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Analysis for the FM/FM/1 Queue with Multiple Working Vacation with N-Policy

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Abstract. We investigate an $FM/FM/1$ queue with multiple working vacation. Using non linear programming method, we obtain some performance measure of interest such as membership of mean queue length, mean waiting time, with N=2. Finally, numerically results are presented to show the effects system parameters.

Keywords: FM/FM/1 Queue; multiple working vacation; mean queue length; mean waiting time

AMS Mathematics Subject Classification (2010): 60K25

1. Introduction

The vacation queue models have been investigated extensively in view of their application in computer systems, communication networks, production managing stops serving customers and may do some additional work or maintain servers during a vacation. Various vacation policies provide more flexibility for optima design and operating control of the system. The details can be seen in the monographs of Takagi [7], Tian and Zhang [8] the surveys of Doshi [2] and Teghem [10]. Lastly, Servi and Finn [3] introduced a class of semi-vacation polices, the server works at the lower rate rather then completely stopping service during a vacation. Such a vacation is called a working vacation (WV). Part of service ability keeps the system operating in a lower speed during a vacation. If service speed degenerates into zero in a working vacation, the working vacation queue becomes a classical vacation queue model. Therefore, the working vacation queue is the generalization of the classical vacation queue and the analysis of this kind of models is more complicated then the previous work.

Servi and Finn [3] studied on $M/M/1$ queue with multiple working vacation, and obtained the PGF of the number of customers in the system and the LST of waiting time distribution, and applied results to performance analysis of gateway router in fiber communication networks. Liu, Xu and Tian gave simple explicit expression of distributions for the stationary queue length and waiting time which have intuitive probability interpretation.

Mery and Gokilavani [2] investigate the performance measure of an $M^X/M/1$ Multiple Working Vacation (MWV) queuing model in a fuzzy environment. George and

Jayalekshmi [4] studied on the analysis of $G/M(n)/1/k$ queuing system with multiple exponential vacations and vacations of fuzzy length. Pavithra and Mary [5] gave the Analysis of $FM/M(a,b)/1/MWV$ queuing model. The analysis the general bulk service queuing model to find the mean queue length probability that the system is in vacation and the probability that the system is in busy state are expressed in terms of crisp value for $FM/M(a,b)/1$ under multiple working vacation with fuzzy numbers. Ramesh et al. [6] constructs the membership function of the system characteristics of a batch-arrival queuing system with multiple servers, in which the batch-arrival rate and customer service rate are all fuzzy numbers. we obtain some performance measure of interest such as membership function of mean queue length, mean waiting time. Finally, numerically results are presented to show the effects system parameters.

2. The model in fuzzy environment

In this section the arrival rate, service rate, busy period, vacation rate are assumed to be fuzzy number $\bar{\lambda}, \bar{\beta}, \bar{\gamma}, \bar{\theta}$ respectively, Now:

$$\bar{\lambda} = \{x, \mu_{\bar{\lambda}}(x) : x \in S(\bar{\lambda})\},\ \bar{\beta} = \{y_1, \mu_{\bar{\beta}}(y_1) : y_1 \in S(\bar{\beta})\},\ \bar{\gamma} = \{y_2, \mu_{\bar{\gamma}}(y_2) : y_2 \in S(\bar{\gamma})\},\ \bar{\theta} = \{z, \mu_{\bar{\theta}}(z) : z \in S(\bar{\theta})\}$$

where $S(\bar{\lambda}), S(\bar{\beta}), S(\bar{\gamma}),$ and $S(\bar{\theta})$ are the universal sets of the arrival rate, service rate, busy period and working vacation respectively.

Define $f(x, y_1, y_2, z)$ as the system performance measures related to the above defined fuzzy queuing model which depends on the fuzzy membership function. $\bar{f}(\bar{\lambda}, \bar{\beta}, \bar{\gamma}, \bar{\theta})$ Applying Zadeh's extension principle [11] the membership function of the performance measure $\bar{f}(\bar{\lambda}, \bar{\beta}, \bar{\gamma}, \bar{\theta})$ can be defined as:

$$\mu_{\bar{f}(\bar{\lambda},\bar{\beta},\bar{\gamma},\bar{\theta})}(D) = \sup_{\substack{x \in S(\bar{\lambda}) \\ y_1 \in S(\bar{\beta}) \\ y_2 \in S(\bar{\gamma}) \\ z \in S(\bar{\theta})}} \{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}}(y_1), \mu_{\bar{\gamma}}(y_2), \mu_{\bar{\theta}}(z) / D\} \qquad (1)$$

If the α -cut of $\bar{f}(\bar{\lambda}, \bar{\beta}, \bar{\gamma}, \bar{\theta})$ degenerate to some fixed value. Then the system performance is a crisp number, otherwise it is a fuzzy number.

Mean queue length

$$E(L) = \frac{\lambda}{(\mu_v - \lambda)}$$

$$+\left[\left[\frac{2\mu_v\lambda + 2(\mu_v)^2 - \mu_v(H)(2\mu_v - (H)(\mu_v - \lambda))}{\lambda\mu_v(\lambda^2\mu_v - \mu_v(H))}\right] + \left[\frac{2(\mu_b\mu_v - \mu_b\lambda)}{2\mu_v\lambda - (\mu_v(H))}\right] + \left[\frac{4\mu_v - (H)}{2\mu_v}\right]\right]^{-1}$$

$$\left[\left[\frac{(\mu_b - \mu_v)2\mu_v - (H)}{2\mu_v\lambda - \mu_v(H)}\right] + 2\left[\frac{2\mu_v 2\mu_b - \mu_v(H)(2\mu_v(2\mu_v\lambda - \mu_v(H)))}{2\mu_v(2\mu_v\lambda - \mu_v(H))}\right]\right]\left[\frac{H}{2\mu_v}\right]$$

$$\times\left[\frac{6\mu_v-2(H)}{2\mu_v}\right]\times\left[\frac{(\mu_b-\mu_v)(H)}{2\mu_v\lambda-\mu_v(H)}\right]\Bigg]$$

where $H=(\lambda+\theta+\mu_v-\sqrt{(\lambda+\theta+\mu_v)^2-4\lambda\mu_v})$

Mean waiting time

$$E(W)=\frac{\mu_v}{(\mu_b\mu_v-\mu_b\lambda)}+E(W_d)$$

$$E(W)=\left[\frac{\mu_v}{(\mu_b\mu_v-\mu_b\lambda)}\right]+\Bigg[\left[\frac{2\mu_v\lambda+2(\mu_v)^2-\mu_v(H)[2\mu_v-(H)](\mu_v-\lambda)}{\lambda\mu_v(\lambda 2\mu_v-\mu_v(H))}\right]$$

$$+\left[\frac{2[\mu_b\mu_v-\mu_b\lambda]}{2\mu_v\lambda-\mu_v(H)}\right]+\left[\frac{4\mu_v-(H)}{2\mu_v}\right]\Bigg]^{-1}$$

$$\left[\left[\frac{(2\mu_v)^2}{(H)^2}\right]\left[\frac{\mu_b-\mu_v(H)}{\lambda 2\mu_v-\mu_v(H)}\right]\left[\frac{4(\mu_v)^2}{\lambda(H)(2\mu_v-(H))}\right]\right]$$

We obtain the membership function some performance measures namely the average of the mean queue length in $\overline{E(L)}$, the mean waiting time $\overline{E(W)}$.

For the system in terms of this membership function are:

$$\mu_{\overline{E(L)}}(M)=\sup_{\substack{x\in S(\overline{\lambda})\\ y_1\in S(\overline{\beta})\\ y_2\in S(\overline{\gamma})\\ z\in S(\overline{\theta})}}\{\min\{\mu_{\overline{\lambda}}(x),\mu_{\overline{\beta}}(y_1),\mu_{\overline{\gamma}}(y_2),\mu_{\overline{\theta}}(z)/M\}$$

where $M=\frac{x}{(y_1-x)}+\Bigg[\left[\frac{2y_1x+2(y_1)^2-y_1.(H)(2y_1-H(y_1-H))}{xy_1(x^2y_1-y_1.(H))}\right]$

$$+\left[\frac{2(y_1y_1)-y_2x}{2y_1x-y_1.(H)}\right]+\left[\frac{4y_1-(H)}{2y_1}\right]\Bigg]$$

$$\left[\left[\frac{(y_2-y_1)2y_1-(H)}{2y_1x-y_1.(H)}\right]+2\left[\frac{2y_1 2y_2-y_1.(H)(2y_1(2y_1x-y_2.(H)))}{2y_1(2y_1x-y_1.(H))}\right]\right.$$

$$\left.\left[\frac{H}{2y_1}\right].\left[\frac{6y_1-2.(H)}{2y_1}\right].\left[\frac{(y_2-y_1).(H)}{2y_1x-y_1.(H)}\right]\right]$$

where $H=(x+z+y_1-\sqrt{(x+z+y_1)^2-4xy_1})$

$$\mu_{\overline{E(W)}}(N) = \sup_{\substack{x\in S(\bar{\lambda}) \\ y_1\in S(\bar{\beta}) \\ y_2\in S(\bar{\gamma}) \\ z\in S(\bar{\theta})}} \{\min\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}}(y_1), \mu_{\bar{\gamma}}(y_2), \mu_{\bar{\theta}}(z) / N\}$$

where $N = \left[\dfrac{y_1}{(y_2 y_2 - y_2 x)}\right] + \left[\left[\dfrac{2y_1 x + 2(y_1)^2 - y_1.(H)[2y_1 - (H)](y_1 - x)}{xy_1(xy_1 - y_1.(H))}\right]\right.$

$$\left. + \left[\frac{2(y_2 y_1 - y_2 x)}{2y_1 x - y_1.(H)}\right]\left[\frac{4y_1 - (H)}{2y_1}\right]\right]^{-1}$$

Using the fuzzy analysis technique explain we can find the membership function of $\overline{E(L)}$ and $\overline{E(W)}$ as a function of the parameter α. Thus the α-cut approach can be used to develop the membership function of $\overline{E(L)}$ and $\overline{E(W)}$ respectively.

3. Performance of measure

The following performance measure are studied for this model in fuzzy environment.

Membership function of average of the mean queue length

We can calculate the lower and upper bounds of the α - cuts of $E(L)$ as, based on Zadeh's extension principle $\mu_{E(L)}(M)$ is the superimum of minimum over $\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}}(y_1), \mu_{\bar{\gamma}}(y_2), \mu_{\bar{\theta}}(z)\}$ to satisfied $\mu_{E(L)}(M) = \alpha,$ where $0 < \alpha \le 1$

The following four cases arise

Case (i) $\mu_{\bar{\lambda}}(x) = \alpha, \mu_{\bar{\beta}}(y_1) \ge \alpha, \mu_{\bar{\gamma}}(y_2) \ge \alpha, \mu_{\bar{\theta}}(z) \ge \alpha$

Case (ii) $\mu_{\bar{\lambda}}(x) \ge \alpha, \mu_{\bar{\beta}}(y_1) = \alpha, \mu_{\bar{\gamma}}(y_2) \ge \alpha, \mu_{\bar{\theta}}(z) \ge \alpha$

Case (iii) $\mu_{\bar{\lambda}}(x) \ge \alpha, \mu_{\bar{\beta}}(y_1) \ge \alpha, \mu_{\bar{\gamma}}(y_2) = \alpha, \mu_{\bar{\theta}}(z) \ge \alpha$

Case (iv) $\mu_{\bar{\lambda}}(x) \ge \alpha, \mu_{\bar{\beta}}(y_1) \ge \alpha, \mu_{\bar{\gamma}}(y_2) \ge \alpha, \mu_{\bar{\theta}}(z) = \alpha$

For case (i) the lower and upper bound of α - cuts of $\overline{E(L)}$ can be obtained through the corresponding parametric non-linear programs as:

$$[\overline{E(L)}]_{\alpha}^{L_1} = \min_{\Omega} M \quad \text{and} \quad [\overline{E(L)}]_{\alpha}^{U_1} = \max_{\Omega} M$$

Similarly, we can calcul

ate the lower and upper bounds of the α -cuts of $\overline{E(L)}$ for the case $(ii), (iii)$ *and* (iv). By considering all the cases simulatuosly the lower and upper bounds of the α -cuts of $\overline{E(L)}$ can be written as: $[\overline{E(L)}]_{\alpha}^{L} = \min_{\Omega} M$ and $[\overline{E(L)}]_{\alpha}^{U} = \max_{\Omega} M$

such that, $x_\alpha^L \le x \le x_\alpha^U$, $y_{1_\alpha}^L \le y_1 \le y_{1_\alpha}^U$, $y_{2_\alpha}^L \le y_2 \le y_{2_\alpha}^U$, $z_\alpha^L \le z \le z_\alpha^U$

If both $(\overline{E(L)})_\alpha^L$ and $(\overline{E(L)})_\alpha^U$ are invertible with respected to α, the left and right shape function $L(M) = [(E(L))_\alpha^L]^{-1}$ and $R(M) = [(E(L))_\alpha^U]^{-1}$ can be derived from which the membership function $\mu_{\overline{E(L)}}(M)$ is constructed as:

$$\mu_{\overline{E(L)}}(M) = \begin{cases} L(M), (E(L))_{\alpha=0}^L \le M \le (E(L))_{\alpha=0}^U \\ 1, \quad (E(L))_{\alpha=1}^L \le M \le (E(L))_{\alpha=1}^U \\ R(M), (E(L))_{\alpha=0}^L \le M \le (E(L))_{\alpha=0}^U \end{cases}$$

Membership function of the mean qwaiting time

We can calculate the lower and upper bounds of the α - cuts of $E(W)$ as, $\mu_{E(W)}(N)$ is the superimum of minimum over $\{\mu_{\bar{\lambda}}(x), \mu_{\bar{\beta}}(y_1), \mu_{\bar{\gamma}}(y_2), \mu_{\bar{\theta}}(z)\}$ are given by

$$[\overline{E(W)}]_\alpha^L = \min_\Omega N \quad \text{and} \quad [\overline{E(W)}]_\alpha^U = \max_\Omega N$$

such that, $x_\alpha^L \le x \le x_\alpha^U$, $y_{1_\alpha}^L \le y_1 \le y_{1_\alpha}^U$, $y_{2_\alpha}^L \le y_2 \le y_{2_\alpha}^U$, $z_\alpha^L \le z \le z_\alpha^U$ .If both $(\overline{E(W)})_\alpha^L$ and $(\overline{E(W)})_\alpha^U$ are invertible with respected to α, then left and right shape function as, $L(N) = [(E(W))_\alpha^L]^{-1}$ and $R(N) = [(E(W))_\alpha^U]^{-1}$
can be derived from which the membership function $\mu_{\overline{E(W)}}(N)$ can be considered as,

$$\mu_{\overline{E(W)}}(N) = \begin{cases} L(N), (E(N))_{\alpha=0}^L \le N \le (E(N))_{\alpha=0}^U \\ 1, \quad (E(N))_{\alpha=1}^L \le N \le (E(N))_{\alpha=1}^U \\ R(N), (E(N))_{\alpha=0}^L \le N \le (E(N))_{\alpha=0}^U \end{cases} \quad (2)$$

4. Numerical study

Suppose the arrival rate $\bar{\lambda}$, the service rate $\bar{\beta}$, busy period $\bar{\gamma}$, the vacation rate $\bar{\theta}$ are assumed to be trapezoidal fuzzy numbers described by

$\bar{\lambda} = [11,12,13,14]$, $\bar{\beta} = [31,32,33,34]$, $\bar{\gamma} = [21,22,23,24]$, $\bar{\theta} = [16,17,18,19]$ per hour respectively.

$$\text{Then, } \lambda(\alpha) = \min_{x \in s(\bar{\lambda})} \left\{ x \in s(\bar{\lambda}), \begin{cases} x - 11, & 1 \le x \le 12 \\ 1 \quad , & 12 \le x \le 13 \ \ge \alpha \\ 14 - x, & 13 \le x \le 14 \end{cases} \right\},$$

$$\max_{x\in s(\bar{\lambda})}\{x\in s(\bar{\lambda}),\begin{cases} x-11, 11\le x\le 12 \\ 1, 12\le x\le 13\ge\alpha \\ 14-x, 13\le x\le 14 \end{cases}.$$

(i,e). $\lambda(\alpha)=[11+\alpha,14-\alpha]$, $\beta(\alpha)=[31+\alpha,34-\alpha]$, $\gamma(\alpha)=[21+\alpha,24-\alpha]$, $\theta(\alpha)=[16+\alpha,19-\alpha]$.

It is clear that when $x=x_\alpha^U, y_1=y_{1_\alpha}^U, y_2=y_{2_\alpha}^U, z=z_\alpha^U$, L attains its maximum value and when $x=x_\alpha^L, y_1=y_{1_\alpha}^L, y_2=y_{2_\alpha}^U, z=z_\alpha^L$, L attains its minimum value.

From the generated for the given input value of $\bar{\lambda},\bar{\beta},\bar{\gamma},\bar{\theta}$ with we infer that

i) For fixed values of $x, y_1 \,\&\, y_2, M$ decreases as z increase.

ii) For fixed values of $x, y_1 \,\&\, z, M$ decreases as y_2 increase.

iii) For fixed values of $x, z \,\&\, y_2, M$ decreases as y_1 increase.

iv) For fixed values of $y_1, y_2 \,\&\, z\ M$ decreases as x increase.

The smallest value of occurs when x-takes its lower bound.
i.e)., $x=11+\alpha$ and y_1, y_2 and z, take their upper bounds given by $y_1=34-\alpha$, $y_2=24-\alpha$ and $z=19-\alpha$ respectively. and maximum value of L occurs when $x=14-\alpha, y_1=31+\alpha, y_2=21+\alpha$, and $z=16+\alpha$.

If both $(E(L))_\alpha^L$ & $(E(L))_U^\alpha$ are invertible with respect to 'α' then the left shape function $L(M)=[(E(L))]^{-1}$ and right shape function $R(M)=[(E(L))_\alpha^L]^{-1}$ can be obtained and from which the membership function $\mu_{\overline{E(L)}}(M)$ can be constructed as:

$$\mu_{\overline{E(L)}}(M)=\begin{cases} L(M), M_1\le M\le M_2 \\ 1, \quad M_2\le M\le M_3 \\ R(M), M_3\le M\le M_4 \end{cases} \tag{3}$$

The values of M_1, M_2, M_3 & M_4 as obtained from (3) are

$$\mu_{\overline{E(L)}}(M)=\begin{cases} L(M),\ 0.4312\le M\le 0.6751 \\ 1, \quad 0.6751\le M\le 0.6801 \\ R(M),\ 0.6801\le M\le 0.8102 \end{cases}$$

The mean waiting time

The smallest value of $E(W)$ occurs when x take its lower bound i,e)., $x=11+\alpha$ & y

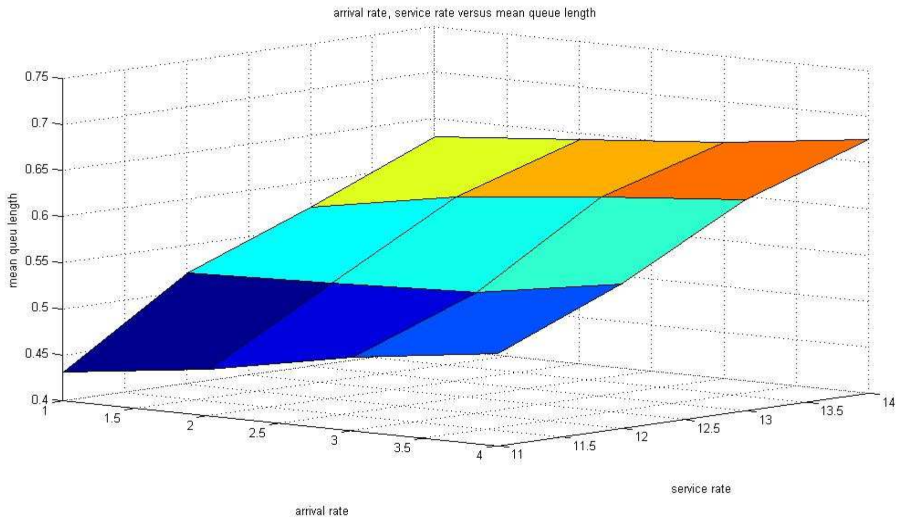
take their upper bounds given by $y_1 = 34-\alpha, y_2 = 24-\alpha, z = 19-\alpha$ respectively and maximum value of $E(W)$ occurswhen $x = 11+\alpha, y_1 = 31+\alpha, y_2 = 21+\alpha, z = 16+\alpha.$ If both $E(W)_\alpha^L$ & $E(W)_\alpha^U$ are invertible with respect to α then left shape function $L(N) = [E(W)_\alpha^L]^{-1}$ and right shape function $R(N) = [E(W)_\alpha^U]^{-1}$ can be obtained, from which the membership function $\mu_{\overline{E(W)}}(N)$ can be written as,

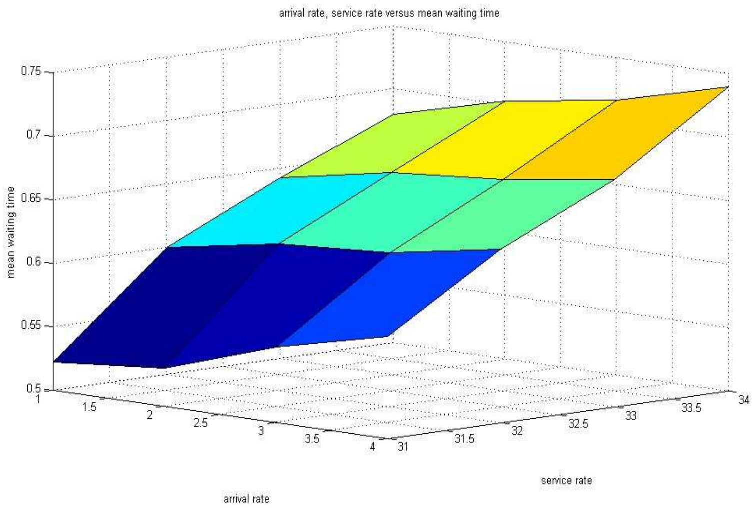
$$\mu_{\overline{E(W)}}(N) = \begin{cases} L(N), N_1 \le N \le N_2 \\ 1, \quad N_2 \le N \le N_3 \\ R(N), N_3 \le N \le N_4 \end{cases} \tag{4}$$

The values of N_1, N_2, N_3 & N_4 as obtained from (4) are

$$\mu_{\overline{E(W)}}(N) = \begin{cases} L(N), 0.5219 \le N \le 0.5812 \\ \\ 1, \quad 0.5812 \le N \le 0.6078 \\ \\ R(N), 0.6078 \le N \le 0.7394 \end{cases}$$

Further by fixed the vacation rate by a crisp value $\overline{\theta} = 16.4$ and $\overline{\gamma} = 21.3$ taking arrival rate $\overline{\lambda} = [11,12,13,14]$, service rate $\overline{\beta} = [31,32,33,34]$ both trapezoidal fuzzy numbers the values of the mean system length are generated and are plotted in from the figure it can be observed that as $\overline{\lambda}$ increases the mean system length increases for the fixed value of the service rate, where as for fixed value of arrival rate, the mean system length decreases as service rate increases. Similar conclusion can be obtained for the case $\overline{\theta} = 18.6$, $\overline{\gamma} = 23.8$ Again for fixed values of taking $\overline{\lambda} = [11,12,13,14],$ $\overline{\beta} = [31,32,33,34]$ the graphs of mean waiting time are drawn in figure 2 respectively, these figure show that as arrival rate increases that waiting time also increases, while the waiting time decreases as the service rate increases in both the case.It is also observed from the data generated that the membership value of the mean queue length is 0.8 and the membership value of the mean waiting time 0.75 when the ranges of arrival rate, service rate, and the vacation rate lie in the intervals $(12,13.4), (33,33.6),$ & $(31.8,32.4)$ respectively.





5. Conclusion

In this paper, we have studied the "Analysis for the FM/FM/1 Queue with Multiple Working Vacation with N-Policy". We have obtained the mean queue length, mean waiting time with N=2. Numerical study of this performance measures for the fuzzy Queues are obtained. Consider the example, an ATM networks, where cell arrivals in a switched virtual channel(SVC) the arrival fuzzy parameter λ, cell transmission time is an service rate y_1. when a SVC finishes cell transmission and becomes empty, we set a period of working vacation in order to economize operating cost and energy consumption, during a working vacation arriving cells can be transmitted at a lower rate $y_2\ (y_2 < y_1)$ immediately. Meanwhile, to avoid switching frequently from a speed to another speed, we set a threshold N to decrease the switching cost. The policy of working vacation takes over cell transmission and save switching cost together, therefore, our model is fitter for practical situation then others.

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