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Optimal Imperfect Production Inventory Model under Joint Effect of Machine Breakdowns and System Reliability

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Abstract: This paper investigates the role of reliability parameter of the imperfect production system with random machine breakdown and stochastic repair time. Here I assume that shifting time of the production system from 'in-control state' to 'out-control state' and the time of machine breakdown are both random. Moreover, the machine repair time is stochastic and independent of the machine break-down rate. To make the research a more realistic one, a relation between reliability parameter and machine breakdown parameter has been established so, that smaller value of reliability parameter indicates long run of production process, lesser imperfect items and negligible number of machine breakdown. Finally, the model is formulated as a profit maximization problem by considering the reliability parameter and production up time as a decision variables. A numerical example is presented to illustrate the impact of profit with different variables in the model. Also, sensitivity analyses are conducted to show how the model reacts to changes in parameters.

Keywords: Imperfect production, machine breakdown, stochastic repair time, reliability of the production system.

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

The classical EMQ model implicitly assumes that the machineries related to production is in perfect condition and all items produced are of perfect quality. Though the production machines have become very sophisticated in the industrial environment, they are not free from deterioration. However, in real manufacturing environments, due to process deterioration or other uncontrollable factors, both production of items of imperfect quality and machine breakdown are inevitable. The EMQ model is one of the most widely used inventory control models when items are in-house produced instead of being purchased from outside suppliers. Research has been carried out to analyse the impact of the random interferences on the EMQ since early 1990s. In the model of [10], demand arrives according to a Poisson process, and production output is also a Poisson process while the facility is producing. The machine's operating time is exponentially distributed and its repair time is distributed arbitrarily. Production operates under an (s, S) policy.

The authors in[7], analyzed a production-inventory model using queuing theory techniques. An unreliable M/G/1 queuing system was considered, and a maintenance process was also introduced in addition to the failure process of the production facilities. For this case, the Laplace transform of the slack capacity was derived. In [4], Groenevelt et al. proposed two production control policies to address the EMQ model with machine failures and they focused on the effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions. The first policy considers that the production of an interrupted lot will not be resumed after a breakdown (i.e., the no resumption or NR policy), while the second assumes that the production of an interrupted lot resumed immediately after production is restored and if the current on-hand inventory is below a certain threshold level (i.e., the abort-resume or AR policy). In the article[8],it was follows that a broad class of production-inventory systems in which a number of producing machines are susceptible to failure following which they must be repaired to make them operative again. The machines' production can also be stopped deliberately due to stocking capacity limitations or any other relevant considerations. In[4], authors has derived some structural properties of the modified lot sizing model with exponential failure. However, they do not give a procedure to determine the optimal lot size. In [6], Chung derived the bounds for the optimal lot size and he obtained some results with these bounds. Kuhn [5] addresses the dynamic lot sizing model with the assumption that the equipment is subject to stochastic breakdowns and considered two different situations. Firstly, after a machine breakdown the setup is totally lost and new setup cost is incurred. Secondly, the cost of resuming the production run after a failure might be substantially lower than the production setup cost. In [11], it was assumed that the number of failures in a production run is Poisson distributed while the repair times are random and can assume any general distribution. Upon failure, the machine will receive immediate service, and the demand will be met from inventory. In the article [20], presented a model of single-machine scheduling problem. The machine is failure-prone and subject to random breakdowns. The processing time is a deterministic sequence that is randomly compressible, which may be from the introduction of new technology or addition of new equipment. In [21,22 ,23], studied the optimal production run time in an EMQ model with imperfect rework and Poisson machine breakdowns under the abort/resume (A/R) control policy. In their proposed system, a random defective rate is assumed and all defective items are reworked at the end of regular production, and there exists a certain percentage of rework failures. The system is subject to random breakdowns and the A/R inventory control policy is adopted when breakdowns occur. Mathematical modeling was used, and theorems related to conditional convexity and bounds of optimal production run times were proposed and proved in their study. A recursive searching algorithm was developed to locate the optimal run time that minimizes the expected production "inventory costs. Some other models concerned with inventory control and machine failure have also been investigated, cf. [1,2,12,15,16,17]. Further effect of stochastic machine breakdown was investigated in [3,13,14]. Again different type of inventory model with different type of demand is studied in [9,18,19].

In this paper, we assume that shifting time of the production system from 'in-control state' to 'out-control state' and the time of machine breakdown are both random. The machine repair time is stochastic and independent of the machine break-down rate. A relation between reliability parameter and machine breakdown parameter has been established so that smaller value of reliability parameter indicates long run of production process, lesser imperfect items and negligible number of machine breakdown. Finally, the model is formulated as a profit maximization problem by considering the reliability parameter and production up time as a decision variables.

2. Assumptions and notations

The following assumptions and notations are adopted for this model.

2.1. Assumptions

(i) The production rate is constant and deterministic.

(ii) From the beginning of the production process, the machinery system is in the 'in-control' state and goes to 'out of control state' when life time of the system is $T = t$, where T follows a probability densityfunction $f(t) = \theta e^{-\theta t}, \theta > 0, 0 < t < \infty$ and produces imperfect quality items .

(iii) Full inspection of the production process is consider. Only the $\beta (o < \beta < 1)$ times of total imperfect quality items are sale at a low price rate(reduction sale). and $(1-\beta)$ times of imperfect quality items are rejected with a certain environmental consciousness cost.

(iv) λ (constant) be the demand rate of perfect quality items throughout the production cycle.

(v) In big-bazaar market or occasional market , with proper advertisement about the product , the demand is exponentially increases with low price rate.So, we consider the demand rate, $\delta = \delta_1 + \delta_2 \, e^{\frac{s_{max}-s_i}{s_i - s_{min}}}$ is for acceptable imperfect quality items. where δ_1 and δ_2 both are positive constant.

(vi) Immediately after the end of production up time, reduction sale of acceptable imperfect items are consider.

(vii) Shortages are not allowed but due to greater machine maintenance time than production down time of perfect item,lost sale will occur.

(viii) In case of machine breakdown ,repair time is stochastic and independent of machine break down.

(ix) For development of the reliability of the production system along with product, we consider production cost related to reliability parameter as $c_p(t,\theta) = c_m + \dfrac{c_d}{\theta p} + \nu p$ .Where c_m and c_d are the material cost and development cost per item per unit time and ν (> 0) is constant.

(x) The time horizon is infinite.

2.2. Notations

p	The production rate is constant and deterministic.
λ	Demand rate of perfect quality items.
δ	Demand rate of acceptable imperfect quality items.
θ	Reliability parameter.
I_p	Inventory of the perfect quality items at time t.
I_d	Inventory of the imperfect quality items at time t.
Q	Total Inventory in the production cycle.
t_1	Time of production stop.
t_p	Time when machine breakdown occur.
t_2	Time of perfect quality items are exhaust.
t_3	Time of imperfect quality items are exhaust.
K	Setup cost per cycle of production system.
c_h	Holding cost per item per unit time.
c_p	Production cost per unit item per unit as a function of reliability parameter.
c_{dd}	Disposal cost per unit item/time.
s_p	Price rate of perfect quality items.
s_i	Price rate(reduced) of acceptable imperfect quality items.
$E(Q)$	Expected inventory holding in the production cycle.
$E(HC)$	Expected inventory holding cost.
$E(P)$	Expected production quantity in the production cycle.
$E(PC)$	Expected production cost.
$E(T)$	Expected duration of the production cycle.
$E(CLS)$	Expected lost sale cost.
$E(DC)$	Expected disposal cost.
$E(TC)$	Expected total cost.
$E(SR)$	Expected sales revenue.
$EP(t_1,\theta)$	Expected profit per unit time.

3. Mathematical formulation of proposed inventory model and discussion

Considering an imperfect production process in which it has two states 'in-control-state' and ' out-of-control-state'.The production process shifted to 'out-of-control' state from 'in -control' state and as a result production of imperfect items aries. Due to long run production process with machine, the system goes to 'out-of-control' state and consider it is as failure , as a result produces more imperfect items than the 'in-control' state.

Let θ be the reliability parameter(design variable) of the machinery system and it is defined as:

$\theta = \dfrac{\text{Number of failure}}{\text{Total number of working hours}}$, that is θ is failure per unit time. For example $\theta = 0.02$, means ,the system goes to ’out of-control’ state one time for a 50 hours working period of production run.Basically θ indicate the hazard rate of the production system , which is combination of random failure and wear-out failure from ’in-control’ state to ’out-of-control’ state.The failure of machinery systems are either due to random failure or wear-out failure, so, reliability of the system can be expressed as $R(t) = R_1(t)R_2(t)$, where $R_1(t)$ and $R_2(t)$ are the reliability due to random failure ,independent of time and reliability due to wear-out . Since the hazard rate due to random failure is independent of time,so, reliability of the system due to random failure with constant hazard rate is $R_1(t) = e^{-\theta t}$. Again by appropriate maintenance policies we can reduce the failure due to wear-out. As a result our study must be concentrate on the failure due to random chance.And therefore corresponding reliability function of the machinery system is defined at time ’t’ as $\mathrm{R(t) = e^{-\theta t}}$, where $\theta > 0, \mathrm{t} > 0$.

$R(t) = 1\text{-}F(t) = 1\text{-}\int_0^t f(t)dt$ .So, $f(t) = \dfrac{dF(t)}{dt} = \theta e^{-\theta t}$, $\theta > 0$ *and* $t > 0$.

Here $f(t)$ and $F(t)$ are the density and distribution function of the life time (i.e, stay in the ’in-control’ state)of the production process .The production process is more reliable if θ is small and due to that the production process may stay long period in the ’in-control’ state. As, a result lesser quantity of imperfect items is produced, which implies θ is the product reliability of the production process it means less imperfect items more reliability of the product. But during long run production process, the percentage of imperfect items increases, in the same time production process shifted to ’out-of-control’ state and causes machinery system of the production process is breakdown.

Since the production of imperfect items and breakdown of the machinery system related to the reliable parameter θ, so, the probability of the imperfect quality of items of total production follow the probability density function $f(t) = \theta e^{-\theta t}$. If p be the production rate ,then the expected rate of imperfect product at time t is $p(1 - e^{-\theta t})$ and consequently the rate of perfect quality item is $pe^{-\theta t}$. In the same time machine breakdown of machinery production system follow the probability density function $\phi(t) = \alpha e^{-\alpha t}$,$\alpha = (1-c)\theta$ or $(1+c)\theta$, $(0 < c < 1)$. When $\alpha = (1-c)\theta$, the mean time of machine breakdown is greater than the mean time of machinery system shifting from ’in-control’ state to ’out-of-control’ state i.e. probably the machine breakdown occur in the ’out-of-control’state.And $\alpha = (1+c)\theta$, indicate the mean time of machine breakdown is lesser than the mean time of machinery system shifting from ’in-control’ state to ’out-of-control’ state i.e. probably the machine is breakdown in the ’in-control’state. Smaller value of θ indicates long run of production process in the ’in-control’ state, lesser imperfect items and negligible number of machine breakdown. The perfect and imperfect quality items are separates by a inspection section. This section select the acceptable quality of imperfect items, β times of the total imperfect quality items which are suitable

for sale at a low price rate(i.e the reduction sale) and $(1-\beta)$ times of imperfect quality items are rejected (scrapped) with environmental consciousness with a certain cost.
For the production cycle, $I_p(t)$ and $I_d(t)$ denotes the inventory level at time t of perfect quality items and imperfect quality items respectively. The inventory levels of perfect quality items increases at a rate $pe^{-\theta t}-\lambda$ and inventory levels of imperfect quality items increase at a rate $p(1-e^{-\theta t})$ up to time $t=t_1$, where t_1 is the production up time. The demand of perfect quality items are meet with a rate λ throughout the production cycle. Immediately after the production run, the acceptable quantity of imperfect items are sale at rate $\delta=\delta_1+\delta_2\ e^{(s_{max}-s_i)/(s_i-s_{min})}$ up to time $t=t_3$, when the imperfect items are exhaust. The behavior of such a unified system is depicted in Figure 1.

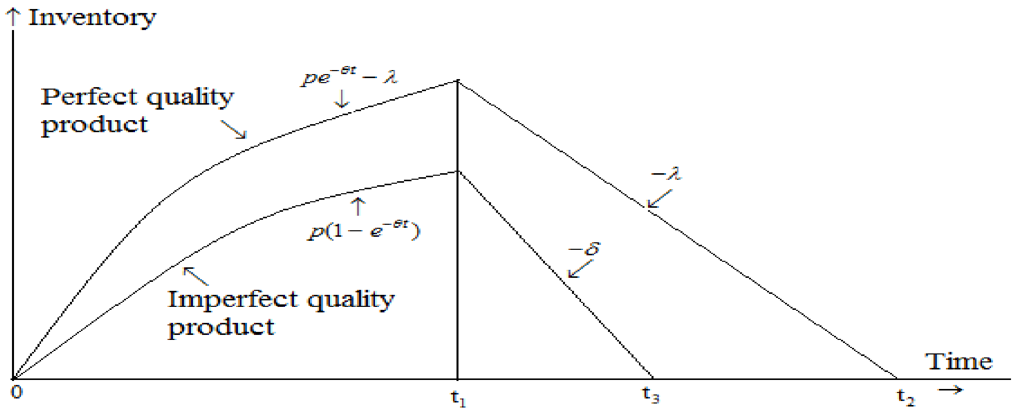

Figure 1: Inventory level of the production system with respect to time.

4. The inventory level are governed by the following differential equations

4.1. Perfect quality inventory

The governing differential equation of the perfect quality iitem is

$$\frac{dI_p(t)}{dt}=\begin{cases} pe^{-\theta t}-\lambda & ;\ 0\le t\le t_1 \\ -\lambda & ;\ t_1\le t\le t_2 \end{cases} \tag{1}$$

with $I_p(0)=0$ and $I_p(t_2)=0$.

4.2. Imperfect quality inventory

The governing differential equation of the imperfect quality item is

$$\frac{dI_d(t)}{dt}=\begin{cases} \beta p(1-e^{-\theta t}) & ;\ 0\le t\le t_1 \\ -\delta & ;\ t_1\le t\le t_3 \end{cases} \tag{2}$$

with $I_d(0)=0$ and $I_d(t_3)=0$.
where β , $(0<\beta<1)$ is the portion of imperfect items ,which are produced during the production run.

From (1) and (2), it follows

$$I_p(t)=\begin{cases}\dfrac{p}{\theta}(1-e^{-\theta t})-\lambda t & ;\ 0\le t\le t_1\\ \dfrac{p}{\theta}(1-e^{-\theta t_1})-\lambda t & ;\ t_1\le t\le t_2\end{cases} \tag{3}$$

$$I_d(t)=\begin{cases}\dfrac{\beta p}{\theta}(e^{-\theta t}+\theta t-1) & ;\ 0\le t\le t_1\\ \dfrac{\beta p}{\theta}(e^{-\theta t_1}+\theta t_1-1)-\delta(t-t_1) & ;\ t_1\le t\le t_3\end{cases} \tag{4}$$

from the boundary condition , $t_2=\dfrac{p}{\theta\lambda}(1-e^{-\theta t_1})$ and $t_3=t_1+\dfrac{p\beta}{\theta\delta}(e^{-\theta t_1}+\theta t_1-1)$

For feasibility, $t_2>t_1$ and $t_3>t_1$ implies $p(1-e^{-\theta t_1})>\theta t_1$ and $\dfrac{p\beta}{\theta}(e^{-\theta t_1}+\theta t_1-1)>0$.

Inventory for the perfect quality items is

$$Q_p=\int_0^{t_1}[\frac{p}{\theta}(1-e^{-\theta t})-\lambda t]\,dt+\int_{t_1}^{t_2}[\frac{p}{\theta}(1-e^{-\theta t_1})-\lambda t]\,dt$$

$$=\frac{p^2}{2\lambda\theta^2}-\frac{p}{\theta^2}+(\frac{p}{\theta^2}-\frac{p^2}{\lambda\theta^2})e^{-\theta t_1}+\frac{pt_1}{\theta}e^{-\theta t_1}+\frac{p^2}{2\lambda\theta^2}e^{-2\theta t_1} \tag{5}$$

Inventory for the imperfect quality items is

$$Q_d=\int_0^{t_1}[\frac{\beta p}{\theta}(e^{-\theta t}+\theta t-1)]\,dt+\int_{t_1}^{t_3}[\frac{\beta p}{\theta}(e^{-\theta t_1}+\theta t_1-1)-\delta(t-t_1)]\,dt$$

$$=(\frac{\beta p}{\theta^2}+\frac{\beta^2p^2}{2\theta^2\delta})+(\frac{\beta p}{2}+\frac{\beta^2p^2}{2\delta})t_1^2-(\frac{\beta p}{\theta}+\frac{\beta^2p^2}{\theta\delta})t_1-(\frac{\beta p}{\theta^2}+\frac{\beta^2p^2}{\theta^2\delta})e^{-\theta t_1}$$

$$+\frac{\beta^2p^2}{\theta\delta}t_1e^{-\theta t_1}+\frac{\beta^2p^2}{2\theta^2\delta}e^{-2\theta t_1} \tag{6}$$

Hence the total inventory is

$$Q=Q_d+Q_p=c_1+c_2t_1+c_3t_1^2+c_4e^{-\theta t_1}+c_5t_1e^{-\theta t_1}+c_6e^{-2\theta t_1} \tag{7}$$

where $c_1=\dfrac{p^2}{2\theta^2}(\dfrac{1}{\lambda}+\dfrac{\beta^2}{\delta})-\dfrac{p(1-\beta)}{\theta^2}$, $c_2=-\dfrac{p\beta}{\theta}(1+\dfrac{\beta p}{\delta})$, $c_3=\dfrac{p\beta}{2}(1+\dfrac{\beta p}{\delta})$

$c_4=\dfrac{p(1-\beta)}{\theta^2}-\dfrac{p^2}{\theta^2}(\dfrac{1}{\lambda}+\dfrac{\beta^2}{\delta})$, $c_5=\dfrac{p}{\theta}(1+\dfrac{\beta^2p}{\delta})$, $c_6=\dfrac{p^2}{2\theta^2}(\dfrac{1}{\lambda}+\dfrac{\beta^2}{\delta})$

4.3.Case 1: Machine breakdown occur and stochastic machine maintenance time exceed production down time

For greater value of reliability parameter , the machinery system of production process breakdown and suppose it is occur at the time point $t=t_p$ and follow the probability density function $f(t_p)=\alpha e^{-\alpha t_p},\alpha>0$, and $t_p>0$.

Then , the Expected inventory holding is

$$Q=\begin{cases} c_1+c_2t_p+c_3t_p^2+c_4e^{-\theta t_p}+c_5t_pe^{-\theta t_p}+c_6e^{-2\theta t_p} & ;\ t_p\le t_1 \\ c_1+c_2t_1+c_3t_1^2+c_4e^{-\theta t_1}+c_5t_1e^{-\theta t_1}+c_6e^{-2\theta t_1} & ;\ t_p\ge t_1 \end{cases} \tag{8}$$

$$E(Q)=\int_0^{t_1}(c_1+c_2t_p+c_3t_p^2+c_4e^{-\theta t_p}+c_5t_pe^{-\theta t_p}+c_6e^{-2\theta t_p})\,f(t_p)\,dt_p$$
$$+\int_{t_1}^{\infty}(c_1+c_2t_1+c_3t_1^2+c_4e^{-\theta t_1}+c_5t_1e^{-\theta t_1}+c_6e^{-2\theta t_1})\,f(t_p)\,dt_p \tag{9}$$

which gives

$$E(Q)=c_1+\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2}+\frac{c_4\alpha}{\theta+\alpha}+\frac{c_5\alpha}{(\theta+\alpha)^2}+\frac{c_6}{2\theta+\alpha}-\frac{2c_3}{\alpha}t_1e^{-\alpha t_1}-(\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2})e^{-\alpha t_1}$$
$$+(\frac{c_4\theta}{\alpha+\theta}-\frac{c_5\alpha}{(\theta+\alpha)^2})e^{-(\theta+\alpha)t_1}+\frac{c_5}{\alpha+\theta}t_1e^{-(\alpha+\theta)t_1}+\frac{2c_6\theta}{\alpha+2\theta}e^{-(2\theta+\alpha)t_1} \tag{10}$$

Hence expected inventory holding cost is

$$E(HC)=c_h[c_1+\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2}+\frac{c_4\alpha}{\theta+\alpha}+\frac{c_5\alpha}{(\theta+\alpha)^2}+\frac{c_6}{2\theta+\alpha}-(\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2})e^{-\alpha t_1}$$
$$-\frac{2c_3}{\alpha}t_1e^{-\alpha t_1}+(\frac{c_4\theta}{\alpha+\theta}-\frac{c_5\alpha}{(\theta+\alpha)^2})e^{-(\theta+\alpha)t_1}+\frac{c_5}{\alpha+\theta}t_1e^{-(\alpha+\theta)t_1}+\frac{2c_6\theta}{\alpha+2\theta}e^{-(2\theta+\alpha)t_1}] \tag{11}$$

Expected production in production run is

$$E(P)=\int_0^{t_1}(pt_p)\alpha e^{-\alpha t_p}\,dt_p\ +\int_{t_1}^{\infty}(pt_1)\alpha e^{-\alpha t_p}\,dt_p=\frac{p}{\alpha}(1-e^{-\alpha t_1}) \tag{12}$$

Expected production cost is

$$E(PC)=c_p\frac{p}{\alpha}(1-e^{-\alpha t_1})=(c_m+\frac{c_d}{p\theta}+\nu p)\frac{p}{\alpha}(1-e^{-\alpha t_1}) \tag{13}$$

4.4. Machine maintenance time is stochastic

When the machine breakdown occur with in the production run time,the corrective maintenance of the machine is required and let machine maintenance time is stochastic which is uniformly distributed in $(0,\mu)$ with density function

$$g(t)=\begin{cases}\frac{1}{\mu} & ;\ 0\le t\le\mu \\ 0 & ;\ elsewhere\end{cases}$$

Therefore Expected duration of the production cycle is

$$E(T)=\int_0^{t_1}t_2\alpha e^{-\alpha t_p}\,dt_p\ +\int_{t_1}^{\infty}t_2\alpha e^{-\alpha t_p}\,dt_p\ +\int_0^{t_1}\int_{t_2}^{\infty}((t-t_2)g(t))\alpha e^{-\alpha t_p}\,dt\ dt_p$$
$$=E(t_2)+\int_0^{t_1}\int_{t_2}^{\infty}(t-t_2)g(t)\alpha e^{-\alpha t_p}\,dt\ dt_p \tag{14}$$

Here t_2 represent the end of the production cycle and 3rd term in the above expression represent the extra time for machine maintenance exceed t_2.

$$E(t_2)=\int_0^{t_1}\frac{p}{\lambda\theta}(1-e^{-\theta t_p})\alpha e^{-\alpha t_p}\,dt_p\ +\int_{t_1}^{\infty}\frac{p}{\lambda\theta}(1-e^{-\theta t_1})\alpha e^{-\alpha t_p}\,dt_p\ =\frac{p}{\lambda(\alpha+\theta)}(1-e^{-(\theta+\alpha)t_1})$$

Again expected extra time for machine repair is

$$E(ET)=\int_0^{t_1}\int_{t=t_2}^{\infty}(t-t_2)g(t)\alpha e^{-\alpha t_p}\,dt\ dt_p$$

$$=\frac{1}{2\mu\lambda^2\theta^2}[(\lambda\mu\theta-p)^2(1-e^{-\alpha t_1})+\frac{p^2\alpha}{2\theta+\alpha}(1-e^{-(2\theta+\alpha)t_1})+\frac{2p\alpha(\lambda\mu\theta-p)}{\theta+\alpha}(1-e^{-(\theta+\alpha)t_1})]$$

Above two relation together gives,

$$E(T)=\frac{1}{2\mu\lambda^2\theta^2}[(\lambda\mu\theta-p)^2(1-e^{-\alpha t_1})+(2p\mu\lambda\theta-\frac{2p^2\alpha}{\theta+\alpha})\ (1-e^{-(\theta+\alpha)t_1}) + \frac{p^2\alpha}{2\theta+\alpha}(1-e^{-(2\theta+\alpha)t_1})] \tag{15}$$

Expected lost sale cost for perfect quality item is

$$E(CLS)=\frac{s_p}{2\mu\lambda\theta^2}[(\lambda\mu\theta-p)^2(1-e^{-\alpha t_1})+(2p\mu\lambda\theta-\frac{2p^2\alpha}{\theta+\alpha})(1-e^{-(\theta+\alpha)t_1}) + \frac{p^2\alpha}{2\theta+\alpha}(1-e^{-(2\theta+\alpha)t_1})] \tag{16}$$

Expected disposal amount of imperfect product in the production process is

$$E(DIS)=(1-\beta)p[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(1-e^{-(\theta+\alpha)t_1})}{\alpha+\theta}] \tag{17}$$

Expected disposal cost for environmental point of view is

$$E(DC)=c_{dd}(1-\beta)p[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(1-e^{-(\theta+\alpha)t_1})}{\alpha+\theta}] \tag{18}$$

Expected total cost $= E(TC)$ = Setup cost(including inspection $\cos(K)+$ Expected Production cost $(E(PC))+$ Expected inventory holding cost $(E(HC))+$ Expected lost sale cost $(E(CLS))+$ Expected disposal cost $(E(DC))$.

$$E(TC)=K+(c_m+\frac{c_d}{p\theta}+\nu p)\frac{p}{\alpha}(1-e^{-\alpha t_1})+c_{dd}(1-\beta)p[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(1-e^{-(\theta+\alpha)t_1})}{\alpha+\theta}]+c_h[c_1+\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2}+\frac{c_4\alpha}{\theta+\alpha}$$
$$+\frac{c_5\alpha}{(\theta+\alpha)^2}+\frac{c_6}{2\theta+\alpha}-(\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2})e^{-\alpha t_1}-\frac{2c_3}{\alpha}t_1e^{-\alpha t_1}+(\frac{c_4\theta}{\alpha+\theta}-\frac{c_5\alpha}{(\theta+\alpha)^2})e^{-(\theta+\alpha)t_1}+\frac{c_5}{\alpha+\theta}t_1e^{-(\alpha+\theta)t_1}+\frac{2c_6\theta}{\alpha+2\theta}$$
$$e^{-(2\theta+\alpha)t_1}]+\frac{s_p}{2\mu\lambda\theta^2}[(\lambda\mu\theta-p)^2(1-e^{-\alpha t_1})+(2p\mu\lambda\theta-\frac{2p^2\alpha}{\theta+\alpha})(1-e^{-(\theta+\alpha)t_1})$$
$$+\frac{p^2\alpha}{2\theta+\alpha}(1-e^{-(2\theta+\alpha)t_1})] \tag{19}$$

4.5. Sales revenue

We consider s_i be the price rate of acceptable imperfect quality items, and s_p be the selling price of perfect item, then sales revenue in the complete cycle is

$$=s_p[\int_0^{t_1}\lambda\,dt\ +\int_{t_1}^{t_2}\lambda\,dt]+s_i\int_{t_1}^{t_3}\delta\,dt=\frac{ps_p}{\theta}(1-e^{-\theta t_1})+s_i\beta p\frac{(e^{-\theta t_1}+\theta t_1-1)}{\theta} \tag{20}$$

If the machine breakdown occurs at $t=t_p$, then expected sales revenue from the complete production cycle is

$$E(SR)=(\int_0^{t_1}\frac{ps_p}{\theta}(1-e^{-\theta t_p})+s_i\beta p\frac{(e^{-\theta t_p}+\theta t_p-1)}{\theta})\alpha e^{-\alpha t_p}\,dt_p\;+(\int_{t_1}^{\infty}\frac{ps_p}{\theta}(1-e^{-\theta t_1})$$

$$+s_i\beta p\frac{(e^{-\theta t_1}+\theta t_1-1)}{\theta})\alpha e^{-\alpha t_p}\,dt_p=\frac{ps_p}{\alpha+\theta}[1-e^{-(\alpha+\theta)t_1}]+s_i\beta p[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(1-e^{-(\theta+\alpha)t_1})}{\alpha+\theta}]$$

Expected profit per unit time $[EP(t_1,\theta)]=$ (Expected sales revenue $-$ Expected total cost) / Expected production cycle

$$EP(t_1,\theta)=\frac{ps_p}{\alpha+\theta}[1-e^{-(\alpha+\theta)t_1}]+s_i\beta p[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(1-e^{-(\theta+\alpha)t_1})}{\alpha+\theta}]-[k+(c_m+\frac{c_d}{p\theta}+\nu p)\frac{p}{\alpha}$$

$$(1-e^{-\alpha t_1})+c_{dd}(1-\beta)p[\frac{(1-e^{-\alpha t_1})}{\alpha}-\frac{(1-e^{-(\theta+\alpha)t_1})}{\alpha+\theta}]+c_h[c_1+\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2}+\frac{c_4\alpha}{\theta+\alpha}+\frac{c_5\alpha}{(\theta+\alpha)^2}$$

$$+\frac{c_6}{2\theta+\alpha}-(\frac{c_2}{\alpha}+\frac{2c_3}{\alpha^2})e^{-\alpha t_1}-\frac{2c_3}{\alpha}t_1e^{-\alpha t_1}+(\frac{c_4\theta}{\alpha+\theta}-\frac{c_5\alpha}{(\theta+\alpha)^2})e^{-(\theta+\alpha)t_1}+\frac{c_5}{\alpha+\theta}t_1e^{-(\alpha+\theta)t_1}$$

$$+\frac{2c_6\theta}{\alpha+2\theta}e^{-(2\theta+\alpha)t_1}]+\frac{s_p}{2\mu\lambda\theta^2}[(\lambda\mu\theta-p)^2(1-e^{-\alpha t_1})+(2p\mu\lambda\theta-\frac{2p^2\alpha}{\theta+\alpha})(1-e^{-(\theta+\alpha)t_1})$$

$$+\frac{p^2\alpha}{2\theta+\alpha}(1-e^{-(2\theta+\alpha)t_1})]]/\frac{1}{2\mu\lambda^2\theta^2}[(\lambda\mu\theta-p)^2(1-e^{-\alpha t_1})+(2p\mu\lambda\theta-\frac{2p^2\alpha}{\theta+\alpha})(1-e^{-(\theta+\alpha)t_1})$$

$$\frac{p^2\alpha}{2\theta+\alpha}(1-e^{-(2\theta+\alpha)t_1})] \tag{21}$$

μ must be greater than t_2, otherwise lost sale does not occur, which gives

$\lambda t_1 < p(1-e^{-\theta t_1}) < \theta\lambda\mu$ (22)

5. Solution

Our objective function (expected profit per unit time) $EP(t_1,\theta)$ is a function of two variable t_1 and θ, when the design parameter of reliability θ is consider as variable. So, our objective is to find the optimal value of t_1 and θ for which the objective function $EP(t_1,\theta)$ is maximum. So that the necessary condition for objective function to be maximised is $\frac{\partial EP(t_1,\theta)}{\partial t_1}=0$ and $\frac{\partial EP(t_1,\theta)}{\partial\theta}=0$ and the sufficient condition $\frac{\partial^2 EP(t_1,\theta)}{\partial t_1^2}<0$ and $\frac{\partial^2 EP(t_1,\theta)}{\partial\theta^2}<0$ and $\frac{\partial^2 EP(t_1,\theta)}{\partial t_1^2}\frac{\partial^2 EP(t_1,\theta)}{\partial\theta^2}-(\frac{\partial^2 EP(t_1,\theta)}{\partial t_1\partial\theta})^2>0$ for $t_1>0$ and $\theta>0$ along with the condition (22) . Now due to highly nonlinearity of $EP(t_1,\theta)$, the first and second derivatives of this with respect to t_1 and θ are very much complicated. Hence it is difficult to determine closed form solution of t_1 and θ for which the objective $EP(t_1,\theta)$ is optimum by analytical method. Using MATHEMATICA, we see that, Expected profit per unit time is concave for $t_1>0$ and $\theta>0$.

6. Numerical examples

Case-1: $(\alpha = (1-c)\theta, 0 < c < 1)$.

The parametric values in the model are as k=10000, p=800, $\lambda = 600$, $\delta_1 = 600$, $\delta_2 = 10$, $s_p = 400$, $s_{max} = 350$, $s_{min} = 200$, $s_i = 250$, $\beta = 0.75$, $\mu = 2$, $c_h = 25$, $c_{dd} = 10$, $c_m = 10$, $c_d = 50$, $\nu = 0.01$, $c = 0.25$. We analyses the effect of expected profit per unit time on optimum production runtime of the production process and on optimum reliability parameter θ.

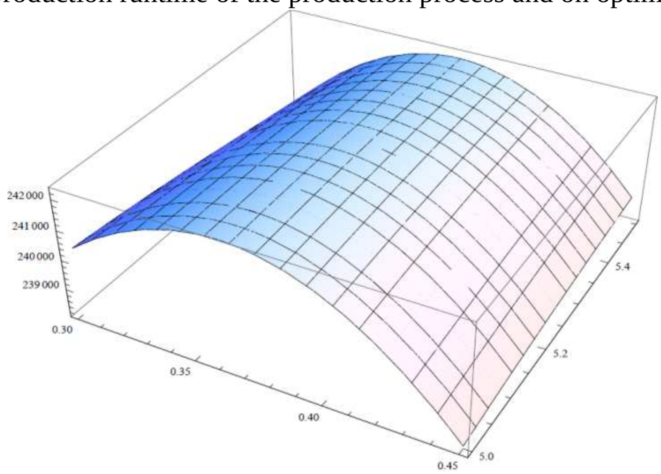

Figure 2: Expected Profit per unit time is concave with respect to both the variable t_1 and θ

Table 1: Expected optimum profit per unit time, optimum production time and Optimum value of θ.

Optimum Production run time(t_1^*)	Optimum Reliability Parameter(θ^*)	Optimum Expected Profit
.26242	0.368111	242311.00

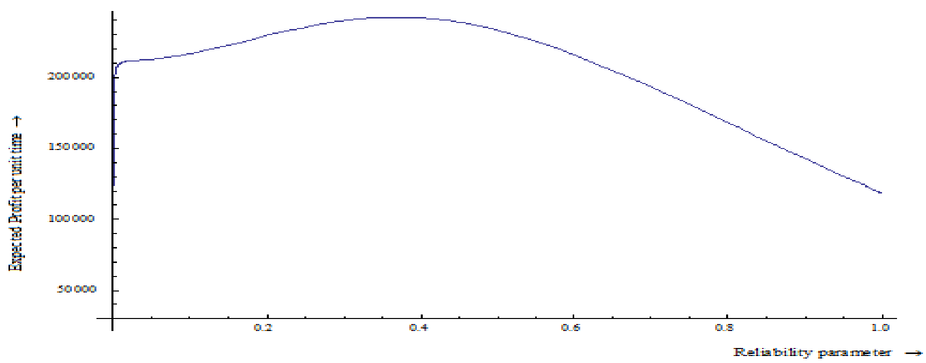

Figure 3: Expected Profit per unit time is decreases with respect to increase of reliability parameter

Table 2: Expected profit per unit time and Production time for changes of reliability parameter.

Reliability Parameter	0.3	0.35	0.36811	0.37	0.4
Production time	4.66876	5.10589	5.26242	5.27853	5.52572
Expected Profit	240599.00	242176.00	242311.00	242310.00	241855.00

Table 3: Expected production cost per item per unit time for changes of reliability parameter.

Reliability Parameter	0.3	0.35	0.36811	0.37	0.4
Expected production cost	42096.2	40899.0	40335.3	40272.7	39189.7

Case-2: $(\alpha = (1+c)\theta,\ \ 0<c<1)$ **,**

For the same parametric values in the model are as k=10000,p=800, λ = 600, δ_1 = 600, δ_2 = 10, s_p = 400, s_{max} = 350, s_{min} = 200, s_i = 250, β = 0.75, μ = 2, c_h = 25, c_{dd} = 10, c_m = 10, c_d = 50, ν = 0.01, c = 0.25. We analyses the effect of expected profit per unit time on optimum production runtime of the production process and on optimum reliability parameter.

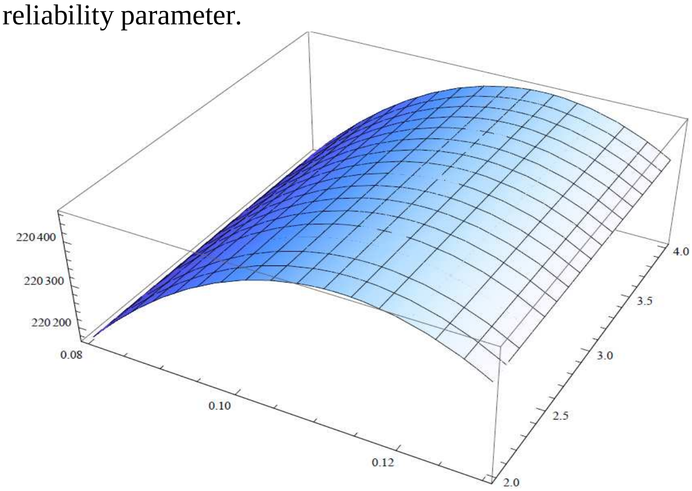

Figure 4: Expected Profit per unit time is concave with respect to both the variable t_1 and θ

Table 4: Expected optimum profit per unit time, optimum production time and optimum value of θ .

Optimum Production run time (t_1^*)	Optimum Reliability Parameter (θ^*)	Optimum Expected Profit
3.01453	0.112945	220462.00

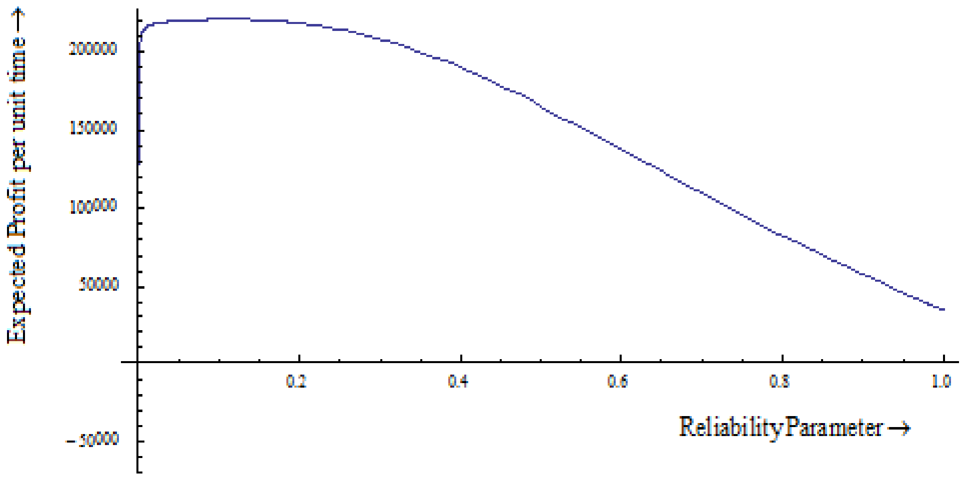

Figure 5: Expected Profit per unit time is decreases with respect to increase of reliability parameter

Table 5: Expected profit per unit time and Production time for changes of reliability parameter.

Reliability Parameter	0.08	0.09	0.112945	0.15	0.2
Production time	2.83391	2.88981	3.01453	3.21872	3.52053
Expected Profit	220293.0	220380.0	220462.0	220236.0	219096.0

Table 6: Expected production cost per item per unit time for changes of reliability parameter.

Reliability Parameter	0.08	0.09	0.112945	0.15	0.2
Expected production cost	37077.9	36896.7	36440.8	35604.5	32969.9

7. Conclusion

In this study, an imperfect production system with random machine breakdown and stochastic repair time are considered. Shifting time of the production system from 'in-control state' to 'out-control state' and the time of machine breakdown are both random. Machine repair time is stochastic and independent of the machine break-down rate. A relation between reliability parameter and machine breakdown parameter has been established and see that smaller value of reliability parameter indicates long run of production process, lesser imperfect items and negligible number of machine breakdown. Finally, the model is formulated as a profit maximization problem and the obtained results can help production planners determine the optimal production run time and optimal value of the reliability parameter. An interesting area for future study would be the effect of variable production rates on this model.

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