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Sum Divisor Cordial Labeling of Theta Graph

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Abstract. A sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, ..., |V|\}$ such that each edge uv assigned the label 1 if 2 divides $f(u) + f(v)$ and 0 otherwise. Further, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that Theta graph T_a, fusion of any two vertices in the cycle of T_a, duplication of any vertex v_i in the cycle of T_a, switching of a central vertex in T_a, path union of two copies of T_a, star of Theta graph are sum divisor cordial graphs.

***Keywords*:** Divisor cordial labeling, sum divisor cordial labeling, fusion, duplication, switching, path union.

***AMS Mathematics Subject Classification (2010)*:** 05C78

1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory we refer to Harary [3]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc.

Varatharajan et al. [11] introduced the concept of divisor cordial labeling. Vaidya and Shah [10] proved that some star and bistar related graphs are divisor cordial labeling. Ujwala Deshmukh and Vahida Y Shaikh [9] proved that Tadpole and olive tree graphs are mean cordial graphs. Rokad and Godasara [7] have discussed the Fibonacci cordial labeling of some special graphs.

For dynamic survey of various graph labeling, we refer to Gallian [1]. Lourdusamy and Patrick [6] introduced the concept of sum divisor cordial labeling. Sugumaran and Rajesh [8] proved that Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph are sum divisor cordial graphs. Ganesan and Balamurugan [2] have discussed the prime labeling of Theta graph. Our primary objective of this paper is to prove that the Theta graph and some graph operations in Theta graph namely fusion, duplication, switching of a central vertex, path union of two copies and the star of Theta graphs are sum divisor cordial graphs.

Definition 1.1. [11] Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u)\,|\,f(v)$ or $f(v)\,|\,f(v)$ and the label 0 otherwise. The function f is called a *divisor cordial labeling* if $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a divisor cordial labeling is called a *divisor cordial graph*.

Definition 1.2. [6] Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G)\rightarrow\{1,2,...,|V(G)|\}$ be a bijection. For each edge $e=uv$, assign the label 1 if either $2\,|\,(f(u)+f(v))$ and assign the label 0 otherwise. The function f is called a *sum divisor cordial labeling* if $|e_f(0)-e_f(1)|\leq 1$. A graph which admits a sum divisor cordial labeling is called a *sum divisor cordial graph*.

Definition 1.3. A *Theta graph* T_a is a block with two non adjacent vertices of degree 3 and all other vertices of degree 2.

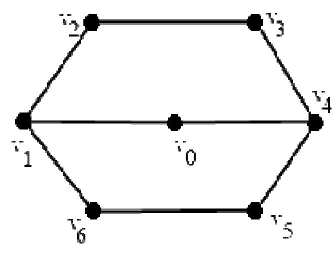

Figure 1: Theta graph

In this paper, we always fix the position of the vertices $v_0, v_1, ..., v_6$ of T_a as indicated in the above figure 1, unless or otherwise specified.

Definition 1.4. Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by *fusing* (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u (or) v in G now incident with x in G_1.

Definition 1.5. *Duplication* of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v'_k with $N(v_k) = N(v'_k)$. In otherwords, a vertex v'_k is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v'_k.

Definition 1.6. A *vertex switching* G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 1.7. [4] Let G be a graph and let $G_1 = G_2 = \cdots = G_n = G$, where $n \geq 2$. Then the graph obtained by adding an edge from each G_i to G_{i+1} $(1 \leq i \leq n-1)$ is called the *path union* of G.

Definition 1.8. [5] Let G be a graph with n vertices. The graph obtained by replacing each vertex of the star $K_{1,n}$ by a copy of G is called a star graph of graph G and it is denoted by G_n^*.

2. Main results

Theorem 2.1. The Theta graph T_a is a sum divisor cordial graph.

Proof: Let $G = T_a$ be a Theta graph with centre v_0. Then $V(G) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E(G) = \{v_i v_{i+1} / 1 \leq i \leq 5\} \cup \{v_0 v_1, v_0 v_4, v_1 v_6\}$. Note that $|V(G)| = 7$ and $|E(G)| = 8$. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v_0) = 3$,

$f(v_i) = i$; $1 \leq i \leq 2$,

$f(v_i) = i+1$; $3 \leq i \leq 6$.

From the above labeling pattern, we have $e_f(0) = e_f(1) = 4$.

Hence $|e_f(0) - e_f(1)| \leq 1$.

Thus G is a sum divisor cordial graph.

Example 2.1. The sum divisor cordial labeling of Theta graph T_a is shown in Figure 2.

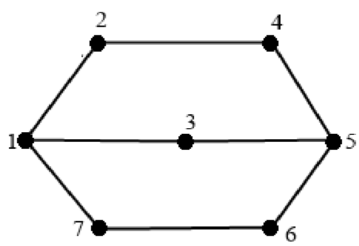

Figure 2: Sum divisor cordial labeling of Theta graph T_a

Theorem 2.2. The fusion of any two vertices in the cycle of T_a is a sum divisor cordial graph.

Proof: Let T_a be a Theta graph with centre v_0. Then $V(T_a)=\{v_0,v_1,v_2,v_3,v_4,v_5,v_6\}$ and $E(T_a)=\{v_iv_{i+1}/1\le i\le 5\}\cup\{v_0v_1,v_0v_4,v_1v_6\}$. Note that $|V(T_a)|=7$ and $|E(T_a)|=8$. Let G be a graph obtained by fusion of two vertices v_2 and v_3 in the cycle of T_a and we call it as vertex v_2 Then $|V(G)|$ = 6 and $|E(G)|$ = 7. We define the vertex labeling $f:V(G)\to\{1,2,...,|V(G)|\}$ as follows.

$f(v_0)=6$, $f(v_6)=3$, $f(v_i)=i$; $i=1,2,4,5$.

From the above labeling pattern, we observe that $e_f(0)=3$ and $e_f(1)=4$.

Hence, $|e_f(0)-e_f(1)|\le 1$.

Thus G is a sum divisor cordial graph.

Example 2.2. The sum divisor cordial labeling of fusion of v_2 and v_3 in T_a is shown in Figure 3.

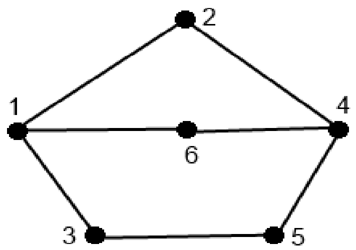

Figure 3: Sum divisor cordial labeling of fusion of v_2 and v_3 in T_a

Theorem 2.3. The duplication of any vertex v_i in the cycle of T_a is a sum divisor cordial graph.

Proof: Let T_a be a Theta graph with centre v_0. Then $V(T_a)=\{v_0,v_1,v_2,v_3,v_4,v_5,v_6\}$ and $E(T_a)=\{v_iv_{i+1}/1\le i\le 5\}\cup\{v_0v_1,v_0v_4,v_1v_6\}$. Note that $|V(T_a)|=7$ and $|E(T_a)|=8$. Let G be a graph obtained from T_a after duplication of the vertex v_i in T_a. Let v_i' be the duplication vertex of v_i in T_a. Clearly $|V(G)|$ = 8 and $|E(G)|$ = 11.

Case 1. Duplication of vertex v_i, where $i=1,2,3,4$ and 6.

We define the vertex labeling $f:V(G)\to\{1,2,...,|V(G)|\}$ as follows.

$f(v_0)=3$, $f(v_i')=8$,

$f(v_i)=i$; $1\le i\le 2$,

$f(v_i)=i+1$; $3\le i\le 6$.

Case 2. Duplication of vertex v_5.

We define the vertex labeling $f:V(G)\to\{1,2,...,|V(G)|\}$ as follows.

$f(v_0)=3$, $f(v_5)=7$, $f(v_5')=8$,

$f(v_i)=i$; $i=1,2,6$,

$f(v_i)=i+1$; $i=3,4$.

From the above two cases, we have $|e_f(0)-e_f(1)|\le 1$.

Hence G is a sum divisor cordial graph.

Example 2.3. The sum divisor cordial labeling of the duplication of the vertex v_1 in T_a is shown in Figure 4.

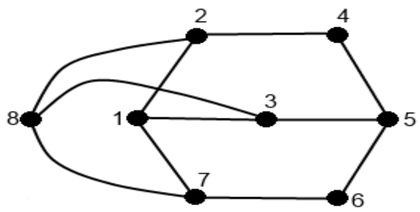

Figure 4: Sum divisor cordial labeling of the duplication of the vertex v_1 in T_a.

Theorem 2.4. The switching of a central vertex in the Theta graph T_a is a sum divisor cordial graph.

Proof: Let T_a be a Theta graph with centre v_0. Then $V(T_a)=\{v_0,v_1,v_2,v_3,v_4,v_5,v_6\}$ and $E(T_a)=\{v_iv_{i+1}/1\le i\le 5\}\cup\{v_0v_1,v_0v_4,v_1v_6\}$. Note that T_a has 7 vertices and 8 edges. Let G be the graph obtained from T_a after switching the central vertex v_0 of T_a. We define the vertex labeling $f:V(G)\to\{1,2,...,|V(G)|\}$ as follows.

$f(v_0)=1$, $f(v_1)=6$, $f(v_6)=7$,

$f(v_i)=i$; $2\le i\le 5$.

From the above labeling pattern, we observe that $e_f(0)=e_f(1)=5$.

Hence $|e_f(0)-e_f(1)|\le 1$.

Thus G is a sum divisor cordial graph.

Example 2.4. The sum divisor cordial labeling of switching of a central vertex v_0 in T_a is shown in Figure 5.

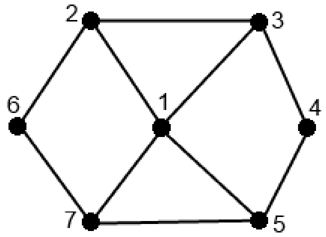

Figure 5: Sum divisor cordial labeling of switching of a central vertex v_0 in T_a.

Theorem 2.5. The graph obtained by path union of two copies of Theta graphs T_a is a sum divisor cordial graph.

Proof: Consider two copies of Theta graphs T_a^1 and T_a^2 respectively.

Then $V(T_a^1)=\{u_0,u_1,u_2,...,u_6\}$ and $E(T_a^1)=\{u_iu_{i+1}/1\le i\le 5\}\cup\{u_0u_1,u_0u_4,u_1u_6\}$.

And $V(T_a^2)=\{v_0,v_1,v_2,...,v_6\}$ and $E(T_a^2)=\{v_iv_{i+1}/1\le i\le 5\}\cup\{v_0v_1,v_0v_4,v_1v_6\}$.

Let G be the graph obtained by the path union of two copies of Theta graph T_a^1 and T_a^2

Then $V(G)=V(T_a^1)\cup V(T_a^2)$ and $E(G)=E(T_a^1)\cup E(T_a^2)\cup\{u_3v_2\}$.

Note that G has 14 vertices and 17 edges.

We define the vertex labeling $f:V(G)\to\{1,2,...,|V(G)|\}$ as follows.

$f(u_0)=3$, $f(v_0)=10$,

$f(u_i)=i$; $i=1,2$,

$f(u_i)=i+1$; $3\le i\le 6$.

$f(v_i)=i+7$; $i=1,2$,

$f(v_i)=i+8$; $3\le i\le 6$.

From the above labeling pattern, we observe that $e_f(0)=9$ and $e_f(1)=8$.

Hence $|e_f(0)-e_f(1)|\le 1$.

Thus G is a sum divisor cordial graph.

Example 2.5. The sum divisor cordial graph of the path union of T_a^1 and T_a^2 is shown in Figure 6.

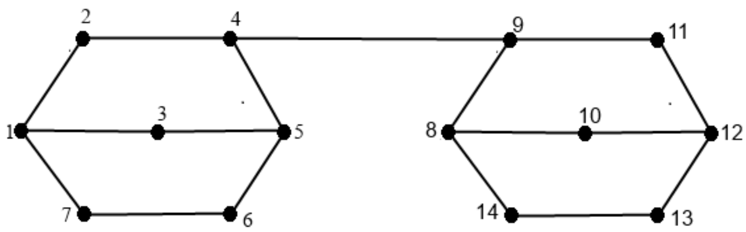

Figure 6: The sum divisor cordial graph of the path union of T_a^1 and T_a^2

Theorem 2.6. The star graph of Theta graph T_a is a sum divisor cordial graph.

Proof: Let $G=T_a^*$ be a star of Theta graph T_a. Then G has 8 copies of Theta graph, namely T_a^i, $i=0,1,2,...,7$. We denote $V(T_a^i)=\{v_0^i,v_1^i,...,v_6^i\}$ and

$$E(T_a^i)=\{v_j^iv_{j+1}^i:1\le j\le 5\}\cup\{v_0^iv_1^i,v_0^iv_4^i,v_1^iv_6^i\}$$

be the vertex and edge sets of T_a^i for each $i=0,1,2,...,7$. Note that G has 56 vertices and 71 edges. We assume that the central copy of $G=T_a^*$ is T_a^0 and the other copies of

$G = T_a^*$ is T_a^k for $1 \le k \le 7$. We define the vertex labeling $f : V(G) \to \{1, 2, ..., 56\}$ as follows.

$f(v_0^0) = 3$,

$f(v_i^0) = i$; $i = 1, 2$,

$f(v_i^0) = i + 1$; $3 \le i \le 6$.

$f(v_0^1) = 10$,

$f(v_i^1) = i + 7$; $i = 1, 2$,

$f(v_i^1) = i + 8$; $3 \le i \le 6$.

$f(v_0^2) = 17$,

$f(v_i^2) = i + 14$; $i = 1, 2$,

$f(v_i^2) = i + 15$; $3 \le i \le 6$.

$f(v_0^3) = 24$,

$f(v_i^3) = i + 21$; $i = 1, 2$,

$f(v_i^3) = i + 22$; $3 \le i \le 6$.

$f(v_0^4) = 31$,

$f(v_i^4) = i + 28$; $i = 1, 2$,

$f(v_i^4) = i + 29$; $3 \le i \le 6$.

$f(v_0^5) = 38$,

$f(v_i^5) = i + 35$; $i = 1, 2$,

$f(v_i^5) = i + 36$; $3 \le i \le 6$.

$f(v_0^6) = 45$,

$f(v_i^6) = i + 42$; $i = 1, 2$,

$f(v_i^6) = i + 43$; $3 \le i \le 6$.

$f(v_0^7) = 52$,

$f(v_i^7) = i + 49$; $i = 1, 2$,

$f(v_i^7) = i + 50$; $3 \le i \le 6$.

From the above labeling pattern, we observe that $e_f(0) = 36$ and $e_f(1) = 35$.

Hence $|e_f(0) - e_f(1)| \le 1$.

Thus G is a sum divisor cordial graph.

Example 2.6. The sum divisor cordial labeling of star of Theta graph is shown in Figure 7.

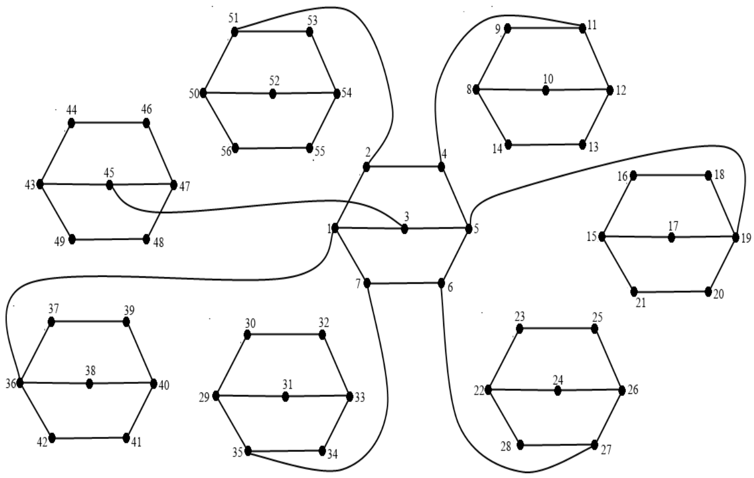

Figure 7: Sum divisor cordial labeling of star of Theta graph

3. Conclusion

In this paper, we investigated the sum divisor cordial graph on a special graph namely Theta graph and proved that Theta graph T_a, fusion of any two vertices in the cycle of T_a, duplication of any vertex v_i in the cycle of T_a, switching of central vertex in T_a, path union of two copies of T_a, star of Theta graphs are sum divisor cordial graphs.

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