Annals of Pure and Applied Mathematics
Vol. 14, No. 2, 2017, 225-229
ISSN: 2279-087X (P), 2279-0888(online)
Published on 30 August 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v14n2a3

Annals of
Pure and Applied Mathematics

On Nano Generalized β Regular Spaces and Nano Generalized β Normal Spaces in Nano Topological Spaces

[1]S.B.Shalini, G. Sindhu and **K.Indirani**

Department of Mathematics, Nirmala College for Women, Coimbatore
Tamil Nadu, India.
[1]Corresponding author. Email: shal.manogar@gmail.com

Received 17 July 2017; accepted 18 August 2017

Abstract. The aim of this paper is to introduce Nano generalized β regular spaces and Nano generalized β normal spaces in Nano topological spaces and we discuss some of its properties.

Keywords: Ng β regular spaces, Ng β normal spaces.

AMS Mathematics Subject Classification (2010): 18B30

1. Introduction

The notion of Nano topology was introduced by Thivagar [4] which was defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and he also defined Nano closed set, Nano interior and Nano closure. Levine [5] introduced generalized closed sets as a generalization of closed sets in topological spaces. Monsef et al. [1] introduced the notion of β-open set in topology, and further investigation of Nano β open sets was given by Gnanambal [3]. Munshi [6] introduced g-regular and g-normal spaces using g-closed sets in topological spaces. In this paper Ng β regular spaces and Ng β normal spaces are introduced and some of its properties are investigated.

2. Preliminaries

Definition 2.1. [4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X)=\{U,\phi,L_R(X),U_R(X),B_R(X)\}$ where $X\subseteq U$. Then it satisfies the following axioms:

i) U and $\phi\in\tau_R(X)$.
ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is called the Nano topology on U with respect to X, $(U,\tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets in U.

Definition 2.2. [4] If $(U,\tau_R(X))$ is a Nano topological space with respect X where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by $N\operatorname{int}(A)$. $N\operatorname{int}(A)$ isthe largest Nano open subset of A.
- The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest Nano closed set containing A.

Definition 2.3. [2] A subset A of $(U,\tau_R(X))$ is called Nano generalized closed set (briefly Ng closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U,\tau_R(X))$.

Definition 2.4. [7] A subset A of Nano topological space $(U,\tau_R(X))$ is called Nano generalized β closed set (briefly Ngβ closed) if $N\beta cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U,\tau_R(X))$.

Definition 2.5. [8] Let $(U,\tau_R(X))$ and $(V,\sigma_{R'}(Y))$ be Nano topological spaces. Then a mapping $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is called Ngβ continuous on U if the inverse image of every Nano open set in V is Ngβ open in U.

Definition 2.6. [8] Let $(U,\tau_R(X))$ and $(V,\sigma_{R'}(Y))$ be Nano topological spaces. Then a mapping $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is called Ngβ irresolute on U if the inverse image of every Ngβ closed set in V is Ngβ closed in U.

Definition 2.7. [8] A map $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is said to be Ngβ closed map on U if the image of every Nano closed set in U is Ngβ closed in V.

Definition 2.8. [8] A map $f:(U,\tau_R(X)) \to (V,\sigma_{R'}(Y))$ is said to be strongly Ngβ closed map on U if the image of every Ngβ closed set in U is Ngβ closed in V.

3. Ngβ regular spaces and Ngβ normal spaces

Definition 3.1. A Nano topological space $(U, \tau_R(X))$ is said to be Ngβ regular space, if for each Nano closed set F and each point $x \notin F$, there exists disjoint Ngβ open sets G and H such that $x \in G$ and $F \subset H$.

Remark 3.2. Every Nano regular space is Ngβ regular space.

Theorem 3.3. If $f : U \to V$ is Nano continuous bijective, Ngβ open function and U is a Nano regular space then V is Ngβ regular.

Proof: Let F be a Nano closed set in V and $y \notin F$. Take $y = f(x)$ for some $x \in U$. Since f is Nano continuous, $f^{-1}(F)$ is Nano closed in U such that $x \notin f^{-1}(F)$. Now U is Nano regular space, there exist disjoint Nano open sets G and H such that $x \in G$ and $f^{-1}(F) \subset H$. That is $y = f(x) \in f(G)$ and $F \subset f(H)$. Since f is Ngβ open function, $f(G), f(H)$ are Ngβ open sets in V and f is bijective, $f(G) \cap f(H) = f(G \cap H) = f(\phi) = \phi$. Therefore V is Ngβ regular space.

Theorem 3.4. If $f : U \to V$ is Nano continuous surjective, strongly Ngβ open function and U is a Ngβ regular then V is also Ngβ regular.

Proof: Let F be a Nano closed set in V and $y \notin F$. Take $y = f(x)$ for some $x \in U$. Since f is Nano continuous surjective, $f^{-1}(F)$ is Nano closed in U such that $x \notin f^{-1}(F)$. Now U is Ngβ regular, there exist disjoint Ngβ open sets G and H such that $x \in G$ and $f^{-1}(F) \subset H$. That is $y = f(x) \in f(G)$ and $F \subset f(H)$. Since f is strongly Ngβ open and surjective, $f(G), f(H)$ are disjoint Ngβ open sets in V. Therefore V is Ngβ regular space.

Theorem 3.5. If $f : U \to V$ is Ngβ continuous, Nano closed injection and V is a Nano regular space then U is Ngβ regular.

Proof: Let F be a Nano closed set in U and $x \notin F$. Since f is Nano closed injection, $f(F)$ is Nano closed set in V such that $f(x) \notin f(F)$. Now V is Nano regular, there exist disjoint Nano open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is Ngβ continuous function, $f^{-1}(G), f^{-1}(H)$ are Ngβ open sets in U. Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$. Hence U is Ngβ regular space.

Theorem 3.6. If $f : U \to V$ is Ngβ irresolute, Nano closed injection and V is a Ngβ regular then U is also Ngβ regular.

Proof: Let F be a Nano closed set in U and $x \notin F$. Since f is Nano closed injection, $f(F)$ is Nano closed set in V such that $f(x) \notin f(F)$. Now V is Nano regular, there exist disjoint Nano open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is Ngβ irresolute, $f^{-1}(G), f^{-1}(H)$ are Ngβ open sets in U. Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$. Hence U is Ngβ regular space.

Definition 3.7. A Nano topological space $(U, \tau_R(X))$ is said to be Ngβ normal space, if for each pair of disjoint Nano closed sets E and F of U, there exists disjoint Ngβ open sets G and H such that $E \subset G$ and $F \subset H$.

Remark 3.8. Every Nano normal space is Ngβ normal space.

Theorem 3.9. If $f : U \to V$ is Nano continuous bijective, Ngβ open function and U is Nano normal space then V is Ngβ normal.

Proof: Let E and F be disjoint Nano closed set in V. Since f is Nano continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint Nano closed in U. Now U is Nano normal space, there exist disjoint Nano open sets G and H such that $f^{-1}(E) \subset G$ and $f^{-1}(F) \subset H$. That is $E \subset f(G)$ and $F \subset f(H)$. Since f is Ngβ open function, $f(G), f(H)$ are Ngβ open sets in V and f is injective, $f(G) \cap f(H) = f(G \cap H) = f(\phi) = \phi$. Therefore V is Ngβ normal space.

Theorem 3.10. If $f : U \to V$ is Ngβ continuous, Nano closed injection and V is a Nano normal space then U is Ngβ normal.

Proof: Let E and F be disjoint Nano closed set in V. Since f is Nano closed injection, $f(E)$ and $f(F)$ are disjoint Nano closed in V. Now V is Nano normal space, there exist disjoint Nano open sets G and H such that $f(E) \subset G$ and $f(F) \subset H$. That is $E \subset f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is Ngβ continuous function, $f^{-1}(G), f^{-1}(H)$ are Ngβ open sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \phi$. Therefore U is Ngβ normal space.

Theorem 3.11. If $f : U \to V$ is Ngβ irresolute, Nano closed injection and V is a Ngβ normal then U is Ngβ normal.

Proof: Let E and F be disjoint Nano closed set in V. Since f is Nano closed injection, $f(E)$ and $f(F)$ are disjoint Nano closed in V. Now V is Ngβ normal space, there exist disjoint Ngβ open sets G and H such that $f(E) \subset G$ and $f(F) \subset H$. That is

$E \subset f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is Ngβ irresolute, $f^{-1}(G), f^{-1}(H)$ are Ngβ open sets in U. Further $f^{-1}(G) \cap f^{-1}(H) = \phi$. Therefore U is Ngβ normal space.

Theorem 3.12. If $f : U \to V$ is Nano continuous bijective, strongly Ngβ open function and U is Ngβ normal then V is also Ngβ normal.

Proof: Let E and F be disjoint Nano closed set in V. Since f is Nano continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint Nano closed in U. Now U is Ngβ normal, there exist disjoint Ngβ open sets G and H such that $f^{-1}(E) \subset G$ and $f^{-1}(F) \subset H$. That is $E \subset f(G)$ and $F \subset f(H)$. Since f is strongly Ngβ open function, $f(G), f(H)$ are Ngβ open sets in V and f is injective, $f(G) \cap f(H) = f(G \cap H) = f(\phi) = \phi$. Therefore V is Ngβ normal space.

4. Conclusion

In this paper, some of the properties of Ngβ regular spaces and Ngβ normal spaces are discussed. This shall be extended in the future research with some applications.

REFERENCES

1. M. E.Abd EL-Monsef, S.N.EL-Deep and R.A.Mahmoud, β-open sets and β-continuous mappings, *Bull. Fac. Sci.*, 12 (1983) 77-90.
2. K.Bhuvaneswari and K.Mythili Gnanapriya, Nano generalised closed sets in nano topological spaces, I*nternational Journal of Scientific and Research Publications*, 4(5) (2014) 1-3.
3. Y.Gnanambal, On Nano β open sets, *Int. Jr. of Engineering*, 1(2) (2015) 1-6.
4. M.L.Thivagar and C.Richard, On Nano forms of weakly open sets, International *Journal of Mathematics and Statistics Invention*, 1(1) (2013) 31 -37.
5. N.Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo*, 19(2) (1970) 89-96.
6. B.M.Munshi, Separation axioms, *Acta Ciencia Indica*, 12 (1986) 140-144.
7. S.B.Shalini and K.Indirani, Characterisation of nano generalized β closed sets in nano topological spaces, *International Journal of Science and Applied Research*, 4(1) (2017) 7-11.
8. S.B.Shalini and K.Indirani, On Nano generalized β continuous functions and Nano generalized β irresolute functions in Nano topological spaces, *IOSR Journal of Mathematics*, 13(1) (2017) 79-86.
9. R.S.Wali and V.Dembre, On pre generalized pre regular weakly open sets and pre generalized pre regular weakly neighbourhoods in topological spaces, *Annals of Pure and Applied Mathematics*, 10(1) (2016) 15-20.