. We observe that all these concepts are preserved under one-one, onto, fuzzy open and fuzzy continuous mappings.

Keywords: Fuzzy topological space, quasi-coincidence, fuzzyR_0 topological space

AMS Mathematics Subject Classification (2010): 54A40

1. Introduction

Chang [5] defined fuzzy topological spaces in 1968 by using fuzzy sets introduced by Zadeh [26] in 1965. Since then extensive work on fuzzy topological spaces has been carried out by many researchers like Gouguen [7], Wong [23, 24], Warren [22], Hutton [10], Lowen [13,14] and others. Separation axioms are important parts in fuzzy topological spaces. Many works [3, 4, 6, 8, 19] on separation axioms have been done by researchers. Among those axioms, fuzzy R_0 topological space is one and it has been already introduced in the literature. There are many articles on fuzzyR_0 topological space which are created by many authors like Wuyts and Lowen [25], Srivastava et al. [20], Ali [1], Hossain and Ali [9] and others.

The purpose of this paper is to further contribute to the development of fuzzy topological spaces especially on fuzzy R_0 topological spaces. In the present paper, we have introduced two notions of fuzzyR_0topological space using quasi-coincidence sense and it is shown that the good extension property is satisfied by our notions. In the next section of this paper, it is also shown that the hereditary, productive, and projective properties hold on our concepts. Finally, we have observed that these notions are preserved under one-one, onto, fuzzy open and fuzzy continuous mappings.

2. **Preliminaries**

In this section, we recall some concepts occurring in the cited papers whichwill be needed in the sequel. In the present paper X and Y always denotenon empty sets and I=[0, 1], I_1=[0, 1).

Definition 2.1. [26] A function *u* from *X* into the unit interval *I* is called a fuzzy set in *X*. For every $x \in X$, $u(x) \in I$is called the grade of membershipof *x* in *u*. Some authors say that *u* is a fuzzy subset of *X* instead of sayingthat *u* is a fuzzy set in *X*. The class of all fuzzy sets from *X*into the closedunit interval *I*will be denoted by I^X.

Definition 2.2. [16] A fuzzy set $u$in X is called a fuzzy singleton if andonly if $u(x) = r, 0 < r \leq 1$for a certain $x \in X$ and $u(y) = 0$ for all points y of X except x. The fuzzy singleton is denoted by $x_r$and $x$is its support.The class of all fuzzy singletons in *X* will be denoted by $S(X)$. If $u \in I^X$ and $x_r \in S(X)$, then we say that $x_r \in u$if and only if $r \leq u(x)$.

Definition 2.3. [11] A fuzzy singleton x_r is said to be quasi-coincidence with *u*, denoted by $x_r q u$ if and only if $u(x) + r > 1$. If x_r is not quasi-coincidence with *u*, we write $x_r \bar{q} u$ and defined as $u(x) + r \leq 1$.

Definition 2.4. [5] Let *f* be a mapping from a set *X* into a set *Y* and *u* be a fuzzy subset of *X*. Then *f* and *u* induce a fuzzy subset *v* of *Y* defined by

$$v(y) = sup\{u(x)\} \text{ if } x \in f^{-1}[\{y\}] \neq \varphi, x \in X$$
$$= 0 \text{ otherwise.}$$

Definition 2.5. [5] Let *f* be a mapping from a set *X* into a set *Y* and *v* be a fuzzy subset of *Y* . Then the inverse of *v* written as $f^{-1}(v)$ is a fuzzy subsetof *X* defined by
$f^{-1}(v)(x) = v(f(x))$, for $x \in X$.

Definition 2.6. [5] Let *I* = [0, 1], *X* be a non empty set and I^X be the collection of all mappings from *X* into *I*, *i.e.* the class of all fuzzy sets in *X*. A fuzzy topology on *X* is defined as a family *t* of members of I^X, satisfying the following conditions. (*i*) $1, 0 \in t$, (*ii*) If $u \in t$for each $i \in \Lambda$, then $\cap_{i \in \Lambda} u_i \in t$, where Λ is an index set. (*iii*) If $u, v \in t$then$u \cap v \in t$.

The pair (*X*, *t*) is called a fuzzy topological space (in short fts) and members of *t* are called *t- open* fuzzy sets. A fuzzy set *v* is called a *t-closed* fuzzy set if $1 - v \in t$.

Definition 2.7. [17] The function $f : (X, t) \to (Y, s)$ is called fuzzy continuous if and only if for every $v \in s, f^{-1}(v) \in t$, the function *f* is called fuzzy homeomorphic if and only if *f* is bijective and both *f* and f^{-1} are fuzzycontinuous.

Definition 2.8. [15] The function $f : (X, t) \to (Y, s)$ is called fuzzy open if and only if for every open fuzzy set *u* in $(X, t), f(u)$ is open fuzzy set in (*Y*, *s*).

Definition 2.9. [12] Let $\{X_i, i \in \Lambda\}$, be any class of sets and let *X* denotes the Cartesian product of these sets, $i.e. X = \Pi_{i \in \Lambda} X_i$. Note that *X* consists of all points $p = < a_i, i \in \Lambda >$, where $a_i \in X_i$. Recall that, for each $j_0 \in \Lambda$, we define the projection π_{j_0} from the product set *X* to the coordinate spaceX_{j_0}, i.e. $\pi_{j_0} : X \to X_{j_0}$ by $\pi_{j_0}(< a_i, i \in \Lambda >) = a_{j_0}$.These projections are used to define the product topology.

Definition 2.10. [23] Let $\{X_i, i \in \Lambda\}$ be a family of nonempty sets. Let $X = \Pi_{i\in\Lambda} \in X_i$ be the usual product of X_i's and let π_i be the projection from X into X_i . Further assume that each X_i is an fuzzy topological space with fuzzy topology t_i. Now, the fuzzy topology generated by $\{\pi_i^{-1}(b_i): b_i \in t_i,\ i \in \Lambda\}$ as a sub basis, is called the product fuzzy topology on X. Clearly if w is a basis element in the product, then there exist $i_1, i_2, i_3, \dots, i_n \in \Lambda$, with $x = (x_i)_{i\in\Lambda} \in X$ such that $w(x) = min\{b_i(x_i) : i = 1,2,3, \dots, n\}$.

Definition 2.11. [18] Let f be a real valued function on a topological space. If $\{x : f(x) > \alpha\}$ is open for every real α, then f is called lower semicontinuous function.

Definition 2.12. [13] Let X be a nonempty set and T be a topology on X. Let $t = \omega(T)$ be the set of all lower semi continuous functions from (X, T) to I (with usual topology). Thus $\omega(T) = \{u \in I^X : u - 1(\alpha, 1] \in T\}$ for each $\alpha \in I_1$. It can be shown that $\omega(T)$ is a fuzzy topology on X.

Let P be the property of a topological space (X, T) and FP be its topological analogue. Then FP is called a 'good extension' of P if and only if the statement (X, T) has P if and only if $(X, \omega(T))$ has FP holds good for every topological space (X, T).

Theorem 2.13. [2] A bijective mapping from an fts (X, t) to an fts (Y, s) preserves the value of a fuzzy singleton (fuzzy point).

Note: Preimage of any fuzzy singleton (fuzzy point) under bijective mapping preserves its value.

3. Main results

In this section, we discuss about our notions and findings. Some well-known properties are discussed here by using our concepts.

Definition 3.1. A fuzzy topological space $(X,\ t)$ is called
(a) $\boldsymbol{F\,R_0(i)}$ if and only if for any pair $x_r,\ y_s \in S(X)$ with $x \neq y$, whenever there exists $u \in t$ with $x_r q u$ and $y_s \bar{q} u$, then there exists $v \in t$ such that $y_s q v$ and $x_r \bar{q} v$.
(b) $\boldsymbol{FR_0(ii)}$ if and only if for any pair $x_r, y_s \in S(X)$ with $x \neq y$, whenever there exists $u \in t$ with $x_r q u$ and $y_s \cap u = 0$, then there exists $v \in t$ such that $y_s q v$ and $x_r \cap v = 0$.

Now, we shall show that our notions satisfy the good extension property.

Theorem 3.2. Let $(X,\ T)$ be a topological space. Consider the following statements:
(1) $(X,\ T)$ be a R_0 topological space.
(2) $(X,\ \omega(T))$ be an $FR_0(i)$ space.
(3) $(X,\ \omega(T))$ be an $FR_0(ii)$ space.
Then the implications are true: $(1) \Leftrightarrow (2)$, $(1) \Leftrightarrow (3)$.

Proof of (1) ⇔ (2): Let (X, T) be a topological space and (X, T) is R_0. We have to prove that $\big(X,\ \omega(T)\big)$ is $FR_0(i)$. Let $x_r,\ y_r$ be fuzzy singletons in X with $x \neq\ y$ and $u \in\ \omega(T)$ with $x_r q u$ and $y_r \bar{q} u$. Now, $x_r q u \Rightarrow u(x) + r > 1 \Rightarrow u(x) > 1 - r \Rightarrow x \in u^{-1}(1-r, 1]$ and $y_r \bar{q} u \Rightarrow u(y) + r \leq 1 \Rightarrow u(y) \leq 1 - r \Rightarrow y \notin u^{-1}(1-r, 1]$

Since (X, T) is R_0 topological space, we have, there exists $V \in T$ such that $y \in V$, $x \notin V$. From the definition of lower semi continuous we have $1_U,\ 1_V \in\ \omega(T)$ and $1_V(y) = 1$, $1_V(x) = 0$. Then $1_V(y) + r > 1 \Rightarrow y_r q 1_V$ and $1_V(x) + r \leq 1 \Rightarrow\ x_r \bar{q} 1_V$. It follows that there exists $1_V \in\ \omega(T)$ such that $y_r q 1_V$ and $x_r \bar{q} 1_V$. Hence (X, ω(T)) is $FR_0(i)$. Thus (1) ⇒ (2) holds.

Conversely, *let* $(X,\ \omega(T))$ be a fuzzy topological space and $(X,\ \omega(T))$ is $FR_0(i)$. We have to prove that $(X,\ T)$ is R_0. Let $x,\ y$ be points in X with $x \neq\ y$ and $U \in T$ with $x \in U$ and $y \notin U$. From the definition of lower semi continuous, we have $1_U \in\ \omega(T)$ and $1_U(x) = 1$, $1_U(y) = 0$. Then $1_U(x) + r > 1 \Rightarrow x_r q 1_U$ and $1_U(y) + r \leq 1 \Rightarrow\ y_r \bar{q} 1_U$. Since $(X,\ \omega(T))$ is $FR_0(i)$ topological space we have, for any fuzzy singletons $x_r,\ y_r$ in X, there exists $v \in\ \omega(T)$ such that $y_r q v$ and $x_r \bar{q} v$.

Now, $y_r q v \Rightarrow\ v(y) + r > 1 \Rightarrow\ v(y) > 1 - r \Rightarrow\ y \in v^{-1}(1-r, 1]$ and $x_r \bar{q} v \Rightarrow v(x) + r \leq 1 \Rightarrow v(x) \leq 1 - r \Rightarrow x \notin v^{-1}(1-r, 1]$
It follows that $\exists\ v^{-1}(1-r, 1] \in\ T$ such that $y \in v^{-1}(1-r, 1]$ and $x \notin v^{-1}(1-r, 1]$. Hence, $(X,\ T)$ is R_0 topological space. Thus (2) ⇒ (1) holds. Similarly, we can prove that (1) ⇔ (3).

Now we shall show that our notions satisfy the hereditary property.

Theorem3.3. Let $(X,\ t)$ be a fuzzy topological space, $A\ \subseteq\ X, t_A = \{u/A : u\ \in\ t\}$, then
(a) $(X,\ t)$ is $FR_0(i)\ \Rightarrow\ (A, t_A)$ is $FR_0(i)$ and
(b) $(X,\ t)$ is $FR_0(ii)\ \Rightarrow\ (A, t_A)$ is $FR_0(ii)$.

Proof of (b): Let $(X,\ t)$ be a fuzzy topological space and $(X,\ t)$ is $FR_0(ii)$. We have to prove that (A, t_A) is $FR_0\ (ii)$. Let $x_r,\ y_s$ be fuzzy singletons in A with $x\ \neq\ y$ and $u\ \in t_A$ with $x_r q u$ and $y_s \cap u = 0$. Since, $A\ \subseteq\ X$ these fuzzy singletons are also fuzzy singletons in X. Since $u\ \in t_A$, we can write $u = v/A$, where $v \in\ t$ with $x_r q v$ and $y_s \cap v = 0$. Also since $(X,\ t)$ is $FR_0(ii)$ fuzzy topological space, we have, there exists $w\ \in\ t$ such that $y_s q w$ and $x_r \cap w = 0$. For $A \subseteq X$, we have $w/A\ \in t_A$. Now, $y_s q w \Rightarrow w(y) + s > 1,\ y\ \in\ X \Rightarrow\ w/A(y) + s > 1,\ y\ \in\ A\ \subseteq\ X \Rightarrow y_s q w/A$ And $x_r \cap w = 0 \Rightarrow w(x) = 0, x \in\ X \Rightarrow (w/A)(x) = 0, x \in A\ \subseteq\ X \Rightarrow x_r \cap (w/A) = 0$. It follows that there exist $w/A\ \in t_A$ such that $y_s q(w/A)$ and $x_r \cap (w/A) = 0$. Hence, (A, t_A) is $FR_0(ii)$. Similarly, we can prove (a).

As our next work we shall show that our notions satisfy the productive and projective properties.

Theorem 3.4. Let (X_i, t_i), $i\ \in\ \Lambda$ be fuzzy topological spaces and $X\ = \Pi_{i \in \Lambda}\ \in X_i$ and $t$be the product topology on X, then
(a) for all $i\ \in\ \Lambda$, (X_i, t_i) is $FR_0(i)$ if and only if $(X,\ t)$ is $FR_0(i)$ and

(b) for all $i \in \Lambda, (X_i, t_i)$ is $FR_0(ii)$ if and only if (X, t) is $FR_0(ii)$.

Proof of (a): Let for all $i \in \Lambda, (X_i, t_i)$ is $FR_0(i)$ space. We have to prove that (X, t) is $FR_0(i)$. Let x_r, y_s be fuzzy singletons in X with $x \neq y$ and $u \in t$ with $x_r q u$ and $y_s \cap u = 0$. Then $(x_i)_r, (y_i)_s$ are fuzzy singletons with $x_i \neq y_i$ for some $i \in \Lambda$ and we can find $u_i \in t_i$ such that $(x_i)_r q u_i, (y_i)_s \bar{q} v_i$. Since (X_i, t_i) is $FR_0(i)$, there exists $v_i \in t_i$ such that $(y_i)_s q v_i$ and $(x_i)_r \bar{q} v_i$. Now, $(y_i)_s q v_i$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$.

Now, $(y_i)_s q v_i \Rightarrow v_i(y_i) + s > 1,\ y \in X \Rightarrow v_i(\pi_i(y)) + s > 1$
$\Rightarrow (v_i \circ \pi_i)(y) + s > 1 \Rightarrow y_s q (v_i \circ \pi_i)$ and

$$(x_i)_r \bar{q} v_i \Rightarrow v_i(x_i) + r \leq 1,\ x \in X \Rightarrow v_i(\pi_i(x)) + r \leq 1$$
$$\Rightarrow (v_i \circ \pi_i)(x) + r \leq 1 \Rightarrow x_r \bar{q} (v_i \circ \pi_i)$$

It follows that there exists $(v_i \circ \pi_i) \in t_i$ such that $y_s q \left(v_i \circ \pi_i\right), x_r \bar{q} (v_i \circ \pi_i)$.
Hence (X, t) is $FR_0(i)$.

Conversely, Let (X, t) be a fuzzy topological space and (X, t) is $FR_0(i)$. We have to prove that $(X_i, t_i),\ i \in \Lambda\ is\ FR_0(i)$. Here let us consider, a_i be a fixed element in X_i. Let
$A_i = \{x \in X = \Pi_{i \in \Lambda} \in X_i \colon x_j = a_j\ for\ some\ i \neq j\}$.Then A_i is a subset of X, and hence (A_i, t_{A_i}) is a subspace of (X, t). Since (X, t) is $FR_0(i)$, so (A_i, t_{A_i}) is $FR_0(i)$. Now, we have A_i is homeomorphic image of X_i. Hence it is clear that for all $i \in \Lambda$, (X_i, t_i) is $FR_0(i)$ space. Thus (a) holds. Similarly, we can prove (b).

Theorem 3.5. Let (X, t) and (Y, s) be two fuzzy topological spaces and $f \colon X \to Y$ be a one-one, onto, fuzzy open and fuzzy continuous map then$(a)\left(X, t\right)$ is $FR_0(i) \Rightarrow \left(Y, s\right)$ is $FR_0(i)$
$(b)(X, t)$ is $FR_0(ii) \Rightarrow (Y, s)$ is $FR_0(ii)$

Proof of (a): Let (X, t) be a fuzzy topological space and (X, t) is $F R_0(i)$. We have to prove that (Y, s) is $FR_0(i)$. Let $x'_r,\ y'_s$ be fuzzy singletons in Y with $x' \neq y'$ and let $u \in s$ with $x'_r\, qu$ and $y'_s\, \bar{q} u$. Since f is onto then there exist $x,\ y \in X$ with $f(x) = x'$, $f(y) = y'$ and $x_r,\ y_s$are fuzzy singletons in X with $x \neq y$as f is one-one. Again since f is fuzzy continuous and $u \in s, f^{-1}(u) \in t$. Now,

$x_r q u \Rightarrow u(x) + r > 1 \Rightarrow u\left(f\left(x'\right)\right) + r > 1 \Rightarrow \left(f^{-1}(u)\right)\left(x'\right) + r > 1 \Rightarrow x'_r\, q f^{-1}(u)$
and $y_s \bar{q} u \Rightarrow u(y) + s \leq 1 \Rightarrow u\left(f\left(y'\right)\right) + s \leq 1 \Rightarrow \left(f^{-1}(u)\right)\left(y'\right) + s \leq 1 \Rightarrow$
$y'_s\, \bar{q} f^{-1}(u)$. Since (X, t) is FR_0 (i) space, there exists $v \in t$ such that $y'_s q v$ and $x'_r \bar{q} v$. Now,

$y'_s q v \Rightarrow v\left(y'\right) + s > 1 \Rightarrow sup\ v\left(y'\right) + s > 1 \Rightarrow (f(v))(y) + s > 1$, where
$$f(v)(y) = \{sup\ v(y') : f(y') = y\}$$
$\Rightarrow\ y_s q f(v)$ and
$x'_r \bar{q} v \Rightarrow v\left(x'\right) + r \leq 1 \Rightarrow (f(v))(x) + r \leq 1$, where $f(v)(x) = \{sup\ v(x') : f(x') = x\} \Rightarrow x_r \bar{q} f(v)$

Since, f is a fuzzy open mapping. Then $f(v) \in s \ as \ v \in t$.
It follows that there exists $f(v) \in s$such that $y_s \, qf\,(v)$ and $x_r\bar{q}f(v)$. Hence it is clear that $(Y,\ s)$ is $FR_0(i)$ space. Similarly, we can prove (*b*).

Theorem 3.6. Let (*X*, *t*) and (*Y*, *s*) be two fuzzy topological spaces and $f : X \to Y$ be a one-one, onto, fuzzy open and fuzzy continuous mapping then,
(*a*) $(Y,\ s)\ is\ FR_0(i) \Rightarrow (X,\ t)\ is\ FR_0(i)$ and
(*b*) $(Y,\ s)\ is\ FR_0(ii) \Rightarrow (X,\ t)\ is\ FR_0(ii)$.

Proof of (*b*): Let (*Y*, *s*) be a fuzzy topological space and (*Y*, *s*) is FR_0 (*ii*). We have to prove that $(X,\ t)$ is $FR_0(ii)$. Let $x_r,\ y_s$ be fuzzy singletons in *X* with $x \neq y$ and let $u \in t$ such that $x_r qu$ and $y_s \cap u = 0$. Then there exist fuzzy singletons $x_r^{'},\ y_s^{'}$ in *Y* with $f(x) = x^{'}, f(y) = y^{'}$ with $x^{'} \neq y^{'}$as f is one-one. Again since f is fuzzy open and $u \in t$, $f(u) \in s$. Now,
$x_r qu \Rightarrow u(x) + r > 1 \Rightarrow \big(f(u)\big)\big(x^{'}\big) + r > 1$, where $f(u)\big(x^{'}\big) = \big\{sup\, u(x) : f(x) = x^{'}\big\} = u(x)$
$\Rightarrow x_r^{'} qf(u)$ and $y_s \cap u = 0 \Rightarrow u(y) = 0 \Rightarrow \big(f(u)\big)\big(y^{'}\big) = 0$, where $f(u)\big(y^{'}\big) = \big\{sup\, u(y) : f(y) = y^{'}\big\} = u(y)$
$\Rightarrow y_s^{'} \cap f(u) = 0$
Since $(Y,\ s)$ is $FR_0(ii)$ space, there exists $v \in s$such that $y_s^{'}\, q\, v\ and\ x_r^{'} \cap v = 0$. Now, $y_s^{'}\, q\, v \Rightarrow v(y^{'}) + s > 1 \Rightarrow v(f(y)) + s > 1 \Rightarrow (f^{-1}(v))(y) + s > 1 \Rightarrow y_s\, q\, f^{-1}(v)$, since f is fuzzy continuous $f^{-1}(v) \in t$ as $v \in s$. And
$x_r^{'} \cap v = 0 \Rightarrow v\big(x^{'}\big) = 0 \Rightarrow v\big(f(x)\big) = 0 \Rightarrow \big(f^{-1}(v)\big)(x) = 0 \Rightarrow x_r \cap \big(f^{-1}(v)\big) = 0$
It follows that there exists $f^{-1}(v) \in t$such that $y_s q f^{-1}(v)$ and $x_r \cap \Big(f^{-1}(v)\Big) = 0$. Hence it is clear that $(X,\ t)$ is $FR_0(ii)$ space. Similarly, we can prove (a).

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