Annals of Pure and Applied Mathematics
Vol. 14, No. 1, 2017, 45-52
ISSN: 2279-087X (P), 2279-0888(online)
Published on 3 July 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v14n1a6

Annals of
Pure and Applied Mathematics

Some New Results on Sum Divisor Cordial Graphs

***A.Sugumaran*[1] and *K.Rajesh*[2]**

Department of Mathematics, Government Arts College
Tiruvannamalai-606603, Tamilnadu, India.
[1]E-mail:sugumaranaruna@gmail.com
Corresponding author. [2]E-mail:k.rajesh3429@gmail.com

Received 14 May 2017; accepted 29 June 2017

Abstract. A sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1,2,...,|V(G)|\}$ such that each edge uv assigned the label 1 if 2 divides $f(u)+f(v)$ and 0 otherwise. Further, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph are sum divisor cordial graphs.

Keywords: Divisor cordial labeling, sum divisor cordial labeling.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory we refer [4]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc. The total cordial labeling concept is further extended to edge magic total labeling, edge trimagic total labeling, 3-equitable and total magic cordial labeling etc.

Girija and Elumalai [3] have discussed edge magic total labeling of some standard graphs. Jayasekaran and Little flower [5] discussed the edge trimagic total labeling of Mobius ladder, book and dragon graphs. 3-Equitable and total magic cordial labeling of some standard graphs are discussed by Avudainagai et al. [1].

Varatharajan et al. [12] introduced the concept of divisor cordial labeling. Vaidya and shah [11] proved that some star and bistars related graphs are divisor cordial labeling. Deshmukh and Shaikh [10] proved that Tadpole and olive tree graphs are mean cordial graphs. Rokad and Godasara [9] have discussed the Fibonacci cordial labeling of some special graphs. For dynamic survey of various graph labeling, we refer to Gallian [2].

Lourdusamy and Patrick [8] introduced the concept of sum divisor cordial labeling. In this paper we investigate the sum divisor cordial labeling behavior of Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph.

Definition 1.1.[12] Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u)\,|\,f(v)$ or $f(v)\,|\,f(v)$ and the label 0 otherwise. The function f is called a *divisor cordial labeling* if $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a divisor cordial labeling is called a *divisor cordial graph.*

Definition 1.2.[8] Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ be a bijection. For each edge $e=uv$, assign the label 1 if either $2\,|\,(f(u)+f(v))$ and assign the label 0 otherwise. The function f is called a *sum divisor cordial labeling* if $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a sum divisor cordial labeling is called a *sum divisor cordial graph.*

Definition 1.3. [6] Consider four copies of cycle C_{4n} $(n\succ 1)$.
Let $V(C_{4n})=\{v_{i,j}:1\leq i\leq 4;1\leq j\leq 4n\}$. We combine the following vertices by a single vertex as $v_{i,4n}=v_{i+1,1}$ for $i=1,2,3$ and $v_{4,4n}=v_{1,1}$. In each cycle we bend the graph 90 degree towards clockwise at the vertices $v_{i,n+1}$, $v_{i,2n+1}$, $v_{i,2n+2}$, $v_{i,3n+1}$ $(i=1,2,3,4)$, then the resulting graph is in the form of *Swastik* and it is denoted by Sw_n. Obviously, $|V(G)|=16n-4$ and $|E(G)|=16n$.

Definition 1.4. [6] Let G be a graph and let $G_1=G_2=\cdots=G_n=G$, where $n\geq 2$. Then the graph obtained by adding an edge from each G_i to G_{i+1} $(1\leq i\leq n-1)$ is called the *path union* of G.

Definition 1.5. [6] For a cycle C_n, each vertex of C_n is replaced by connected graphs $G_1,G_2,...,G_n$ and is known as *cycle of graphs.* We shall denote it by $C(G_1,G_2,...,G_n)$.

If we replace each vertex of C_n by a graph G, then we denote such *cycle of graphs* as $C(n.G)$.

Definition 1.6. [7] Let C_4 be a cycle with vertex set $\{v_1, v_2, v_3, v_4\}$. The *Jelly fish* graph $J(m,n)$ is obtained by joining v_1 and v_3 by an edge, also appending m pendant edges at v_2 and n pendant edges at v_4.

2. Main results

Theorem 2.1. A Swastik graph Sw_n is a sum divisor cordial graph.

Proof: Let $v_{k,i}$ $(1 \le k \le 4; 1 \le i \le 4n)$ be the vertices of k^{th} copy of C_{4n} in Swastik graph Sw_n, where $v_{k,4n} = v_{k+1,1}$ $(1 \le k \le 3)$ and $v_{4,4n} = v_{1,1}$.

We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v_{1,1}) = f(v_{4,4n}) = 1, \quad f(v_{1,4n}) = f(v_{2,1}) = 2,$

$f(v_{3,1}) = f(v_{2,4n}) = 4, \quad f(v_{4,1}) = f(v_{3,4n}) = 3,$

$f(v_{1,i}) = i + 3; \; i = 2, 3, ..., 4n-1,$

$f(v_{2,i}) = 4n + 1 + i; \; i = 2, 3, ..., 4n-1,$

$f(v_{3,i}) = 2(4n+i) - 2; \; i = 2, 3, ..., 2n,$

$f(v_{3,i}) = 2(2n+i) - 1; \; i = 2n+1, ..., 4n-1,$

$f(v_{4,i}) = 2(6n+i) - 5; \; i = 2, 3, ..., 2n,$

$f(v_{4,i}) = 2(4n+i) - 2; \; i = 2n+1, ..., 4n-1.$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \le 1$.

Hence a Swastik graph Sw_n is a sum divisor cordial graph.

Example 2.1. The sum divisor cordial labeling of Swastik graph Sw_2 is shown in Figure 1.

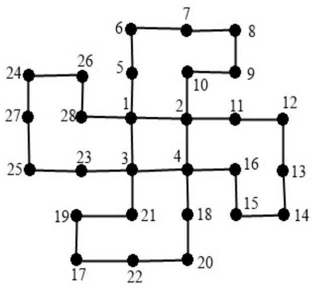

Figure 1: Sum divisor cordial labeling of Swastik graph Sw_2

Theorem 2.2. The path union of finite copies of the Swastik graph Sw_n is a sum divisor cordial graph.

Proof: Let $G = P(k.Sw_n)$ be the path union of k copies for the Swastik graph Sw_n, where $k \geq 2$ is a positive integer. We denote the r^{th} copy of Sw_n by Sw_n^r, $(1 \leq r \leq k)$. Let $V(Sw_n^r) = \{v_{i,j}^r : 1 \leq i \leq 4; 1 \leq j \leq 4n\}$, where $1 \leq r \leq k$. Note that each copy of Sw_n has $p = 16n - 4$ vertices and $q = 16n$ edges. Join the vertices $v_{4,2n+1}^r$ with $v_{4,2n+1}^{r+1}$ for $r = 1, 2, ..., k-1$ by an edge to form the path union of k copies of the Swastik graph. Finally, we observe that $|V(G)| = k(16n-4)$ and $|E(G)| = (k-1) + k.16n$.

We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v_{1,1}^r) = (r-1)p + 1 = f(v_{4,4n}^r)$; $1 \leq r \leq k$,

$f(v_{2,1}^r) = (r-1)p + 2 = f(v_{1,4n}^r)$; $1 \leq r \leq k$,

$f(v_{3,1}^r) = (r-1)p + 4 = f(v_{2,4n}^r)$; $1 \leq r \leq k$,

$f(v_{4,1}^r) = (r-1)p + 3 = f(v_{3,4n}^r)$; $1 \leq r \leq k$,

$f(v_{1,j}^r) = (r-1)p + 1 + j + 2; \quad j = 2, 3, ..., 4n-1,$

$f(v_{2,j}^r) = (r-1)p + 4n + 1 + j; \quad j = 2, 3, ..., 4n-1,$

$f(v_{3,j}^r) = (r-1)p + 2(4n + j) - 2; \quad j = 2, 3, ..., 2n,$

$f(v_{3,j}^r) = (r-1)p + 2(2n + j) + 1; \quad j = 2n+1, ..., 4n-1.$

The vertices of $v_{4,j}^r$ $(1 \leq r \leq k)$ are labeled from the following two cases:

Case 1: When $r \equiv 1 (\text{mod } 4)$ (or) $r \equiv 2 (\text{mod } 4)$.

$f(v_{4,j}^r) = (r-1)p + 2(6n + j) - 4; \quad j = 2, 3, ..., 2n,$

$f(v_{4,j}^r) = (r-1)p + 2(4n + j) - 3; \quad j = 2n+1, ..., 4n-1.$

Case 2: When $r \equiv 0 (\text{mod } 4)$ (or) $r \equiv 3 (\text{mod } 4)$.

$f(v_{4,j}^r) = (r-1)p + 2(4n + j) + 3; \quad j = 2, 3, ..., 2n,$

$f(v_{4,j}^r) = (r-1)p + 2(3n + j) + 2; \quad j = 2n+1, ..., 4n-1.$

From the above labeling pattern, we observe that $e_f(0) = e_f(1)$, when n is odd and $e_f(1) = e_f(0) + 1$, when n is even. Hence $|e_f(0) - e_f(1)| \leq 1$.
Thus, G is a sum divisor cordial graph.

Example 2.2. The sum divisor cordial labeling of path union of 3 copies of Swastik graph Sw_2 is shown in Figure 2.

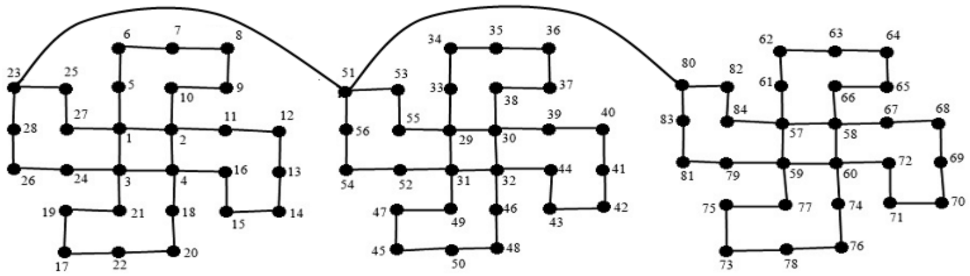

Figure 2: Sum divisor cordial labeling of path union of 3 copies of Swastik graph Sw_2

Corollary 2.2.1. A cycle of k copies of Swastik graph $C(k.Sw_n)$ is a sum divisor cordial graph, where k is odd.

Proof: Let $G = C(k.Sw_n)$ be a cycle of Swastik graph Sw_n, where k is odd positive integer. First we consider the path union of k copies of Sw_n. By Theorem 2.2, the path union of k copies of Sw_n is a sum divisor cordial graph and $e_f(0) = e_f(1)$, since k is odd. Now we obtain the cycle of k copies of Sw_n from the path union of k copies of Sw_n by joining an additional edge connecting the vertices $v^1_{4,2n+1}$ and $v^k_{4,2n+1}$. The label of this particular edge does not affect the condition $|e_f(0) - e_f(1)| \leq 1$. Hence G is a sum divisor cordial graph.

Example 2.2.1. The sum divisor cordial labeling of $C(3.Sw_2)$ is shown in Figure 3.

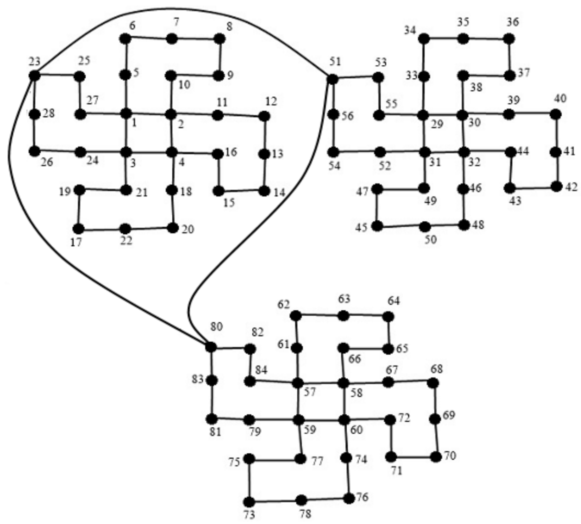

Figure 3: sum divisor cordial labeling of $C(3.Sw_2)$

Corollary 2.2.2. A cycle of k copies of Swastik graph $C(k.Sw_n)$ is a sum divisor cordial graph, where $k \equiv 0 (\text{mod}\, 4)$.

Proof: Let $k = 4m$, for some positive integer m. By Theorem 2.2, the path union of k copies of Sw_n is a sum divisor cordial graph and $e_f(1) = e_f(0)+1$, since k is even. We obtain the cycle of k copies of Sw_n from the path union of Sw_n by joining an additional edge e (say) connecting the vertices $v^1_{4,2n+1}$ and $v^k_{4,2n+1}$. By definition of f (as defined in Theorem 2.2), $f(v^1_{4,2n+1})$ labeled as an odd integer, but $f(v^k_{4,2n+1})$ labeled as an even integer. Hence the additional edge e takes the value 0. Thus in graph G, $e_f(0) = e_f(1) = \frac{1}{2}\left[(k-1)+k.16n\right]$.

Hence, $C(k.Sw_n)$ is a sum divisor cordial graph.

Remark 2.2. The graph $C(k.Sw_n)$ is not a sum divisor cordial graph, if $k = 2(\bmod 4)$. Note that the path union of k copies of Sw_n is a sum divisor cordial graph, and $e_f(1) = e_f(0)+1$, since k is even. To obtain a cycle of k copies of Sw_n, the new edge e (say) is included, which connects the vertices $v^1_{4,2n+1}$ and $v^k_{4,2n+1}$. By definition of f (as defined in Theorem 2.2), the vertex $v^1_{4,2n+1}$ is labeled an odd integer and the vertex $v^k_{4,2n+1}$ is labeled an odd integer. Hence the new edge e is labeled as 1. In this case, the graph $C(k.Sw_n)$ has satisfying $e_f(1) = e_f(0)+2$. Hence $C(k.Sw_n)$ is not a sum divisor cordial graph.

Theorem 2.3. The Jelly fish $J(n,n)$ is a sum divisor cordial labeling for $n \geq 1$.

Proof: Let $G = J(n,n)$. Let $V_1 = \{v_i : 1 \leq i \leq 4\}$ be the vertices of **c**ycle C_4 and let $V_2 = \{p_i, q_i : 1 \leq i \leq n\}$ be the pendant vertices, which are appending at v_2 and v_4 respectively. Then G has $2n+4$ vertices and $2n+5$ edges.

We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v_1) = 1$, $f(v_2) = 3$, $f(v_3) = 2$, $f(v_4) = 4$,

$f(p_i) = i+4$; $i = 1, 2, ..., n$,

$f(q_i) = n+i+4$; $i = 1, 2, ..., n$.

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.

Hence, G is a sum divisor cordial graph.

Example 2.3. The sum divisor cordial labeling of Jelly fish $J(4,4)$ is shown in Figure 4.

Theorem 2.4. Petersen graph is a sum divisor cordial labeling.

Proof: Let u_1, u_2, u_3, u_4, u_5 be the internal vertices and let $u_6, u_7, u_8, u_9, u_{10}$ be the external vertices of Petersen graph such that each u_i is adjacent to u_{i+5}, $1 \leq i \leq 5$.

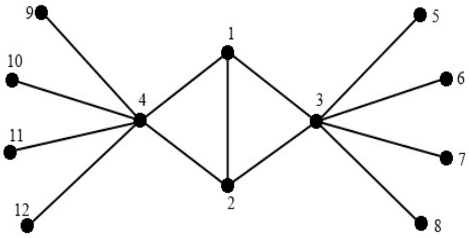

Figure 4: Sum divisor cordial labeling of Jelly fish $J(4,4)$

We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.
$f(u_i) = i$; $1 \le i \le 5$, $f(u_6) = 10$, $f(u_i) = i-1$; $7 \le i \le 10$.
From the above labeling pattern, we have $|e_f(0) - e_f(1)| \le 1$.
Hence Petersen graph is a sum divisor cordial graph.

Example 2.4. The sum divisor cordial labeling of Petersen graph is shown in Figure 5.

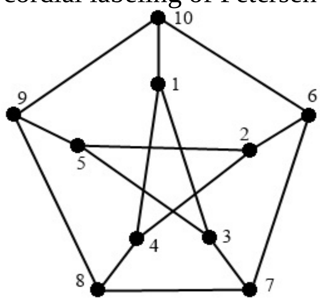

Figure 5: Sum divisor cordial labeling of Petersen graph

3. Conclusion

In this paper, we have proved that Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph are sum divisor cordial graphs.

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