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Topological Indices of the Total Graph of Subdivision Graphs

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Abstract. In this paper, we compute topological indices of the total graphs of the tadpole graphs, wheel graphs and ladder graphs using the subdivision concept, which extend the results of Ranjini et al. (2011).

Keywords: Topological index, line graphs, total graphs, subdivision graphs, tadpole graphs, wheel graphs, ladder graphs

AMS Mathematics Subject Classification (2010): 05C05, 05C07

1. Introduction

Unless stated otherwise all graphs considered here are finite, simple and undirected. Let G be such a graph with vertex (node) set V(G) (|V(G)| = n) and edge set E(G) (|E(G)| = m). The *degree* (or valency) of a vertex v denoted by d_v is the number of edges incident to v. The *line graph*, [7], of the graph G, written L(G), is the simple graph whose vertices are the edges of G, with ef ϵ E(L(G)) when e and f have a common end point in G. An extension of the notion of line graph is the graph valued function known as a *total graph*. The *total graph* [7] of G, denoted by T(G), is the graph whose adjacent vertices corresponds to the union of the set of vertices and edges of G, with two vertices of T(G) is being adjacent if and only if the corresponding element are adjacent or incident in G. The Subdivision graph [7], S(G) is the graph attained from G by replacing each of its edge by a path of length 2. The sum of all the degrees of all the vertices of a graph is equal to twice the total number of its edges is called the *Handshaking Lemma*.

A number can uniquely identify a graph. A *Topological index* is a numeric number invariant under the isomorphism of graphs and associated to a graph which completely describes the topology of the graph. The significance of the topological indices is usually associated with quantitative structure-property and relationship (QSPR) and quantitative structure-activity relationship (QSAR).

The study of topological indices was effectively employed in 1947 in chemistry by Wiener [13]. He introduced a distance – based topological indices called Wiener index.The first degree-based connectivity index for graphs developed by the chemist

Randić under the name "branching index" [9] as $R(G) = [\ \Sigma\ (d_u\ d_v)\ ]^{-1/2}$, where uv is an edge in G.

Later, this index was generalized for any real number α and known as *generalized Randić index*:

$$R_\alpha(G) = \sum_{u \in V(G)} (d_u.d_v)^\alpha. \quad (1)$$

Li and Zhao proposed the *first general Zagreb index* in [6]:

$$M_\alpha(G) = \sum_{u \in V(G)} (d_u)^\alpha. \quad (2)$$

Zhou and Trinajistic introduced the *general sum-connectivity index* [14]:

$$\chi_\alpha(G) = \sum_{u \in V(G)} (d_u + d_v)^\alpha. \quad (3)$$

Estrada et al introduced the *atom-bond connectivity (ABC) index* in [2]. The ABC index of a G is

$$ABC(G) = \sum_{u \in V(G)} [\ (d_u + d_v - 2)]^{1/2}\ [(d_u\ .\ d_v)\ ]^{-1/2} \quad (4)$$

Vukicevic et al proposed the *geometric arithmetic (GA) index* in [12]. The GA index of G is defined as

$$GA(G) = \sum_{u \in V(G)} 2[\ d_u\ .\ d_v\ ]^{1/2}\ [d_u + d_v\ ]^{-1} \quad (5)$$

where, d_u is the degree of each vertex in V(G) and uv every edge in E(G).

Recently in [1] Durgi et al found Zagreb indices of semi total point graphs of some graphs. In [3,4,5], Kulli have introduced several degree based new topological indices and studied the same for several nanostructures.

2. Topological indices of total graphs and line graph of subdivision graphs

In 2011, Ranjini et al. calculated the explicit expressions for the Shultz indices of the subdivision graphs of the tadpole, wheel, helm and ladder graphs [10]. They also studied the Zagreb indices of the line graphs of subdivision graphs of tadpole, wheel and ladder graphs in [11]. Motivated by the results of [8,10,11], we compute the topological indices of total graph of subdivision graphs of tadpole graph, wheel graph and ladder graph.

Table 1: The edge partition of the graph $T(S(W_{n+1}))$.

(d_u,d_v), uv ϵ E(G)	(4,5)	(4,6)	(4,n+2)	(4,2n)	(5,5)	(5,6)	(5,n+2)	(n+2,n+2)	(2n,n+2)
Number of Edges	3n	3n	n	n	4n	3n	n	(n (n-1))/2	n

Table 2: The edge partition of the graph $T(S(L_n))$.

(d_u,d_v) where uv ϵ E(G)	(4,4)	(4,5)	(4,6)	(5,5)	(5,6)
Number of Edges	30	(6n-8)	(6n-12)	(6n-12)	(9n-20)

Table 3: The edge partition of the graph $T(S(T_{n,k}))$.

(d_u,d_v) where uv ϵ E(G)	(2,3)	(2,4)	(3,4)	(4,4)	(4,5)	(4,6)	(5,5)	(5,6)
Number of Edges	1	1	2	(8n+8k-18)	6	3	3	3

Proposition 3.1. Let G be the total graph of subdivision graph of W_{n+1}, n > 4. Then
$R_\alpha(G) = 2^\alpha[n.\ 5^\alpha.\ 3^{\alpha+1} + n^{\alpha+1}(n+2)^\alpha] + 2^{3\alpha}[n.3^{\alpha+1} + n^{\alpha+1}] + 2^{2\alpha}.n.[\ 3.\ 5^\alpha + (n+2)^\alpha] +$
$\quad + n.2^2.5^{2\alpha} + n.5(n+2)^\alpha + (1/2)(n(n-1)(n^2+4n+4)^\alpha)$.
$M_\alpha(G) = n.6^\alpha + n.\ 3.\ 5^\alpha + n^\alpha.\ 2^\alpha + n.\ 2^{2\alpha+1} + n(n+2)^\alpha$.
$\chi_\alpha(G) = n[3(3^{2\alpha}+2^\alpha.5^\alpha+11^\alpha)+(2n+4)^\alpha[(n+6)^\alpha+(1/2)(n-1)]+(n+6)^\alpha+(n+7)^\alpha+(3n+2)^\alpha$
$\quad +2^{\alpha+1}.\ 5^\alpha]$.
$ABC(G) = \sqrt{3}n + (1/2)[\ 3n.\sqrt{(7/5)} + n(\sqrt{(n+4)}/\sqrt{(n+2)}) + n\sqrt{(n-1)} + n\sqrt{(2n+2)}$
$\quad ((n-1)/(n+2))] + ((8n)/5)\sqrt{2} + ((3n)/10)\sqrt{3} + n[(\sqrt{(n+5)}/(\sqrt{(5n+10)} +$
$\quad + (\sqrt{3}n)/(\sqrt{2}n^2+4n)]$.
$GA(G) = n[(4\sqrt{5})/3+(6\sqrt{30})/11+(6\sqrt{6})/5+(1/(n+6)[4\sqrt{(n+2)}+2\sqrt{(5n+10)}]$
$\quad +(1/(2n+4)(n^2-n-2)] + (1/(n+2))(2n\sqrt{2}n + \sqrt{3}n+2)]$.
Proof: In G, there are 7n+1 nodes, among which 3n nodes of valency five, 2n nodes of valency four, n nodes of valency six, n nodes of valency n+2 and one node of valency 2n. Thus by Handshaking lemma the total number of edges in G is $(n^2+33n)/2$. Hence we get the edge partition, based on the valency of the nodes as shown in Table 1, by applying formulas (1)-(5) to the data in Table 1 and by calculation, we obtained the required results.

Corollary 3.1. Let G be the total graph of the subdivision graph of W_{3+1}. Then
$R_\alpha(G) = 2^{3\alpha+2}3^{\alpha+1} + 2.3^2.5^{2\alpha} + 5^\alpha[2^2.\ 3^{\alpha+1} + 3.2^{2\alpha+2}]$

$M_\alpha(G) = 3^\alpha.2^{2\alpha+1} + 2.3[5^\alpha + 2^{2\alpha}]$

$\chi_\alpha(G) = 3.5^\alpha[2^{\alpha+2} + 3.2^{\alpha+1}] + 2^2.\ 3[11^{\alpha+}3^{2\alpha}]$

$ABC(G) = 2^2\sqrt{3} + (6/5)[\sqrt{30} + \sqrt{35} + 6\sqrt{2}]$

$GA(G) = 18 + (24/5)\sqrt{6} + (24/11)\sqrt{30} + (16/3)\sqrt{5}$.

Corollary 3.2. Let G be the total graph of the subdivision graph of W_{4+1}. Then

$R_\alpha(G) = 6^{2\alpha+1} + 2^2(2^2.\ 5^{2\alpha} + 2^{5\alpha}) + 3^\alpha[2^{4\alpha+2} + 2^{3\alpha+4} + 5^\alpha(2^{\alpha+4} + 6^{2\alpha+2})]$
$M_\alpha(G) = 2^{3\alpha} + 2^2[3.\ 2^{\alpha+1} + 3.\ 5^\alpha + 2^{2\alpha+1}]$
$\chi_\alpha(G) = 5^\alpha.2^{\alpha+5} + 7^\alpha.\ 2^{\alpha+2} + 11^\alpha.2^4 + 3^\alpha[3.2^{2\alpha+1} + 2^{2\alpha+2} + 2^2.\ 3^{\alpha+1}]$
$ABC(G) = 2 + \sqrt{5} + (8/3)[\sqrt{10} + 2\sqrt{3}] + (2/5)[4\sqrt{30} + 3\sqrt{35} + 16\sqrt{2}]$
$GA(G) = 22 + (2/3)[\sqrt{32} + 4\sqrt{20}] + (16/5)\sqrt{24} + (4/7)\sqrt{48} + (32/11)\sqrt{30}$.

Proposition 3.2. Let G be the total graph of subdivision graph of L_n. Then
$R_\alpha(G) = 3.5.2^{4\alpha+1} + 5^{2\alpha}[9n-20] + 5^{\alpha.}.\ 2^{2\alpha+1\,[}3n-4] + 2^{3\alpha+2}\ 3^{\alpha+1}[n-2]$.
$M_\alpha(G) = 2^{2\alpha}[3n+10] + [n-2][2.3.\ 5^\alpha + 3^\alpha.2^{\alpha+1}]$.
$\chi_\alpha(G) = 2(3n-4)[3^{2\alpha}+11^\alpha] + 5^\alpha[2^\alpha(9n-20) + 2^{\alpha+1}(3n-6)] + 3.5.\ 2^{3\alpha+1.}$.
$ABC(G) = \sqrt{3}(2n-4) + (15/2)\sqrt{6} + (1/5)[\ 2\sqrt{2}(9n-2) + (3n-4)\sqrt{35} + (3n-6)\sqrt{30}]$.
$GA(G) = 30 + (9n-20) + (4/9)\sqrt{5}(6n-8) + (4/5)\sqrt{6}(3n-4) + (4/11)\sqrt{30}(3n-4)$.
Proof: In G, there are 11n-6 nodes, among which 3n+10 nodes of valency four, 6n-12 nodes of valency five and 2n-4 nodes of valency six. Thus by, Handshaking lemma there are 27n-22 edges in G. Hence we get the edge partition, based on the valency of the nodes as shown in Table 2, by applying formulas (1) - (5) to the data in Table 2 and by calculation, we obtained the required results**.**

Corollary 3.3. Let G be the total graph of subdivision graph of L_2. Then
$R_\alpha(G) = 2^{4\alpha+4}$
$M_\alpha(G) = 2^{2\alpha+4}$
$\chi_\alpha(G) = 2^{3\alpha+4}$
$ABC(G) = 4\sqrt{6}$
$GA(G) = 64.$

Proposition 3.3. Let G be the total graph of subdivision graph of $T_{n,k}$. Then
$R_\alpha(G) = 2^\alpha[3^\alpha + 3^{\alpha+1}.\ 5^\alpha]+\ 2^{3\alpha}[3^{\alpha+1} +1] + 2^{2\alpha+1}[3^\alpha + 3.\ 5^\alpha] + 2^{4\alpha+1}[4n+4k-9]+ 3.5^{2\alpha}$
$M_\alpha(G) = 2^\alpha + 3.5^\alpha + 3^\alpha[1+2^\alpha]+ 2^{2\alpha+1}[2n+2k-3]$
$\chi_\alpha(G) = 3.11^\alpha+5^\alpha + 2\ [3^\alpha+7^\alpha +3^{2\alpha+1}\ ] + 3.\ 5^\alpha\ .2^{\alpha+1} + 2^{3\alpha+1}[4n+4k-9]$
$ABC(G) = \sqrt{2}+\sqrt{3}+(\sqrt{5}\sqrt{3})/3\ + (\sqrt{6}/2)[4n+4k-9]+ (1/5)\ [3\sqrt{35}+6\sqrt{2}+(3/2)\sqrt{30}]$
$GA(G) = 3+2[4n+4k-9] + (2/3)(\sqrt{2}+4\sqrt{5})+(2/5)\ (\sqrt{6}+6) +(1/7)(2^3\sqrt{3})+(6/11)\ (\sqrt{30})$
Proof: In G, there are (4n+4k) nodes, among which 4n+4k-9 nodes of valency four, three nodes of valency five, one vertex of valency two, one node of valency three and a node of valency six. Thus by Handshaking lemma there are (8n+8k+1) edges in G. Hence we get the edge partition, based on the valency of the nodes as shown in Table 3, by applying formulas (1) - (5) to the data in Table 3 and by calculation, we obtained the required results.

3. Conclusion

In this paper, we compute some degree based topological indices such as generalized Randić index, first general Zagreb index, geometric arithmetic index, atom-bond connectivity index and general sum-connectivity index of the total graph of the tadpole graphs, wheel graphs and ladder graphs using the subdivision concept.

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