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Generalized Minimal Closed Sets in Bitopological Spaces

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Abstract. In this paper, we introduce and characterize generalized minimal closed sets in bitopological spaces and study some of their properties. A subset A of X is said to be (τ_i, τ_j)- generalized minimal closed (briefly (τ_i, τ_j)- g-m_i closed) set in a bitopological space if τ_j -cl (A) $\subseteq$ U whenever A $\subseteq$U and U is τ_i- minimal open set in (X; τ_1, τ_2).

***Keywords**:* τ_i minimal open set, τ_i-maximal closed set, (τ_i, τ_j)- g- closed set, (τ_i, τ_j)- ω-closed set, (τ_i, τ_j)- g-m_a open set

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1. Introduction and preliminaries

The triple (X; τ_1, τ_2) where X is a set and τ_1 and τ_2 are two topologies on X is a bitopological space. Kelly [5] initiated the systematic study of such spaces. After the work of Kelly [5] various authors [2,3,7,8] turned their attention to generalization of various concepts of topology by considering bitopological spaces. The concept of generalized closed sets in bitopological spaces was introduced and investigated by T [7].

Throughout this chapter (X; τ_1, τ_2) denote non empty bitopological spaces on which no separation axioms are assumed unless otherwise mentioned and the fixed integers i, j $\in$ {1,2}.

We recall the following definitions, which are useful in the sequel.

Definition 1.1. Let i, j $\in$ {1, 2} be fixed integers. In a bitopological space (X; τ_1, τ_2), a subset A of X is said to be

(i) (τ_i, τ_j)- g-closed set [7] if τ_j -cl (A) $\subseteq$ U whenever A $\subseteq$U and U is τ_i- open set.
(ii) (τ_i, τ_j)-g-open set iff A^c is (τ_i, τ_j)- g-closed set.
(iii) (τ_i, τ_j)- ω-closed set [6] if τ_j - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - semi open set in (X, τ).
(iv) (τ_i, τ_j)- ω-open set [6] iff A^c is (τ_i, τ_j)- ω-closed set.

Definition 1.2. Let i, j $\in$ {1, 2} be fixed integers. In a bitopological space (X; τ_1, τ_2), a proper nonempty (τ_i, τ_j)-g-open set A of (X; τ_1, τ_2) is said to be
(i) (τ_i, τ_j)-minimal g-open (resp. (τ_i, τ_j)-minimal g-closed) set if any (τ_i, τ_j) -g-open (respectively (τ_i, τ_j) -g- closed) subset of (X; τ_1, τ_2) which is contained in A, is either A or ϕ.
(ii) (τ_i, τ_j) - maximal g-open (resp. (τ_i, τ_j) - maximal g-closed) set if any (τ_i, τ_j)-g- open (respectively (τ_i, τ_j) –g- closed) subset of (X; τ_1, τ_2) which contains A, is either A or X.

2. Generalized minimal closed sets in bitopological spaces
In this section, we introduce and investigate generalized minimal closed sets in bitopological spaces.
Definition 2.1. Let i, j $\in$ {1, 2} be fixed integers. In a bitopological space (X; τ_1, τ_2), a subset A of X is said to be (τ_i, τ_j)- generalized minimal closed (briefly (τ_i, τ_j)- g-m_i closed) set if τ_j -cl (A) $\subseteq$ U whenever A $\subseteq$U and U is τ_i- minimal open set in (X; τ_1, τ_2).

Remark 2.2. By setting τ_1=τ_2 in the Definition 2.1, a (τ_i, τ_j)- g-m_i closed set is a g-m_i closed set in a topological space.

Theorem 2.3. Let i, j $\in$ {1, 2} be fixed integers. Every (τ_i, τ_j)- g-mi closed set in a bitopological space (X; τ_1, τ_2) is a (τ_i, τ_j)- g- closed set.
Proof: Let A $\subset$X be any (τ_i, τ_j)- g-m_i closed set in (X; τ_1, τ_2). By Definition 2.1 τ_j-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a τ_i- minimal open set. But every minimal open set is an open set. Therefore τ_j-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a τ_i-open set. Hence A is a (τ_i, τ_j)- g-closed set in (X; τ_1, τ_2).

Remark 2.4. Converse of the Theorem 2.3 need not be true.

Example 2.5. Let X= {a, b, c, d} with τ_1= {ϕ, {a}, {a, b}, {c, d}, {a, c, d}, X} and
τ_2 = {ϕ, {a}, {b}, {c},{a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d},{b, c, d}, X}.
(τ_1, τ_2)- g-m_i closed sets: {ϕ, {a}, {c}, {d}, {c, d}}.
(τ_2, τ_1)- g-m_i closed sets: {ϕ, {b}}.
(τ_1, τ_2)-g-closed sets = {ϕ, {a}, {c}, {d},{a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X}.
(τ_2, τ_1)-g-closed sets = {ϕ, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, X}

Theorem 2.6. Let i, j $\in$ {1, 2} be fixed integers. Every (τ_i, τ_j)- g-m_i closed set in a bitopological space (X; τ_1, τ_2) is a (τ_i, τ_j) - ω-closed set.
Proof: Let A $\subset$X be any (τ_i, τ_j)- g-m_i closed set in (X; τ_1, τ_2). By Definition 2.1 τ_j-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a τ_i- minimal open set. But every minimal open set is an open set and hence is a semi open set. Therefore τ_j-cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a τ_i-semi open set. Hence A is a (τ_i, τ_j)- ω-closed set in (X; τ_1, τ_2).

Remark 2.7. Converse of the above Theorem 2.6 need not be true.

Example 2.8. Let X= {a, b, c, d} with τ_1= {ϕ, {b}, {a, b}, {c, d}, {b, c, d}, X} and
τ_2 = {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c},{a, b, d},{b, c, d}, X}.
(τ_1, τ_2)- g-m_i closed sets: {ϕ, {c}, {d}, {c, d}}.
(τ_2, τ_1)- g-m_i closed sets: {ϕ, {a}}.
(τ_1, τ_2)-ω-closed sets = {ϕ, {a}, {c}, {d}, {a, c}, {a, d},{b, c}, {b, d}, {c, d},{a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X}.
(τ_2, τ_1)-ω-closed sets = {ϕ, {a}, {a, b}, {c, d}, {a, c, d}, X}.

Proposition 2.9. Let i, j $\in$ {1, 2} be fixed integers. If A is a τ_j- minimal closed subset of a bitopological space (X; τ_1, τ_2), then A is a (τ_i, τ_j)- g-m_i closed set in (X; τ_1, τ_2).
Proof: Let A $\subseteq$ U, such that U is a τ_i-minimal open set. By hypothesis A is a τ_j- minimal closed subset of (X; τ_1, τ_2), then A is a τ_j – closed subset of (X; τ_1, τ_2), so that τ_j -cl(A)= A. Therefore, τ_j -cl(A)$\subseteq$ A, whenever A $\subseteq$ U and U is a τ_i- minimal open set in (X; τ_1, τ_2). Hence A is a (τ_i, τ_j)- g-m_i closed set in (X; τ_1, τ_2).

Remark 2.10. If $\tau_1 \subset \tau_2$ in (X; τ_1, τ_2) then,(τ_2, τ_1)- g-m_i closed sets $\not\subset$ (τ_1, τ_2)- g-m_i closed sets.

Example 2.11. Let X= {a, b, c, d} with τ_1= {ϕ, {a}, {a, b}, {c, d},{a, c, d}, X} and
τ_2 = {ϕ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, {b, c, d}, X}.
(τ_1, τ_2)- g-m_i closed sets: {ϕ, {a}, {c}, {d}, {c, d}}.
(τ_2, τ_1)- g-m_i closed sets: {ϕ, {b}, {c}, {d}, {c, d}}.

Theorem 2.12. Let i, j $\in$ {1, 2} be fixed integers. If A is a (τ_i, τ_j)- g-mi closed set in a bitopological space (X; τ_1, τ_2) and A $\subseteq$ B $\subseteq$ τ_j -cl(A) then B is a (τ_i, τ_j) - g-m_i closed set in a bitopological space (X; τ_1, τ_2).
Proof: Let B be any set such that B $\subseteq$ U and U is a τ_i- minimal open set in (X; τ_1, τ_2).
Given that A $\subseteq$ B $\subseteq$ τ_j –cl (A) (i)
Since A $\subseteq$ B $\subseteq$ U, then A $\subseteq$ U where U is a τ_i- minimal open set. But A is a (τ_i, τ_j)- g-m_i closed set, by Definition 2.1, τ_j-cl (A) $\subseteq$ U whenever A$\subseteq$U and U is a τ_i-minimal open set in (X; τ_1, τ_2) From (i) A$\subseteq$B $\subseteq$ τ_j–cl (A), implies B $\subseteq$$\tau_j$- cl (A) which implies τ_j-cl (B) $\subseteq$ τ_j- cl (τ_j -cl (A))= τ_j-cl(A). That is τ_j- cl (B) $\subseteq$ τ_j -cl (A). But τ_j-cl (A) $\subseteq$ U. Therefore, τ_j- cl (B) $\subseteq$ U whenever B $\subseteq$ U and U is a τ_i- minimal open set in (X; τ_1, τ_2). Hence B is a (τ_i, τ_j)- g-m_i closed set in (X; τ_1, τ_2).

Theorem 2.13. Let i, j $\in$ {1, 2} be fixed integers. If A is a (τ_i, τ_j)- g-mi closed set in a bitopological space (X; τ_1, τ_2), then τ_j- cl (A) – A contains no nonempty τ_i- maximal closed subset.
Proof: Let F be a τ_i -maximal closed subset of cl (A) – A. Then F^c is a τ_i -minimal open set. Let A be such that A $\subseteq$ F^c where F^c is a τ_i- minimal open set in (X; τ_1, τ_2). Since A is a (τ_i, τ_j)- g-m_i closed set, by the Definition 2.1, τ_j- cl (A) $\subseteq$ F^c whenever A $\subseteq$ F^c and F^c is a τ_i - minimal open set in (X; τ_1, τ_2). So F $\subseteq$ [τ_j- cl (A)]^c.
On the other hand F $\subseteq$ τ_j- cl (A).Therefore F $\subseteq$ [τ_j- cl (A)]^c $\cap$ τ_j- cl (A) = ϕ.

Therefore F = ϕ.

Theorem 2.14. Let i, j $\in$ {1, 2} be fixed integers. If A is a (τ_i, τ_j)- g-m_i closed set in a bitopological space (X; τ_1, τ_2) , then τ_j- cl (A) − A contains no nonempty τ_i - closed subset.

Proof: Let A be a (τ_i, τ_j)- g-m_i closed set in (X; τ_1, τ_2) and F be a nonempty τ_i - closed set contained in τ_j- cl (A)–A. So F $\subseteq$ τ_j- cl (A)–A = τ_j- cl (A) $\bigcap$ A^c. Then F $\subseteq$ τ_j- cl (A) and F $\subseteq$ A^c. Now F $\subseteq$ A^c means A $\subseteq$ F^c where F^c is an open set. Since every (τ_i, τ_j)- g-m_i closed set in a bitopological space (X; τ_1, τ_2) is a (τ_i, τ_j)- g- closed set, A is a (τ_i, τ_j)- g-closed set. Then by the Definition [7], τ_j- cl (A) $\subseteq$ F^c whenever A $\subseteq$ F^c and F^c is an open set in (X; τ_1, τ_2), so that F $\subseteq$ [τ_j- cl (A)]^c On the other hand F $\subseteq$ τ_j- cl (A), so that F $\subseteq$ [τ_j- cl (A)]^c$\bigcap$ τ_j- cl (A) = ϕ. Therefore F = ϕ.

Corollary 2.15. Let i, j $\in$ {1, 2} be fixed integers. A (τ_i, τ_j)- g-m_i closed set A in a bitopological space (X; τ_1, τ_2) is τ_j- closed iff τ_j- cl (A) − A is τ_i - closed .

Proof: Let A be any (τ_i, τ_j)- g-m_i closed set in a bitopological space (X; τ_1, τ_2) which is a τ_j-closed set so that τ_j-cl (A) = A, then τ_j- cl (A)–A = ϕ.Therefore τ_j- cl (A) − A is a τ_i - closed set.

Conversely, let A be any (τ_i, τ_j)- g-m_i closed set in a bitopological space (X; τ_1, τ_2) such that τ_j- cl (A) − A is a τ_i-closed set. Since τ_j- cl (A)–A is a subset of itself and is a τ_i-closed set, by the Theorem 2.14, τ_j- cl (A)-A = ϕ, so that A = τ_j- cl (A). Therefore A is a τ_j- closed set.

Proposition 2.16. Let i, j $\in$ {1, 2} be fixed integers. If A is an τ_i minimal open set and a (τ_i, τ_j)- g- m_i closed set in a bitopological space (X; τ_1, τ_2) then A is a τ_j- closed set.

Proof: Since A $\subseteq$ A and as A is a τ_i -minimal open and a (τ_i, τ_j)- g-m_i closed set, we have cl (A) $\subseteq$ A. Therefore τ_j-cl (A) = A. Hence A is a τ_j-closed set.

Theorem 2.17. Let i, j $\in$ {1, 2} be fixed integers. If A is a (τ_i, τ_j)- g-m_i closed set in a bitopological space (X; τ_1, τ_2), then for each x $\in$ τ_j-cl (A), τ_j-cl {x} $\bigcap$ A $\neq$ ϕ.

Proof: Let A be any (τ_i,τ_j)-g-m_i closed set in a bitopological space (X; τ_1, τ_2), such that for each x $\in$ τ_j-cl (A), τ_j-cl {x} $\bigcap$ A = ϕ and [τ_j-cl{x}]^c is a τ_i-minimal open set. Then A $\subseteq$ [τ_j-cl ({x})]^c where [τ_j-cl ({x})]^c is a τ_i-minimal open set in (X; τ_1, τ_2). But A is a (τ_i,τ_j)-g-m_i closed set. By the Definition 2.1τ_j-cl (A) $\subseteq$ [τ_j-cl ({x})]^c. This is a contradiction to the fact that x $\in$ τ_j-cl (A). Therefore τ_j-cl {x} $\bigcap$ A $\neq$ ϕ.

Lemma 2.18. If Y$\subseteq$X is any subspace of a bitopological space (X;τ_1,τ_2) and U is any τ_i-minimal open set in (X;τ_1,τ_2) then Y $\bigcap$ U is a $\tau_{i\text{-}Y}$ minimal open set.

Proof: Let U be a τ_i-minimal open set in a bitopological space (X;τ_1,τ_2) such that Y $\bigcap$ U is not a $\tau_{i\text{-}Y}$ minimal open set in Y. Then there exists an $\tau_{i\text{-}Y}$ open set G$\neq$ Y in Y such that G $\subseteq$ Y$\bigcap$U where G = Y$\bigcap$H and H is an τ_i-open set in (X;τ_1,τ_2). Now Y$\bigcap$H $\subseteq$ Y$\bigcap$U implies H $\subseteq$U. This contradicts the fact that U is a τ_i-minimal open set. Therefore Y$\cap$U is a $\tau_{i\text{-}Y}$ minimal open set.

Theorem 2.19. Let i, j $\in$ {1, 2} be fixed integers. If A $\subseteq$ Y $\subseteq$ X and A is (τ_i, τ_j)-g-m_i closed set in a bitopological space (X; τ_1, τ_2), then A is (τ_i, τ_j)- g-m_i closed relative to Y.
Proof: Let $\tau_{i\text{-}Y}$ be the restriction of τ_i to Y and O be any τ_i -minimal open set in (X; τ_1, τ_2), then by the Lemma 2.18 Y $\cap$ O is $\tau_{i\text{-}Y}$ minimal open set. Let A $\subseteq$ Y $\cap$ O, where Y $\cap$ O is a $\tau_{i\text{-}Y}$ minimal open set implies A $\subseteq$ O and O is a τ_i -minimal open set in (X; τ_1, τ_2). But A is a (τ_i, τ_j)-g-m_i closed set in (X; τ_1, τ_2). By the Definition 2.1 τ_j- cl (A) $\subseteq$ O whenever A $\subseteq$ O and O is a τ_i-minimal open set in (X; τ_1, τ_2). It follows that Y $\cap$ τ_j-cl (A) $\subseteq$ Y $\cap$ O that is τ_{j-Y} -cl(A)$\subseteq$ Y $\cap$ O whenever A $\subseteq$ Y $\cap$ O and Y $\cap$ O is a τ_{i-Y} minimal open set in Y. Therefore A is (τ_i, τ_j)- g-m_i closed relative to Y.

Lemma 2.20. If A is a τ_i-minimal open set and B is an τ_i-open set in (X;τ_1,τ_2) then either A $\cap$ B=ϕ or A$\cap$ B is a τ_i-minimal open set in (X; τ_1, τ_2).
Proof: Let A be any τ_i-minimal open set and B be an τ_i-open set in (X;τ_1,τ_2) such that A $\cap$ B = ϕ, then there is nothing to prove. But if A $\cap$ B $\neq$ ϕ, then we have to prove that A $\cap$ B is a τ_i-minimal open set in (X;τ_1,τ_2). Now A $\cap$ B $\neq$ ϕ means A $\cap$ B $\subseteq$ A and A $\cap$ B $\subseteq$ B. For A $\cap$ B $\subseteq$ A and since A is a τ_i-minimal open set, by the definition of a minimal open set, either A $\cap$ B = ϕ or A $\cap$ B = A. But A $\cap$ B $\neq\phi$, then A $\cap$ B = A which implies that A $\cap$ B is a τ_i-minimal open set in (X;τ_1,τ_2).

Theorem 2.21. Let i, j $\in$ {1, 2} be fixed integers. If B $\subseteq$ A $\subseteq$ X such that B is (τ_i, τ_j)- g-m_i closed relative to A and that A is an τ_i-open and (τ_i, τ_j)- g-m_i closed set in (X;τ_1,τ_2) then B is a (τ_i, τ_j)- g-m_i closed set in (X;τ_1,τ_2).
Proof: Let B $\subseteq$ U such that U is a τ_i-minimal open set in (X;τ_1,τ_2).Given B $\subseteq$ A $\subseteq$ X, so B $\subseteq$ A $\cap$ U and A is an τ_i-open set in X. Then by the Lemma 2.20 A $\cap$ U is a τ_i-minimal open set in X. Now A $\cap$ U $\subseteq$ A $\subseteq$ X, then A $\cap$ U is a τ_i-minimal open set in A by the Lemma 2.18. Therefore B $\subseteq$ A $\cap$ U and A $\cap$ U is a τ_i-minimal open set in A. By hypothesis A $\cap$ τ_j-cl (B) $\subseteq$ A $\cap$ U implies τ_j- cl (B) $\subseteq$ U. Hence B is a (τ_i, τ_j)- g-m_i closed set in (X;τ_1,τ_2).

Definition 2.22. Let i, j $\in$ {1, 2} be fixed integers. In a bitopological space (X;τ_1,τ_2), a subset A of X is said to be a (τ_i, τ_j)- generalized maximal open (briefly (τ_i, τ_j)- g- m_a open) set iff A^c is a (τ_i, τ_j)- generalized minimal closed set.

Theorem 2.23. Let i, j $\in$ {1, 2} be fixed integers. A subset A of a bitopological space (X;τ_1,τ_2) is a (τ_i, τ_j)-g-m_a open set iff F$\subseteq\tau_j$-int A whenever F $\subseteq$ A and F is a τ_i-maximal closed set in (X;τ_1,τ_2).

Proof: Let A be any (τ_i, τ_j)-g-m_a open in $(X;\tau_1,\tau_2)$ such that $F \subseteq A$ and F is a τ_i-maximal closed set in $(X;\tau_1,\tau_2)$, then by the Definition 2.22 A^c is a (τ_i, τ_j)- g-m_i closed set in $(X;\tau_1,\tau_2)$. That is A^c is a (τ_i, τ_j)- g-m_i closed set whenever $A^c \subseteq F^c$ and F^c is a τ_i-minimal open set. Therefore by the Definition 2.1 τ_j-cl $(A^c) \subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is a τ_i-minimal open set. Then $(\tau_j\text{-int } A)^c \subseteq F^c$, which implies $F \subseteq$ int A. Conversely, let A be any subset of X such that $F \subseteq \tau_j$-int A whenever $F \subseteq A$ and F is a τ_i-maximal closed set in $(X;\tau_1,\tau_2)$. Then $(\tau_j\text{-int } A)^c \subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is a τ_i-minimal open set. We have τ_j-cl $(A^c) \subseteq F^c$ whenever $A^c \subseteq F^c$ and F^c is a τ_i-minimal open set. Therefore by the Definition 2.1 A^c is a (τ_i, τ_j)-g-m_i closed set. Thus A is a (τ_i, τ_j)- g-m_a open set in $(X;\tau_1,\tau_2)$.

Theorem 2.24. Let i, j $\in$ {1, 2} be fixed integers. Every (τ_i, τ_j)- g-m_a open set is a (τ_i, τ_j)-g-open set in a bitopological space $(X;\tau_1,\tau_2)$.
Proof: Let A be a (τ_i, τ_j)- g-m_a open set in $(X;\tau_1,\tau_2)$. Then A^c is a (τ_i, τ_j)-g-m_i closed set and by the theorem 2.3 A^c is a (τ_i, τ_j)-g- closed set. Therefore A is a (τ_i, τ_j)- g-open set in $(X;\tau_1,\tau_2)$.

Remark 2.25. Converse of the Theorem 2.24 need not be true.

Example 2.26. Let X= {a, b, c, d} with τ_1= {ϕ, {a}, {a, b}, {c, d},{a, c, d}, X} and
τ_2 = {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d},{a, b, c}, {a, b, d},{b, c, d}, X}.
(τ_1, τ_2)- g-m_a open sets: {{a, b}, {a, b, c}, {a, b, d}, {b, c, d}, X}.
(τ_2, τ_1)- g-m_a open sets: {{a, c, d}, X}.
(τ_1, τ_2)-g-open sets = {ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c},{b, d}, {c, d}, {a, b, c}, {a, b, d}, {b, c, d}, X}.
(τ_2, τ_1)-g-open sets = {ϕ, {a}, {b}, {a, b}, {c, d}, {a, c, d}, X}

Theorem 2.27. Let i, j $\in$ {1, 2} be fixed integers. Every (τ_i, τ_j)- g-m_a open set is a (τ_i, τ_j)-ω-open set in a bitopological space $(X;\tau_1,\tau_2)$.
Proof: Let A be a (τ_i, τ_j)- g-m_a open set in $(X;\tau_1,\tau_2)$. Then A^c is a (τ_i, τ_j)-g-m_i closed set and by the theorem 2.6 A^c is a (τ_i, τ_j)- ω- closed set. Therefore A is a (τ_i, τ_j)- ω-open set in $(X;\tau_1,\tau_2)$.

Remark 2.28. Converse of the Theorem 2.27 need not be true.

Example 2.29. Let X= {a, b, c, d} with τ_1= {ϕ, {b}, {a, b}, {c, d},{b, c, d}, X} and
τ_2 = {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d},{b, c, d}, X}.
(τ_1, τ_2)- g-m_a open sets: {{a, b}, {a, b, c}, {a, b, d}, X}.
(τ_2, τ_1)- g-m_a open sets: {{b, c, d}, X}.
(τ_1, τ_2)- ω-open sets = {ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c},{b, d}, {a, b,c}, {a, b, d}, {b, c, d}, X}.
(τ_2, τ_1)- ω-open sets = {ϕ, {b}, {a, b}, {c, d}, {b, c, d}, X}.

Theorem 2.30. Let i, j $\in$ {1, 2} be fixed integers. If τ_j- int A $\subseteq$ B $\subseteq$ A and A is a (τ_i, τ_j)-g-m_a open set in a bitopological space (X;τ_1,τ_2), then B is a (τ_i, τ_j)- g-m_a open set in (X;τ_1,τ_2).

Proof: Given τ_j-int A $\subseteq$ B $\subseteq$ A and A is a (τ_i, τ_j)-g-m_a open set in (X;τ_1,τ_2). Then $A^c \subseteq B^c \subseteq (\tau_j\text{-int } A)^c$ and A^c is a (τ_i, τ_j)- g-m_i closed set. That is $A^c \subseteq B^c \subseteq \tau_j\text{-cl}(A^c)$ and A^c is a (τ_i, τ_j)- g-m_i closed set. By the Theorem 2.13 B^c is a (τ_i, τ_j)- g-m_i closed set. Thus by the Definition 2.22 B is a (τ_i, τ_j)- g-m_a open set in (X;τ_1,τ_2).

Theorem 2.31. Let i, j $\in$ {1, 2} be fixed integers. If a set A is any (τ_i, τ_j)- g-m_a open set in a bitopological space (X;τ_1,τ_2),then O = X whenever O is an τ_i-open set and τ_j-int (A) $\bigcup$ A^c $\subseteq$ O.

Proof: Let A be any (τ_i, τ_j)-g-m_a open set in (X;τ_1,τ_2) and O be an τ_i-open set such that τ_j-int (A) $\bigcup$ A^c $\subseteq$ O. Then A^c is a (τ_i, τ_j)-g-m_i closed set and O^c is a τ_i-closed set such that $O^c \subseteq [\tau_j\text{-int}(A) \bigcup A^c]^c = (\tau_j\text{-int } A)^c \bigcap (A^c)^c = \tau_j\text{-cl}(A^c) - A^c$. Since A^c is a (τ_i, τ_j)-g-m_i closed set and O^c is a τ_i-closed set, by the Theorem 2.13 $\tau_j\text{-cl}(A^c) - A^c$ contains no nonempty closed subset, which implies $O^c = \phi$. Hence O = X.

Remark 2.32. Converse of the Theorem 2.32 need not be true.

Example 2.33. In Example 2.26 let A = {a, c}. The only τ_1-open set containing τ_2-int (A) $\bigcup$ A^c is X, but A is not a (τ_1, τ_2)- g- m_a open set.

Theorem 2.34. Let i, j$\in${1, 2} be fixed integers. If A $\subseteq$ Y $\subseteq$ X and A is a (τ_i, τ_j)- g-m_a open set in a bitopological space (X;τ_1,τ_2), then A is a (τ_i, τ_j)- g- m_a open set relative to Y.

Proof: Let $A^c \subseteq Y \bigcap O$ such that Y $\bigcap$ O is $\tau_{i\text{-}Y}$ minimal open set and O is τ_i- minimal open set in Then $A^c \subseteq O$. By hypothesis A^c is a (τ_i, τ_j)-g-m_i closed set. Therefore $\tau_j\text{-cl}(A^c) \subseteq O$, $Y \bigcap \tau_j\text{-cl}(A^c) \subseteq Y \bigcap O$. Hence A^c is (τ_i, τ_j)-g-m_i closed relative to Y which implies A is (τ_i, τ_j)-g-m_a open relative to Y.

Theorem 2.35. Let i, j$\in${1, 2} be fixed integers. If A $\subseteq$ B $\subseteq$ X and A is (τ_i, τ_j)-g- m_a open relative to B and B is (τ_i, τ_j)-g- m_a open set in (X;τ_1,τ_2), then A is (τ_i, τ_j)-g-m_a open relative to Y.

Proof: From [1] it is followed that A is (τ_i, τ_j)-g-m_a open relative to Y.

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