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Cone S-Metric Space and Fixed Point Theorems of Contractive Mappings

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Abstract. In this paper, we discuss the concept of generalized S-Metric space of Cone S-Metric space and prove some contractive conditions of unique fixed point.

Keywords: Common fixed point, cone S metric space, cone normed space cone C –class function.

AMS Mathematics Subject Classification (2010): 47H10, 54H25

1. Introduction and mathematical preliminaries

In 2007, Huang and Zhang [10] introduced the concept of cone metric spaces and fixed point theorems of contraction mappings; Any mapping T of a complete cone metric space X into itself that satisfies, for some $0 \leq k < 1$, the inequality $d(Tx, Ty) \leq kd(x, y)$ $\forall x, y \in X$ has a unique fixed point. In 2012, Sedghi et al. [6] introduced the concept of generalization of fixed point theorems in S-metric spaces. Rahman and Sarwar [7] are discussed in fixed point results of Altman integral type mappings in S-metric space. In recently, Ozgur and Tas [8] are discuss new contractive conditions of integral type on complete S-metric spaces. In 2017, Gholidahneh, et al. [12] are introduce the notion of integral type contractive mapping with respect to ordered s-metric spaces and coupled common fixed point theorems of integral type contraction.

In this paper, we discuss the concept of cone S metric space for some contraction of fixed point theorems.

Definition1.1. [10] Let E be a Banach space. A subset P of E is called a cone if and only if:

1. P is closed, nonempty and $P \neq 0$
2. $ax + by \in P$ for all $x, y \in P$ and nonnegative real numbers a, b
3. $P \cap (-P) = \{0\}$.

Given a cone $P \subset E$, we define a partial ordering $\preccurlyeq$ with respect to P by $x \preccurlyeq y$ if and only if $y - x \in P$. We will write $x < y$ to indicate that $x \preccurlyeq y$ but $x \neq y$, while x, y

will stand for $y - x \in intP$, where $intP$ denotes the interior of P. The cone P is called normal if there is a number $K > 0$ such that $0 \preccurlyeq x \preccurlyeq y$ implies $||x|| \preccurlyeq K\,||y|$ for all $x, y \in E$. The least positive number satisfying the above is called the normal constant.

The cone P is called regular if every increasing sequence which is bounded from above is convergent. That is, if $\{x_n\}$ is sequence such that $x_1 \preccurlyeq x_2 \preccurlyeq \cdots \preccurlyeq x_n \cdots \preccurlyeq y$for some $y \in E$, then there is $x \in E$ such that $\| x_n - x \| \to 0$ as $n \to 0$. Equivalently the cone P is regular if and only if every decreasing sequence which is bounded from below is convergent. It is well known that a regular cone is a normal cone. Suppose E is a Banach space, P is a cone in E with $intP \neq 0$ and $\preccurlyeq$is partial ordering with respect to P.

Example 1.2. [4] Let $K > 1$ be given. Consider the real vector space with

$$E = \{ax + b: a, b \in R; x \in [1 - \frac{1}{k}, 1]\}$$

with supremum norm and the cone

$$P = \{ax + b : a \geq 0, b \geq 0\}$$

in E. The cone P is regular and so normal.

Definition 1.3. [10] Let X be a nonempty set. Suppose the mapping $d: X \times X \to E$ satisfies

1. $d(x, y) \geq 0$, and $d(x, y) = 0$ if and only if $x = y$ $\forall x, y \in X$,
2. $d(x, y) = d(y, x)$, $\forall x, y \in X$,
3. $d(x, y) \leq d(x, z) + d(z, y)$, $\forall x, y, z \in X$,

Then (X, d) is called a cone metric space simply CMS.

Lemma 1.4. [11] Every regular cone is normal.

Example 1.5. [10] Let $E = R^2$

$$P = \{(x, y): x, y \geq 0\}$$

$X = R$ and $d: X \times X \to E$ such that

$$d(x, y) = (|\, x - y \,|, \alpha \mid x - y \mid)$$

where $\alpha \geq 0$ is a constant. Then(X, d) is a Cone metric space.

Definition 1.6. [9] Let $X \neq \emptyset$ be any set and $S: X \times X \times X \to [0, \infty)$ be a function satisfying the following conditions for all $u, v, z, a \in X$.

1. $S(u, v, z) \geq 0$
2. $S(u, v, z) = 0$ if and only if $u = v = z$.
3. $S(u, v, z) \leq S(u, u, a) + S(v, v, a) + S(z, z, a)$

Then the function S is called an S-metric on X and the pair (X, S) is called an S-metric space simply SMS.

Example 1.7. [12] Let X be a non empty set, d is ordinary metric space on X , then $S(x, y, z) = d(x, z) + d(y, z)$ is an S- metric on X.

Definition 1.8. Suppose that E is a real Banach space, then P is a cone in E with $intP \neq \emptyset$, and $\preccurlyeq$is partial ordering with respect to P. Let X be a nonempty set, a function

$d: X \times X \times X \to E$ is called a cone S metric on X if it satisfies the following conditions with

1. $S(u,v,z) \succcurlyeq 0$
2. $S(u,v,z) = 0$ if and only if $u = v = z$.
3. $S(u,v,z) \preccurlyeq S(u,u,a) + S(v,v,a) + S(z,z,a)$

Then the function S is called an cone S-metric on X and the pair (X,S) is called an cone S-metric space simply CSMS.

Example 1.9. Let $E = R^2$, $P = \{(x,y): x,y \succcurlyeq 0\}$, $X = R$ and $d: X \times X \times X \to E$ such that then $S(x,y,z) = (d(x,z) + d(y,z), \alpha(d(x,z) + d(y,z))), (\alpha \succ 0)$ is an cone S-metric on X.

Lemma 1.10. Let (X,S) be an cone S-metric space. Then we have $S(u,u,v) = S(v,v,u)$.

Definition 1.11. Let (X,S) be an cone S-metric space .

1. A sequence $\{u_n\}$ in X converges to u if and only if $S(u_n,u_n,u) \to 0$ as $n \to \infty$. That is, there exists $n_0 \in N$ such that for all $n \succcurlyeq n_0, S(u_n,u_n,u) \ll c$ for each $c \in E, 0 \ll c$. We denote this by $\lim_{n\to\infty} u_n = u$ or $\lim_{n\to\infty} S(u_n,u_n,u) = 0$.
2. A sequence $\{u_n\}$ in X is called a Cauchy sequence if $S(u_n,u_n,u_m) \to 0$ as $n,m \to \infty$. That is, there exists $n_0 \in N$ such that for all $n,m \succcurlyeq n_0$, $S(u_n,u_n,u_m) \ll c$ for each $c \in E, 0 \ll c$.
3. The cone S-metric space (X,S) is called complete if every Cauchy sequence is convergent.

In the following lemma we see the relationship between a cone metric and an cone S-metric.

Lemma 1.12. Let (X,d) be a cone metric space. Then the following properties are satisfied:

1. $S(u,v,z) = d(u,z) + d(v,z)$ for all $u,v,z \in X$ is an cone S-metric on X.
2. $u_n \to u$ in $\{X,d\}$ if and only if $u_n \to u$ in (X,S_d) :
3. $\{u_n\}$ is Cauchy in $\{X,d\}$ if and only if $\{u_n\}$ is Cauchy in (X,S_d) :
4. $\{X,d\}$ is complete if and only if (X,S_d) is complete.

2. Main result

Theorem 2.1. Let (X,S) be a complete cone S-metric space and P be a normal cone with normal constant K. Suppose the mapping $T: X \to X$ satisfies the following conditions:

$$S(Tu,Tu,Tv) \preccurlyeq h\ S(u,u,v) \tag{2.1}$$

for all $u,v \in X$. Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$

Proof: Let $u_0 \in X$ and the sequence $\{u_n\}$ be defined as $T^n u_0 = u_n$. Suppose that $u_n \neq u_{n+1}$ for all n. Using the inequality (2.1), we obtain

$$S(u_n,u_n,u_{n+1}) \preccurlyeq h\, S(u_{n-1},u_{n-1},u_n) \preccurlyeq \cdots \preccurlyeq h^n\ S(u_0,u_0,u_1) \tag{2.2}$$

If we take limit for $n \to \infty$, using the inequality (2.2) we get

$$\lim_{n\to\infty} S(u_n,u_n,u_{n+1}) = 0,$$

Since $h \in (0,1)$. The $\varepsilon \succ 0$ implies $\lim_{n\to\infty} S(u_n, u_n, u_{n+1}) = 0$.

Now we show that the sequence $\{u_n\}$ is a Cauchy sequence.Assume that $\{u_n\}$ is not Cauchy. Then there exists an $0 \ll \varepsilon$ and subsequences $\{m_k\}$ and $\{n_k\}$ such that $m_k \prec n_k \prec m_{k+1}$ with

$$S\left(u_{m_k}, u_{m_k}, u_{n_k}\right) \succcurlyeq \epsilon \tag{2.3}$$

And

$$S(u_{m_k}, u_{m_k}, u_{n_{k-1}}) \prec \varepsilon \tag{2.4}$$

Hence using Lemma (1.10), we have

$$S\left(u_{m_{k-1}}, u_{m_{k-1}}, u_{n_{k-1}}\right) \preccurlyeq 2S\left(u_{m_{k-1}}, u_{m_{k-1}}, u_{m_k}\right) + S\left(u_{n_{k-1}}, u_{n_{k-1}}, u_{m_k}\right)$$
$$\prec 2\, S(u_{m_{k-1}}, u_{m_{k-1}}, u_{m_k}) + \epsilon$$

And

$$lim_{k\to\infty} S\left(u_{m_k} - 1, u_{m_k} - 1, u_{m_k} - 1\right) \preccurlyeq \epsilon \tag{2.5}$$

Using the inequalities (2.1), (2.3) and (2.5), we obtain

$$\epsilon \preccurlyeq S\left(u_{m_k}, u_{m_k}, u_{n_k}\right) \preccurlyeq S\left(u_{m_k} - 1, u_{m_k} - 1, u_{n_k} - 1\right) \preccurlyeq h\,\epsilon$$

which is a contradiction with our assumption since $h \in (0,1)$. So the sequence $\{u_n\}$ is Cauchy. Using the completeness hypothesis, there exists $w \in X$ such that $\lim_{n\to\infty} T^n u_0 = w$.

From the inequality (2.1) we find

$$S(Tw, Tw, u_n + 1) = S(Tw, Tw, u_n) \preccurlyeq h\ S(w, w, u_n)$$

Therefore, $\lim_{n\to 0} \left\| S(Tw, Tw, u_n + 1) \right\| \preccurlyeq h\, K \left\| S(Tw, Tw, w) \right\|$

$$\lim_{n\to 0} \left\| S(Tw, Tw, w) \right\| \preccurlyeq h\, K \left\| S(Tw, Tw, w) \right\|$$

Since $h \in (0,1)$, then $S(Tw, Tw, w) = 0$ which is implies that $S(Tw, Tw, w) \ll 0$ thus $Tw = w$.

To prove T has unique fixed point.

Let w, w_1 be two fixed points of T such that $w \neq w_1$. Taking $u = w$ and $v = w_1$ in equation (2.1) we have

$$S(w, w, w_1) = S(Tw, Tw, w_1) \preccurlyeq h\, S(w, w, w_1)$$

where $h \in (0,1)$, which is implies

$$S(w, w, w_1) = 0$$

Thus $w = w_1$.

Corollary 2.2. Let (X, S) be a complete cone S-metric space and P be a normal cone with normal constant K. Suppose the mapping $T: X \to X$ satisfies the following conditions:

$$S(Tu, Tu, Tv) \preccurlyeq h\ S(u, u, v) \tag{2.6}$$

for all $u, v \in X$. Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$.

Theorem 2.3. Let (X, S) be a complete cone S-metric space and P be a normal cone with normal constant K. Suppose the mapping $T: X \to X$ satisfies the following conditions:

$$S(Tu, Tu, Tv) \preccurlyeq h_1 S(u, u, v) + h_2 S(Tu, Tu, v) + h_3 S(Tv, Tv, u) + h_4 max\{S(Tu, Tu, v), S(Tv, Tv, u)\} \tag{2.7}$$

for all $u, v \in X$ with non negative real numbers $h_i (i \in \{1,2,3,4\}$ satisfyingmax$\{h_1 + 3h_3 + 2h_4, h_1 + h_2 + h_3\} \prec 1$.

Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$.

Proof: Let $u_0 \in X$ and the sequence $\{u_n\}$ be defined as $\lim_{n\to\infty} T^n u_0 = u_n$ Suppose that $u_n \neq u_{n+1}$ for all n. Using the inequality (2.7), the condition (S3) and Lemma (1.10), we get

$$S(u_n, u_n, u_{n+1}) \; = \; S(Tu_{n-1}, Tu_{n-1}, Tu_n)$$
$$\preccurlyeq \; h_1 S(u_{n-1}, u_{n-1}, u_n) + h_2 S(u_n, u_n, u_n) + h_3 S(u_{n+1}, u_{n+1}, u_{n-1})$$
$$+ h_4 \max\{S(u_n, u_n, u_{n-1}), S(u_{n+1}, u_{n+1}, u_n)\}$$
$$= \; h_1\, S(u_{n-1}, u_{n-1}, u_n) \; + h_3\, S(u_{n+1}, u_{n+1}, u_{n-1})$$
$$+ h_4 \max\{S(u_n, u_n, u_{n-1}), S(u_{n+1}, u_{n+1}, u_n)\}$$
$$\preccurlyeq h_1 S(u_{n-1}, u_{n-1}, u_n) + h_3\{2S(u_{n+1}, u_{n+1}, u_n) + h_3\, S(u_{n-1}, u_{n-1}, u_n)$$
$$+ h_4\, S(u_{n+1}, u_{n+1}, u_{n-1}) \; + h_4\, S(u_{n+1}, u_{n+1}, u_n)$$
$$= (h_1 + h_3 + h_4) + h_4\, S(u_{n-1}, u_{n-1}, u_n) \; + (2h_3 + h_4) + h_4\, S(u_n, u_n, u_{n+1})$$

which implies

$$\mathrm{S}(u_n, u_n, u_{n+1}) \preccurlyeq \frac{h_1 + h_3 + h_4}{1 - 2h_3 - h_4} S(u_{n-1}, u_{n-1}, u_n) \tag{2.8}$$

If we put $h = \frac{h_1 + h_3 + h_4}{1 - 2h_3 - h_4}$ then we find $h \prec 1$ since $h_1 + 3h_3 + 2h_4 \prec 1$. Using the inequality (2.8), we have

$$\lim_{n\to 0} ||S(u_n, u_n, u_{n+1})|| \; \preccurlyeq \; h^n\, K\, ||\, S(u_0, u_0, u_1)||(2.9)$$

If we take limit for $n \to \infty$, using the inequality (2.9), we get

$$\lim_{n\to\infty} S(u_n, u_n, u_{n+1}) = 0$$

since $h \in (0,1)$. For all $\varepsilon \succ 0$ implies $\lim_{n\to\infty} S(u_n, u_n, u_{n+1}) = 0$. By the similar arguments used in the proof of theorem (2.1), we see that the sequence $\{u_n\}$ is Cauchy. Then there exists $w \in X$ such that $\lim_{n\to\infty} T^n u_0 = w$, since (X, S) is a complete cone S-metric space. From the inequality (9) we find

$$S(u_n, u_n, Tw) \; = \; S(Tu_{n-1}, Tu_{n-1}, Tw)$$
$$\preccurlyeq h_1\, S(u_{n-1}, u_{n-1}, Tw) \; + h_2\, S(u_n, u_n, w) \; + h_3\, S(Tw, Tw, u_{n-1})$$
$$+ h_4 \max\{S(u_n, u_n, u_{n-1}), S(Tw, Tw, w)\}$$

Therefore, $\lim_{n\to 0} \left\| S(u_n, u_n, Tw) \right\| \preccurlyeq (h_3 + h_4) K ||\, S(Tw, Tw, w)||$

$$||S(Tw, Tw, w)|| \; \preccurlyeq \; (h_3 + h_4) K ||\, S(Tw, Tw, w)\; ||\, ,$$

since $h_3 + h_4 \prec 1$. Then $S(Tw, Tw, w) = 0$ which implies that $S(Tw, Tw, w) \ll 0$ thus $Tw = w$.

Prove that T has unique fixed point. Let w, w_1 be two fixed points of T such that $w \neq w_1$. Taking $u = w$ and $v = w_1$ in equation (2.7), we have

$$S(w, w, w_1) \; = \; S(Tw, Tw, w_1)$$
$$\preccurlyeq \; h_1 \;\; S(w, w, w_1) + h_2 \;\; S(w, w, w_1) + h_3\, S(w_1, w_1, w)$$
$$+ \; h_4 max\{S(w, w, w), S(w_1, w_1, w_1)\}$$

which implies

$$S(w, w, w_1) \preccurlyeq \; (h_1 + h_2 + h_3)\, S(w, w, w_1)$$

Then we obtain

$$S(w, w, w_1) = 0$$

that is, $w = w_1$ since $h_1 + h_2 + h_3 \prec 1$. Consequently, T has a unique fixed point $w \in X$.

Corollary 2.4. Let (X, S) be a complete cone S-metric space and P be a normal cone with normal constant K.Suppose the mapping $T: X \to X$ satisfies the following conditions:

$$S(Tu, Tu, Tv) \preccurlyeq h_1 S(u, u, v) + h_2 S(Tu, Tu, v) + h_3 S(Tv, Tv, u) + h_4 \, max\,\{S(Tu, Tu, v), S(Tv, Tv, u)\}$$

for all $u, v \in X$ with non-negative real numbers $h_i (i \in \{1,2,3,4)$ satisfying $\max\{h_1 + 3h_3 + 2h_4, h_1 + h_2 + h_3\} \prec 1$, Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$.

Theorem 2.5. Let (X, S) be a complete cone S-metric space and P be a normal cone with normal constant K. Suppose the mapping $T: X \to X$ satisfies the following conditions:

$$S(Tu, Tu, Tv) \preccurlyeq h_1 S(u, u, v) + h_2 S(Tu, Tu, u) + h_3 S(Tu, Tu, v) + h_4 S(Tv, Tv, u) + h_5 S(Tv, Tv, v) + h_6 \max\{S(u, u, v), S(Tu, Tu, u), S(Tu, Tu, v), S(Tv, Tv, u), S(Tv, Tv, v)\} \quad (2.10)$$

for all $u, v \in X$ with non negative real numbers $h_i (i \in \{1,2,3,4,5,6)$ satisfying $\max\{h_1 + h_2 + 3h_4 + h_5 + 3h_6, h_1 + h_3 + h_4 + h_6\} \prec 1$ Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$.

Proof: Let $u_0 \in X$ and the sequence $\{u_n\}$ be defined as $\lim_{n\to\infty} T^n u_0 = u_n$ Suppose that $u_n \neq u_{n+1}$ for all n. Using the inequality (2.7) the condition (S3) and Lemma (2.10) we get

$$S(u_n, u_n, u_{n+1}) = S(Tu_{n-1}, Tu_{n-1}, Tu_n)$$
$$\preccurlyeq h_1 S(u_{n-1}, u_{n-1}, u_n) + h_2 S(u_n, u_n, u_{n-1}) + h_3 S(u_n, u_n, u_n) + h_4 S(u_{n+1}, u_{n+1}, u_{n-1}) + h_5 S(u_{n+1}, u_{n+1}, u_n)$$
$$+ h_6 \max\{S(u_{n-1}, u_{n-1}, u_n), S(u_n, u_n, u_{n-1}), S(u_n, u_n, u_n), S(u_{n+1}, u_{n+1}, u_{n-1}), S(u_{n+1}, u_{n+1}, u_n)\}$$
$$\preccurlyeq (h_1 + h_2 + h_4 + h_6) S(u_{n-1}, u_{n-1}, u_n) + (2h_4 + h_5 + 2h_6) S(u_{n+1}, u_{n+1}, u_n)$$

which implies

$$S(u_n, u_n, u_n + 1) \preccurlyeq \left(\frac{h_1 + h_2 + h_4 + h_6}{1 - 2h_4 - h_5 - 2h_6}\right) S(u_{n-1}, u_{n-1}, u_n) \quad (2.11)$$

If we put $h = \frac{h_1 + h_2 + h_4 + h_6}{1 - 2h_4 - h_5 - 2h_6}$ then we find $h \preccurlyeq 1$ since $h_1 + h_2 + 3h_4 + h_5 + 3h_6 \prec 1$. Using the inequality (13) we have

$$\lim_{n\to\infty} \left||S(u_n, u_n, u_{n+1})|\right| \preccurlyeq h^n K \,||\, S(u_0, u_0, u_1)| \quad (2.12)$$

If we take limit for $n \to \infty$, using the inequality (2.12), we get

$$\lim_{n\to 0} S(u_n, u_n, u_{n+1}) = 0$$

since $h \in (0,1)$ The $\varepsilon \succ 0$ implies $\lim_{n\to\infty} S(u_n, u_n, u_{n+1}) = 0$ By the similar arguments used in the proof of Theorem (2.4), we see that the sequence $\{u_n\}$ is Cauchy. Then there exists $w \in X$ such that $\lim_{n\to\infty} T^n u_0 = w$, since (X, S) is a complete cone S-metric space. From the inequality (2.12) we find

$$S(u_n, u_n, Tw) = S(Tu_n - 1, Tu_n - 1, Tw)$$
$$\preccurlyeq h_1 S(u_{n-1}, u_{n-1}, w) + h_2 S(u_n, u_n, u_{n-1}) + h_3 S(u_n, u_n, w)$$
$$+ h_4 S(Tw, Tw, u_{n-1}) + h_5 S(Tw, Tw, w)$$
$$+ h_6 max\{S(u_{n-1}, u_{n-1}, w), S(u_n, u_n, u_{n-1}), S(u_n, u_n, w), S(Tw, Tw, u_{n-1}), S(Tw, Tw, w)\}$$

Therefore, $\lim_{n\to 0} \left||S(u_n, u_n, Tw)|\right| \preccurlyeq (h_4 + h_5 + h_6) K ||S(Tw, Tw, w||$, since $h_4 + h_5 + h_6 \prec 1$.Then $S(Tw,Tw,w) = 0$ which implies that $S(Tw,Tw,w) \ll 0$ thus $Tw = w$.

Prove that T has unique fixed point. Let w, w_1 be two fixed points of T such that $w \neq w_1$. Taking $u = w$ and $v = w_1$ in equation (2.10), we have

$$S(w,w,w_1) = S(Tw,Tw,Tw_1)$$
$$\preccurlyeq h_1 S(w,w,w_1) + h_2 S(w,w,w) + h_3 S(w,w,w_1) + h_4 S(w_1,w_1,w) + h_5 S(w_1,w_1,w_1) + h_6 \max\{S(w,w,w_1), S(w,w,w), S(w,w,w_1), S(w_1,w_1,w), S(w_1,w_1,w_1)\},$$

which implies

$$S(w,w,w_1) \preccurlyeq (h_1 + h_3 + h_4 + h_6) S(w,w,w_1)$$

Then we obtain

$$S(w,w,w_1) = 0,$$

that is, $w = w_1$ since $h_1 + h_3 + h_4 + h_6 \prec 1$. Consequently, T has a unique fixed point $w \in X$.

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