

Fixed Point Theorem for Occasionally Weakly Compatible Mappings in Modified Intuitionistic Fuzzy Metric Space

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Abstract. In this paper, we prove a common fixed point theorem for occasionally weakly compatible mappings in modified intuitionistic fuzzy metric space. Consequently, our result improves and sharpens many known common fixed point theorems available in the existing literature of metric fixed point theory.

Keywords: Modified intuitionistic fuzzy metric space, common fixed point, fuzzy metric space, t -representable norms, occasionally weakly compatible mappings.

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1. Introduction

The concept of fuzzy set was introduced in 1965 by Zadeh [22]. Since then, with a view to utilize this concept in topology and analysis, many authors have extensively developed by Park [15] which is now known as intuitionistic fuzzy metric space wherein notions of continuous t -norm and continuous t -conorm are employed. Fixed point theory is one of the most fruitful and effective tools in mathematics which has enormous applications in several branches of science especially in chaos theory, game theory, theory of differential equation, etc. Intuitionistic fuzzy metric notion is also useful in modeling some physical problems wherein it is necessary to study the relationship between two probability functions as noticed by Gregori et al. [13]. For instance, it has a concrete physical visualization in the context of two slit experiment as the foundation of E -infinity theory of high energy physics whose details are available in El Naschie in [7, 8, 9]. As noticed by Gregori et al. [13], the topology induced by intuitionistic fuzzy metric coincides with the topology induced by fuzzy metric, Saadati et al. [20] reframed the idea of intuitionistic fuzzy metric spaces and proposed a new notion under the name

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of modified intuitionistic fuzzy metric spaces by introducing the of continuous t –representable.

In this paper, we prove a common fixed point theorem for occasionally weakly compatible mappings in modified intuitionistic fuzzy metric space. Consequently, our result improves and sharpens many known common fixed point theorems available in the existing literature of metric fixed point theory.

2. Preliminaries

Definition 2.1. A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t – norm if $*$ satisfies following conditions:

- (i) $*$ is commutative and associative
- (ii) $*$ is continuous;
- (iii) $a * 1 = a$ for all $a \in [0, 1]$;
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Definition 2.2. [9] A binary operation \diamond : $[0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t –norm if \diamond is satisfying the following conditions

- (a) \diamond is commutative and associative;
- (b) \diamond is continuous;
- (c) $a \diamond 0 = a$ for all $a \in [0,1]$;
- (d) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Lemma 2.1. [9] Consider the set L^* and operation \leq_{L^*} defined by:

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0, 1]^2 \text{ and } x_1 + x_2 \leq 1\},$$

$$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1$$

And $x_2 \geq y_2$, for every $(x_1, x_2), (y_1, y_2) \in L^*$.
then (L^*, \leq_{L^*}) is a complete lattice.

Definition 2.3. [1] An intuitionistic fuzzy set $\mathcal{A}_{\zeta, \eta}$ in a universe U is an object $\mathcal{A}_{\zeta, \eta} = \{(\zeta_{\mathcal{A}}(u), \eta_{\mathcal{A}}(u)) | u \in U\}$, where, for all $u \in U$, $\zeta_{\mathcal{A}}(u) \in [0, 1]$ and $\eta_{\mathcal{A}}(u) \in [0, 1]$ are called the membership degree and the non-membership degree, respectively of u in $\mathcal{A}_{\zeta, \eta}$ and furthermore they satisfy $\zeta_{\mathcal{A}}(u) + \eta_{\mathcal{A}}(u) \leq 1$.

Definition 2.4. For every $z_{\alpha} = (x_{\alpha}, y_{\alpha}) \in L^*$ we define

$$\vee (z_{\alpha}) = (\sup(x_{\alpha}), \inf(y_{\alpha})).$$

Since $z_{\alpha} \in L^*$ then $x_{\alpha} + y_{\alpha} \leq 1$ so $\sup(x_{\alpha}) + \inf(y_{\alpha}) \leq \sup(x_{\alpha} + y_{\alpha}) \leq 1$, that is $\vee (z_{\alpha}) \in L^*$. We denote its units by $0_{L^*} = (0, 1)$ and $1_{L^*} = (1, 0)$.

Classically, a triangular norm $*$ = T on $([0,1], \leq)$ is define as an increasing, commutative, associative mapping $T: [0,1]^2 \rightarrow [0,1]$ satisfying $T(1, x) = 1 * x = x$, for all $x \in [0,1]$. A triangular conorm S = \diamond is defined as an increasing, commutative, associative mapping $S: [0,1]^2 \rightarrow [0,1]$ satisfying $S(0, x) = 0 \diamond x = x$, for all $x \in [0,1]$. Using the lattice (L^*, \leq_{L^*}) these definitions can be straightforwardly extended.

Definition 2.5. [9, 14] A triangular norm (t –norm) on L^* is a mapping $\mathcal{T} : (L^*)^2 \rightarrow L^*$ satisfying the following conditions:

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- 1) $(\forall x \in L^*)(\mathcal{T}(x, 1_{L^*}) = x)$, (boundary condition)
- 2) $(\forall (x, y) \in (L^*)^2)(\mathcal{T}(x, y) = \mathcal{T}(y, x))$, (commutativity)
- 3) $(\forall (x, y, z) \in (L^*)^3)(\mathcal{T}(x, \mathcal{T}(y, z)) = \mathcal{T}(\mathcal{T}(x, y), z))$, (associativity),
- 4) $(\forall (x, x', y, y') \in (L^*)^4)(x \leq_{L^*} x')$ and $(y \leq_{L^*} y' \Rightarrow \mathcal{T}(x, y) \leq_{L^*} \mathcal{T}(x', y'))$.
(monotonicity).

Definition 2.6. [9, 14] A continuous t -norm \mathcal{T} on L^* is called continuous t -representable if and only if there exist a continuous t -norm $*$ and a continuous t -conorm \diamond on $[0, 1]$ such that, for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$,

$$\mathcal{T}(x, y) = (x_1 * y_1, x_2 \diamond y_2).$$

Now define a sequence \mathcal{T}^n recursively by $\mathcal{T}^1 = \mathcal{T}$ and

$$\mathcal{T}^n(x^{(1)}, \dots, x^{(n+1)}) = \mathcal{T}(\mathcal{T}^{n-1}(x^{(1)}, \dots, x^{(n)}), x^{(n+1)})$$

For $n \geq 2$ and $x^{(i)} \in L^*$.

We say the continuous t -representable norm is natural and write \mathcal{T}_n whenever $\mathcal{T}_n(a, b) = \mathcal{T}_n(c, d)$ and $a \leq_{L^*} c$ implies $b \leq_{L^*} d$.

Definition 2.7. A negator on L^* is any decreasing mapping $\mathcal{N} : L^* \rightarrow L^*$ satisfying $\mathcal{N}(0_{L^*}) = 1_{L^*}$ and $\mathcal{N}(1_{L^*}) = 0_{L^*}$. If $\mathcal{N}(\mathcal{N}(x)) = x$, for all $x \in L^*$, then \mathcal{N} is called an involutive negator. A negator on $[0, 1]$ is a decreasing mapping $N : [0, 1] \rightarrow [0, 1]$ satisfying $N(0) = 1$ and $N(1) = 0$. N_S denotes the standard negator on $[0, 1]$ defined as $N_S(x) = 1 - x$ for all $x \in [0, 1]$.

Definition 2.8. Let M, N are fuzzy sets from $X^2 \times (0, +\infty)$ to $[0, 1]$ such that $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$. The 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be a Modified Intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, \mathcal{T} is a continuous t -representable norm and $\mathcal{M}_{M,N}$ is a mapping $X^2 \times (0, +\infty) \rightarrow L^*$ (an intuitionistic fuzzy set, see Definition 3) satisfying the following conditions for every $x, y \in X$ and $t, s > 0$:

- 1) $\mathcal{M}_{M,N}(x, y, t) >_{L^*} 0_{L^*}$;
- 2) $\mathcal{M}_{M,N}(x, y, t) = 1_{L^*}$ if and only if $x = y$;
- 3) $\mathcal{M}_{M,N}(x, y, t) = \mathcal{M}_{M,N}(y, x, t)$;
- 4) $\mathcal{M}_{M,N}(x, y, t + s) \geq_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(x, z, t), \mathcal{M}_{M,N}(z, y, s))$;
- 5) $\mathcal{M}_{M,N}(x, y, \cdot) : (0, \infty) \rightarrow L^*$ is continuous.

In this case $\mathcal{M}_{M,N}$ is called an intuitionistic fuzzy metric space. Here,

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)).$$

In the sequel, we will call $(X, \mathcal{M}_{M,N}, \mathcal{T})$ to be just a Modified Intuitionistic fuzzy metric space.

Remark 2.1. [18] In a Modified Intuitionistic fuzzy metric space

$(X, \mathcal{M}_{M,N}, \mathcal{T})$, $M(x, y, \cdot)$ is non decreasing and $N(x, y, \cdot)$ is non increasing for all $x, y \in X$. Hence $\mathcal{M}_{M,N}(x, y, t)$ is non decreasing with respect to t for all $x, y \in X$.

Example 2.1. [11] Let (X, d) be a metric space.

Define $\mathcal{T}(a, b) = \{a_1 b_1, \min(a_2 + b_2, 1)\}$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \left(\frac{ht^n}{ht^n + md(x, y)}, \frac{md(x, y)}{ht^n + md(x, y)} \right),$$

for all $h, m, n, t \in \mathbb{R}^+$. Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is a modified intuitionistic fuzzy metric space.

Example 2.2. [1] Let $X = \mathbb{N}$.

Define $\mathcal{T}(a, b) = \{(\max(0, a_1 + b_1 - 1), a_2 + b_2 - a_2 b_2)\}$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \begin{cases} \left(\frac{x}{y}, \frac{y-x}{y} \right) & \text{if } x \leq y \\ \left(\frac{y}{x}, \frac{x-y}{x} \right) & \text{if } y \leq x \end{cases}$$

for all $x, y \in X$ and $t > 0$. Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is a modified intuitionistic fuzzy metric space.

Definition 2.9. [11] Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified intuitionistic fuzzy metric space and $\{x_n\}$ be a sequence in X .

1. A sequence $\{x_n\}$ is said to be convergent to $x \in X$ in the modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ and denoted by $x_n \xrightarrow{\mathcal{M}_{M,N}} x$ if $\mathcal{M}_{M,N}(x_n, x, t) \rightarrow 1_{L^*}$ as $n \rightarrow \infty$ for every $t > 0$.
2. A sequence $\{x_n\}$ in a modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is called a Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mathcal{M}_{M,N}(x_n, y_m, t) >_{L^*} (N_s(\varepsilon), \varepsilon)$, and for each $n, m \geq n_0$; here N_s is the standard negator.
3. A modified intuitionistic fuzzy metric space is said to be complete if and only if every Cauchy sequence in this space is convergent. Henceforth, we assume that \mathcal{T} is

a continuous t -norm on the lattice \mathcal{L} such that for every $\mu \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$, there exists $\lambda \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$ such that $\mathcal{T}^{n-1}(\mathcal{N}(\lambda), \dots, \mathcal{N}(\lambda)) \geq_{L^*} \mathcal{N}(\mu)$.

Lemma 2.2. [18] Let $\mathcal{M}_{M,N}$ be a modified intuitionistic fuzzy metric. Then for any $t > 0$, $\mathcal{M}_{M,N}(x, y, t)$ is nondecreasing with respect to t in (L^*, \leq_{L^*}) for all $x, y \in X$.

Lemma 2.3. [3] Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified intuitionistic fuzzy metric space and define $E_{\lambda, \mathcal{M}_{M,N}} : X^2 \rightarrow \mathbb{R}^+ \cup \{0\}$ by

$$E_{\lambda, \mathcal{M}_{M,N}}(x, y) = \inf\{t > 0 : \mathcal{M}_{M,N}(x, y, t) >_{L^*} \mathcal{N}(\lambda)\}$$

for each $\lambda \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$ and $x, y \in X$ here, \mathcal{N} is an involutive negator. Then we have

(i) For any $\mu \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$, there exists $\lambda \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$ such that

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$E_{\mu, \mathcal{M}_{M,N}}(x_1, x_n) \leq E_{\lambda, \mathcal{M}_{M,N}}(x_1, x_2) + E_{\lambda, \mathcal{M}_{M,N}}(x_2, x_3) + \cdots + E_{\lambda, \mathcal{M}_{M,N}}(x_{n-1}, x_n)$ for any $x_1, x_2, x_3, \dots, x_n \in X$.

(ii) The sequence $\{x_n\}_{n \in \mathbb{N}}$ in X is convergent to x with respect to modified intuitionistic fuzzy metric $\mathcal{M}_{M,N}$ if and only if $E_{\lambda, \mathcal{M}_{M,N}}(x_n, x) \rightarrow 0$. Also the sequence $\{x_n\}$ is a Cauchy sequence with respect to modified intuitionistic fuzzy metric $\mathcal{M}_{M,N}$ if and only if it is a Cauchy sequence with $E_{\lambda, \mathcal{M}_{M,N}}$.

Proof: For (i), by the continuity of t -norms, for every $\mu \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$, we can find a $\lambda \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$ such that $\mathcal{T}(\mathcal{N}(\lambda), \mathcal{N}(\lambda)) \geq_{L^*} \mathcal{N}(\mu)$. By Definition 2.8 (iv) we have

$$\begin{aligned} & \mathcal{M}_{M,N}(x, y, E_{\lambda, \mathcal{M}_{M,N}}(x_1, x_2) + E_{\lambda, \mathcal{M}_{M,N}}(x_2, x_3) + \cdots + E_{\lambda, \mathcal{M}_{M,N}}(x_{n-2}, x_{n-1}) \\ & \quad + E_{\lambda, \mathcal{M}_{M,N}}(x_{n-1}, x_n) + n\delta) \\ \geq_{L^*} & \mathcal{T}(\mathcal{M}_{M,N}(x, z, E_{\lambda, \mathcal{M}_{M,N}}(x_1, x_2) + E_{\lambda, \mathcal{M}_{M,N}}(x_2, x_3) + \cdots + E_{\lambda, \mathcal{M}_{M,N}}(x_{n-3}, x_{n-2}) \\ & \quad + \frac{n\delta}{2}), \mathcal{M}_{M,N}(z, y, E_{\lambda, \mathcal{M}_{M,N}}(x_{n-2}, x_{n-1}) + E_{\lambda, \mathcal{M}_{M,N}}(x_{n-1}, x_n) \\ & \quad + \frac{n\delta}{2})) \geq_{L^*} \mathcal{T}(\mathcal{N}(\lambda), \mathcal{N}(\lambda)) \geq_{L^*} \mathcal{N}(\mu) \end{aligned}$$

for every $\delta > 0$, which implies that

$$E_{\mu, \mathcal{M}_{M,N}}(x_1, x_n) \leq E_{\lambda, \mathcal{M}_{M,N}}(x_1, x_2) + E_{\lambda, \mathcal{M}_{M,N}}(x_2, x_3) + \cdots + E_{\lambda, \mathcal{M}_{M,N}}(x_{n-1}, x_n) + n\delta$$

Since $\delta > 0$ was arbitrary, we have

$$E_{\mu, \mathcal{M}_{M,N}}(x_1, x_n) \leq E_{\lambda, \mathcal{M}_{M,N}}(x_1, x_2) + E_{\lambda, \mathcal{M}_{M,N}}(x_2, x_3) + \cdots + E_{\lambda, \mathcal{M}_{M,N}}(x_{n-1}, x_n).$$

For (ii), we have $\mathcal{M}_{M,N}(x_n, x, \eta) >_{L^*} \mathcal{N}(\lambda) \Leftrightarrow E_{\lambda, \mathcal{M}_{M,N}}(x_n, x) < \eta$ for every $\eta > 0$.

Lemma 2.4. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified intuitionistic fuzzy metric space. If $\mathcal{M}_{M,N}(x_n, x_{n+1}, t) \geq_{L^*} \mathcal{M}_{M,N}(x_0, x_1, \frac{t}{k^n})$ for some $k < 1$ and $n \in \mathbb{N}$ then $\{x_n\}$ is a Cauchy sequence.

Proof: For every $\lambda \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$ and $x_n \in X$, we have

$$\begin{aligned} & E_{\lambda, \mathcal{M}_{M,N}}(x_n, x_{n+1}, t) \\ & = \inf\{t > 0 : \mathcal{M}_{M,N}(x_n, x_{n+1}, t) >_{L^*} \mathcal{N}(\lambda)\} \\ & \leq \inf\{t > 0 : \mathcal{M}_{M,N}(x_0, x_1, \frac{t}{k^n}) >_{L^*} \mathcal{N}(\lambda)\} \\ & = \inf\{k^n t : \mathcal{M}_{M,N}(x_0, x_1, t) >_{L^*} \mathcal{N}(\lambda)\} \\ & = k^n \inf\{t > 0 : \mathcal{M}_{M,N}(x_0, x_1, t) >_{L^*} \mathcal{N}(\lambda)\} \\ & = k^n E_{\lambda, \mathcal{M}_{M,N}}(x_0, x_1, t) \end{aligned}$$

From lemma (2.3) for every $\mu \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$ there exists $\lambda \in L^* \setminus \{0_{L^*}, 1_{L^*}\}$, such that

$$\begin{aligned} & E_{\mu, \mathcal{M}_{M,N}}(x_n, x_m) \\ & \leq E_{\lambda, \mathcal{M}_{M,N}}(x_n, x_{n+1}) + E_{\lambda, \mathcal{M}_{M,N}}(x_{n+1}, x_{n+2}) + \cdots + E_{\lambda, \mathcal{M}_{M,N}}(x_{m-1}, x_m) \\ & \leq k^n E_{\lambda, \mathcal{M}_{M,N}}(x_0, x_1) + k^{n+1} E_{\lambda, \mathcal{M}_{M,N}}(x_0, x_1) + \cdots + k^{m-1} E_{\lambda, \mathcal{M}_{M,N}}(x_0, x_1) \\ & = E_{\lambda, \mathcal{M}_{M,N}}(x_0, x_1) \sum_{j=n}^{m-1} k^j \rightarrow 0. \end{aligned}$$

Hence sequence $\{x_n\}$ is a Cauchy sequence.

Definition 2.10. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified intuitionistic fuzzy metric space. \mathcal{M} is said to be continuous on $X \times X \times]0, \infty[$ if

$$\lim_{n \rightarrow \infty} \mathcal{M}_{M,N}(x_n, y_n, t_n) = \mathcal{M}_{M,N}(x, y, t),$$

Whenever $\{(x_n, y_n, t_n)\}$ is a sequence in $X \times X \times]0, \infty[$ which converges to a point $(x, y, t) \in X \times X \times]0, \infty[$ that is

$$\lim_{n \rightarrow \infty} \mathcal{M}_{M,N}(x_n, x, t) = \lim_{n \rightarrow \infty} \mathcal{M}_{M,N}(y_n, y, t) = 1_{L^*}$$

$$\text{and } \lim_{n \rightarrow \infty} \mathcal{M}_{M,N}(x, y, t_n) = \mathcal{M}_{M,N}(x, y, t).$$

Lemma 2.5. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0, 1)$, $\mathcal{M}_{M,N}(x, y, kt) \geq_{L^*} \mathcal{M}_{M,N}(x, y, t)$. Then $x = y$

Proof: For every $\lambda \in L \setminus \{0_{L^*}, 1_{L^*}\}$ and $x, y \in X$, we have

$$\begin{aligned} & E_{\lambda, \mathcal{M}_{M,N}}(x, y, t) \\ &= \inf\{t > 0 : \mathcal{M}_{M,N}(x, y, t) >_{L^*} \mathcal{N}(\lambda)\} \\ &\leq \inf\{t > 0 : \mathcal{M}_{M,N}\left(x, y, \frac{t}{k}\right) >_{L^*} \mathcal{N}(\lambda)\} \\ &= \inf\{kt : \mathcal{M}_{M,N}(x, y, t) >_{L^*} \mathcal{N}(\lambda)\} \\ &= k \inf\{t > 0 : \mathcal{M}_{M,N}(x, y, t) >_{L^*} \mathcal{N}(\lambda)\} \\ &= k E_{\lambda, \mathcal{M}_{M,N}}(x, y, t) \end{aligned}$$

Therefore $E_{\lambda, \mathcal{M}_{M,N}}(x, y, t) = 0$. Hence $x = y$.

Definition 2.11. Let f and g be mappings from a modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ into itself. The maps f and g are said to be weakly commuting if $\mathcal{M}_{M,N}(ASz, SAz, t) \geq_{L^*} \mathcal{M}_{M,N}(Az, Sz, t)$ for all $z \in X, t < 0$.

Definition 2.12. A pair of self mappings (f, g) of modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be compatible if

$$\lim_{n \rightarrow \infty} \mathcal{M}_{M,N}(fgx_n, gfx_n, t) = 1_{L^*}$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z$ for some $z \in X$.

Definition 2.13. Two self mappings f and g are called non compatible if there exist at least one sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z$ for some $z \in X$ but either $\lim_{n \rightarrow \infty} \mathcal{M}_{M,N}(fgx_n, gfx_n, t) \neq 1_{L^*}$ or the limit does not exist for all $z \in X$.

Definition 2.14. A pair of self mappings (f, g) of modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be weakly compatible if they commute at coincidence point that is if $fu = gu$ for some $u \in X$, then $fgu = gfu$.

Definition 2.15. A pair of self mappings (f, g) of modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be occasionally weakly compatible if the pair (f, g)

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commutes at least one coincidence point that is there exists at least one point $x \in X$, such that $fx = gx$ and $fgx = gfx$.

Lemma 2.6.: Let X be a set, f and g be occasionally weakly compatible self maps on X of a modified intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$. If f and g have unique point of coincidence that is $w = fx = gx$ for $x \in X$, then w is the unique common fixed point of f and g .

3. The main results

Theorem 3.1. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a complete modified intuitionistic fuzzy metric space and let A, B, S, T, P and Q be self mappings of X . Let the pairs (P, AB) and (Q, ST) be occasionally weakly compatible. If there exist $k \in (0, 1)$ such that

$$\mathcal{M}_{M,N}(Px, Qy, kt) \geq_{L^*} \min \left\{ \begin{array}{l} \mathcal{M}_{M,N}(ABx, STy, t), \mathcal{M}_{M,N}(Qy, ABx, t), \mathcal{M}_{M,N}(Px, STy, t), \mathcal{M}_{M,N}(ABx, Px, t), \\ \frac{a \mathcal{M}_{M,N}(Px, Qy, t) + b \mathcal{M}_{M,N}(Px, STy, t)}{a \mathcal{M}_{M,N}(ABx, Qy, t) + b \mathcal{M}_{M,N}(ABx, STy, t)}, \\ \frac{c \mathcal{M}_{M,N}(ABx, Qy, t) + d \mathcal{M}_{M,N}(ABx, STy, t)}{c \mathcal{M}_{M,N}(Qy, STy, t) + d}, \\ \frac{e \mathcal{M}_{M,N}(ABx, Px, t) + f \mathcal{M}_{M,N}(Qy, STy, t)}{e + f}, \\ \frac{a \mathcal{M}_{M,N}(Px, STy, t) + b \mathcal{M}_{M,N}(ABx, Qy, t) + c \mathcal{M}_{M,N}(ABx, STy, t)}{a + b + c} \end{array} \right\} \quad (1)$$

For all $x, y \in X$ all $t > 0$ and $a, b, c, d, e, f \geq 0$ with a & b , c & d and e & f cannot be simultaneously 0, then there exist a unique point $w \in X$ such that $Pw = ABw = w$ and a unique point $z \in X$ such that $Qz = STz = z$. Moreover $w = z$, so that there is a unique common fixed point of A, B, S, T, P and Q .

Proof: Let the pairs (P, AB) and (Q, ST) be occasionally weakly compatible, so there are points $x, y \in X$ such that $Px = ABx$ and $Qy = STy$. We claim $Px = Qy$. If not, by inequality (1)

$$\mathcal{M}_{M,N}(Px, Qy, kt) \geq_{L^*} \min \left\{ \begin{array}{l} \mathcal{M}_{M,N}(ABx, STy, t), \mathcal{M}_{M,N}(Qy, ABx, t), \mathcal{M}_{M,N}(Px, STy, t), \mathcal{M}_{M,N}(ABx, Px, t), \\ \frac{a \mathcal{M}_{M,N}(Px, Qy, t) + b \mathcal{M}_{M,N}(Px, STy, t)}{a \mathcal{M}_{M,N}(ABx, Qy, t) + b \mathcal{M}_{M,N}(ABx, STy, t)}, \\ \frac{c \mathcal{M}_{M,N}(ABx, Qy, t) + d \mathcal{M}_{M,N}(ABx, STy, t)}{c \mathcal{M}_{M,N}(Qy, STy, t) + d}, \\ \frac{e \mathcal{M}_{M,N}(ABx, Px, t) + f \mathcal{M}_{M,N}(Qy, STy, t)}{e + f}, \\ \frac{a \mathcal{M}_{M,N}(Px, STy, t) + b \mathcal{M}_{M,N}(ABx, Qy, t) + c \mathcal{M}_{M,N}(ABx, STy, t)}{a + b + c} \end{array} \right\}$$

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$$\begin{aligned}
 & \left. \begin{aligned}
 & \mathcal{M}_{M,N}(Px, Qy, t), \mathcal{M}_{M,N}(Qy, Px, t), \mathcal{M}_{M,N}(Px, Qy, t), \mathcal{M}_{M,N}(Px, Px, t), \\
 & \frac{a \mathcal{M}_{M,N}(Px, Qy, t) + b \mathcal{M}_{M,N}(Px, Qy, t)}{a \mathcal{M}_{M,N}(Px, Qy, t) + b \mathcal{M}_{M,N}(Px, Qy, t)}, \\
 & \frac{c \mathcal{M}_{M,N}(Px, Qy, t) + d \mathcal{M}_{M,N}(Px, Qy, t)}{c \mathcal{M}_{M,N}(Qy, Qy, t) + d}, \\
 & \frac{e \mathcal{M}_{M,N}(Px, Px, t) + f \mathcal{M}_{M,N}(Qy, Qy, t)}{e + f}, \\
 & \frac{a \mathcal{M}_{M,N}(Px, Qy, t) + b \mathcal{M}_{M,N}(Px, Qy, t) + c \mathcal{M}_{M,N}(Px, Qy, t)}{a + b + c}
 \end{aligned} \right\} \\
 & =_{L^*} \min \left\{ \begin{aligned}
 & \mathcal{M}_{M,N}(Px, Qy, t), \mathcal{M}_{M,N}(Qy, Px, t), \mathcal{M}_{M,N}(Px, Qy, t), 1_{L^*}, \\
 & 1_{L^*}, \\
 & \mathcal{M}_{M,N}(Px, Qy, t), \\
 & 1_{L^*}, \\
 & \mathcal{M}_{M,N}(Px, Qy, t)
 \end{aligned} \right\} \\
 & =_{L^*} \mathcal{M}_{M,N}(Px, Qy, t)
 \end{aligned}$$

Therefore $Px = Qy$ that is $Px = ABx = Qy = STy$. Suppose that there is another point z such that $Pz = ABz$ then by inequality (1) we have $Pz = ABz = Qz = STz$. So $Px = Pz$ and $w = Px = ABx$ is the unique point of coincidence of P and AB . By lemma 2.6, w is the only common fixed point of P and AB . Similarly there is a unique point $z \in X$ such that $z = Qz = STz$.

Assume that $w \neq z$. We have, by inequality (1)

$$\begin{aligned}
 & \mathcal{M}_{M,N}(w, z, kt) = \mathcal{M}_{M,N}(Pw, Qz, kt) \geq_{L^*} \min \\
 & \left. \begin{aligned}
 & \mathcal{M}_{M,N}(w, z, t), \mathcal{M}_{M,N}(z, w, t), \mathcal{M}_{M,N}(w, z, t), \mathcal{M}_{M,N}(w, w, t), \\
 & \frac{a \mathcal{M}_{M,N}(w, z, t) + b \mathcal{M}_{M,N}(w, z, t)}{a \mathcal{M}_{M,N}(w, z, t) + b \mathcal{M}_{M,N}(w, z, t)}, \\
 & \frac{c \mathcal{M}_{M,N}(w, z, t) + d \mathcal{M}_{M,N}(w, z, t)}{c \mathcal{M}_{M,N}(z, z, t) + d}, \\
 & \frac{e \mathcal{M}_{M,N}(w, w, t) + f \mathcal{M}_{M,N}(z, z, t)}{e + f}, \\
 & \frac{a \mathcal{M}_{M,N}(w, z, t) + b \mathcal{M}_{M,N}(w, z, t) + c \mathcal{M}_{M,N}(w, z, t)}{a + b + c}
 \end{aligned} \right\} \\
 & \geq_{L^*} \min \left\{ \begin{aligned}
 & \mathcal{M}_{M,N}(w, z, t), \mathcal{M}_{M,N}(z, w, t), \mathcal{M}_{M,N}(w, z, t), 1_{L^*}, \\
 & \mathcal{M}_{M,N}(w, z, t), \\
 & \mathcal{M}_{M,N}(w, z, t), \\
 & 1_{L^*}, \\
 & \mathcal{M}_{M,N}(w, z, t)
 \end{aligned} \right\} \\
 & =_{L^*} \mathcal{M}_{M,N}(w, z, t)
 \end{aligned}$$

Therefore we have $z = w$, by lemma 2.6, z is a common fixed point of A, B, S, T, P and Q .

For uniqueness, let u be another common fixed point of A, B, S, T, P and Q . Then

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$$\begin{aligned}
 & \mathcal{M}_{M,N}(z, u, kt) = \mathcal{M}_{M,N}(Pz, Qu, kt) \\
 & \geq_{L^*} \min \left\{ \begin{array}{l} \mathcal{M}_{M,N}(ABz, STu, t), \mathcal{M}_{M,N}(Qu, ABz, t), \mathcal{M}_{M,N}(Pz, STu, t), \mathcal{M}_{M,N}(ABz, Pz, t), \\ \frac{a \mathcal{M}_{M,N}(Pz, Qu, t) + b \mathcal{M}_{M,N}(Pz, STu, t)}{a \mathcal{M}_{M,N}(ABz, Qu, t) + b \mathcal{M}_{M,N}(ABz, STu, t)}, \\ \frac{c \mathcal{M}_{M,N}(ABz, Qu, t) + d \mathcal{M}_{M,N}(ABz, STu, t)}{c \mathcal{M}_{M,N}(Qu, STu, t) + d}, \\ \frac{e \mathcal{M}_{M,N}(ABz, Pz, t) + f \mathcal{M}_{M,N}(Qu, STu, t)}{e + f}, \\ \frac{a \mathcal{M}_{M,N}(Pz, STu, t) + b \mathcal{M}_{M,N}(ABz, Qu, t) + c \mathcal{M}_{M,N}(ABz, STu, t)}{a + b + c} \end{array} \right\} \\
 & =_{L^*} \min \left\{ \begin{array}{l} \mathcal{M}_{M,N}(z, u, t), \mathcal{M}_{M,N}(u, z, t), \mathcal{M}_{M,N}(z, u, t), \mathcal{M}_{M,N}(z, z, t), \\ \frac{a \mathcal{M}_{M,N}(z, u, t) + b \mathcal{M}_{M,N}(z, u, t)}{a \mathcal{M}_{M,N}(z, u, t) + b \mathcal{M}_{M,N}(z, u, t)}, \\ \frac{c \mathcal{M}_{M,N}(z, u, t) + d \mathcal{M}_{M,N}(z, u, t)}{c \mathcal{M}_{M,N}(u, u, t) + d}, \\ \frac{e \mathcal{M}_{M,N}(z, z, t) + f \mathcal{M}_{M,N}(u, u, t)}{e + f}, \\ \frac{a \mathcal{M}_{M,N}(z, u, t) + b \mathcal{M}_{M,N}(z, u, t) + c \mathcal{M}_{M,N}(z, u, t)}{a + b + c} \end{array} \right\} \\
 & =_{L^*} \min \left\{ \begin{array}{l} \mathcal{M}_{M,N}(z, u, t), \mathcal{M}_{M,N}(u, z, t), \mathcal{M}_{M,N}(z, u, t), 1_{L^*}, \\ \mathcal{M}_{M,N}(z, u, t), \\ \mathcal{M}_{M,N}(z, u, t), \\ 1_{L^*}, \\ \mathcal{M}_{M,N}(z, u, t) \end{array} \right\} \\
 & =_{L^*} \mathcal{M}_{M,N}(z, u, t)
 \end{aligned}$$

Therefore by lemma 2.6 we have $z = u$.

4. Conclusions

In this paper, we prove a common fixed point theorem for occasionally weakly compatible mappings in modified intuitionistic fuzzy metric space. Our result is more interesting and useful for other researchers.

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