Annals of Pure and Applied Mathematics
Vol. 14, No. 3, 2017, 449-455
ISSN: 2279-087X (P), 2279-0888(online)
Published on 26 October 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v14n3a11

Annals of
Pure and Applied Mathematics

Edge Version of *F*-index, General Sum Connectivity Index of Certain Nanotubes

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli @gmail.com

Received 14 October 2017; accepted 23 October 2017

Abstract. Chemical graph theory is a branch of graph theory whose focus of interest is to finding topological indices of chemical graphs which correlate well with chemical properties of the chemical molecules. In this paper, we define the edge version of *F*-index, the edge version of the general sum connectivity index of a graph and compute exact formulas for some families of nanotubes.

Keywords: *F*- index, hyper-Zagreb index, sum connectivity index, nanotubes.

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*12

1. Introduction

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices have been considered in Theoretical Chemistry and have some applications, especially in *QSPR/QSAR* research [1, 2].

The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The degree of an edge $e=uv$ in G is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. The line graph $L(G)$ of a graph G is the graph whose vertex set correponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. We refer to [3] for undefined term and notation.

The F-index of a graph G is defined as

$$F(G)=\sum_{u\in V(G)} d_G(u)^3 \qquad \text{or} \qquad F(G)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right].$$

This index was introduced in [4]. In [5], Furtula and Gutman studied this index and called it forgotten topological index

In [6], Shirdel et al. introduced the hyper-Zagreb index of a graph G. It is defined as

$$HM(G)=\sum_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]^2.$$

In [7], Zhou and Trinajstić introduced the sum connectivity index of a graph G and it is defined as

$$X(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)+d_G(v)}}.$$

The general sum connectivity index was introduced by Zhou and Trinajstić in [8]. This index is defined as

$$M_1^a(G)=\sum_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]^a.$$

where a is a real number.

The above mentioned indices were also studied in [9,10]. Recently many other topological indices were studied, for example, in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

We define the edge version of F-index, edge version of sum connectivity index, edge version of hyper-Zagreb index, edge version of general sum connectivity index of a molecular graph G as follows:

The edge version of the F-index of a graph G is defined as

$$F_e(G)=\sum_{uv\in E(L(G))}d_{L(G)}(e)^3.$$

or $$F_e(G)=\sum_{ef\in E(L(G))}\left[d_{L(G)}(e)^2+d_{L(G)}(f)^2\right].$$

The edge version of sum connectivity index of a graph G is defined as

$$X_e(G)=\sum_{ef\in E(L(G))}\frac{1}{\sqrt{d_{L(G)}(e)+d_{L(G)}(f)}}.$$

The edge version of hyper-Zagreb index of a graph G is defined as

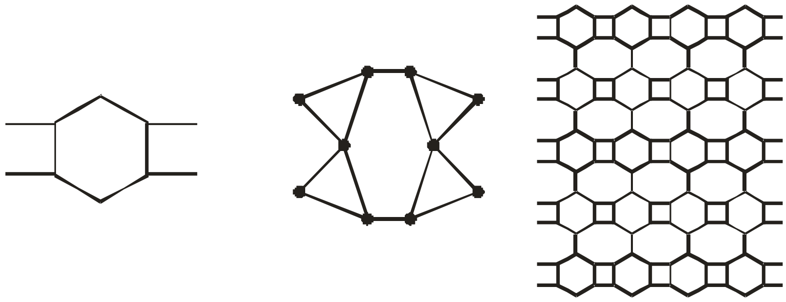
$$HM_e(G)=\sum_{ef\in E(L(G))}\left[d_{L(G)}(e)+d_{L(G)}(f)\right]^2.$$

The edge version of theses indices were studied, for example, in [26, 27].

In this paper, we compute the edge version of F-index, edge version of sum connectivity index, hyper Zagreb index and edge version of general sum connectivity index of $TUC_4C_6C_8[p, q]$ nanotubes, $TUSC_4C_8(S)[p, q]$ nanotubes. For more information about these nanotubes see [28].

2. Results for $TUC_4C_6C_8[p, q]$ nanotube

We consider the graph of 2-dimensional lattice of $TUC_4C_6C_8[p, q]$ nanotube with p columns and q rows. The graph of 2-dimensional lattice of $TUC_4C_6C_8[1,1]$ nanotube is shown in Figure 1 (a). The line graph of $TUC_4C_6C_8[1,1]$ nanotube is shown in Figure 1(b). Also the graph of $TUC_4C_6C_8[4,5]$ is shown in Figure 1 (c).



(a) (b) (c)

Figure 1:

Let G be the graph of 2-dimensional lattice of $TUC_4C_6C_8[p,q]$ nanotube. By calculation, we obtain that the line graph of $TUC_4C_6C_8[p, q]$ has $18pq - 4p$ edges. In $L(TUC_4C_6C_8[p, q])$, there are three types of edges based on the degree of the vertices of each edge. Thus by calculation, we obtain edge partitions of $L(TUC_4C_6C_8[p, q])$ based on the sum of degrees of the end vertices of each edge as given in Table 1.

$d_{L(G)}(e), d_{L(G)}(f)\backslash ef \in E(L(G))$	(3, 3)	(3, 4)	(4,4)
Number of edges	$2p$	$8p$	$18pq - 14p$

Table 1: Edge partition of $L(G)$

In the following theorem, we compute the exact value of $F_e(TUC_4C_6C_8[p,q])$.

Theorem 1. Let G be the graph of $TUC_4C_6C_8[p, q]$ nanotube. Then

$$F_e\left(TUC_4C_6C_8\left[p,q\right]\right) = 576pq - 212p.$$

Proof: We have $F_e(G) = \sum_{ef \in E(L(G))} \left[d_{L(G)}(e)^2 + d_{L(G)}(f)^2\right].$

By using Table 1, we have

$$F_e(G) = \left(3^2 + 3^2\right)2p + \left(3^2 + 4^2\right)8p + \left(4^2 + 4^2\right)(18pq - 14p)$$
$$= 576pq - 212p.$$

In the next theorem, we compute the exact value of the edge version of the general sum connectivity index of $TUC_4C_6C_8[p,q]$ nanotube.

Theorem 2. Let G be the graph of $TUC_4C_6C_8[p, q]$ nanotube. Then

$$M_{1e}^a\left(TUC_4C_6C_8\left[p,q\right]\right) = 18 \times 8^a pq + \left(2 \times 6^a + 8 \times 7^a - 14 \times 8^a\right)p. \quad (1)$$

Proof: We have $M_{1e}^a(G) = \sum_{ef \in E(L(G))} \left[d_{L(G)}(e) + d_{L(G)}(f)\right]^a.$

By using Table 1, we have

$$M_{1e}^a(G) = (3+3)^a 2p + (3+4)^a 8p + (4+4)^a (18pq - 14p)$$

$$=18\times 8^a pq+\left(2\times 6^a+8\times 7^a-14\times 8^a\right)p.$$

We obtain the following results by using Theorem 2.

Corollary 2.1. The edge version of the sum connectivity index of $TUC_4C_6C_8[p, q]$ nanotube is given by

$$M_{1e}^a\left(TUC_4C_6C_8\left[p,q\right]\right)=\frac{9}{\sqrt{2}}pq+\left(\frac{2}{\sqrt{6}}+\frac{8}{\sqrt{7}}-\frac{7}{\sqrt{2}}\right)p.$$

Proof: Put $a=-\frac{1}{2}$ in equation (1), we get the desired result.

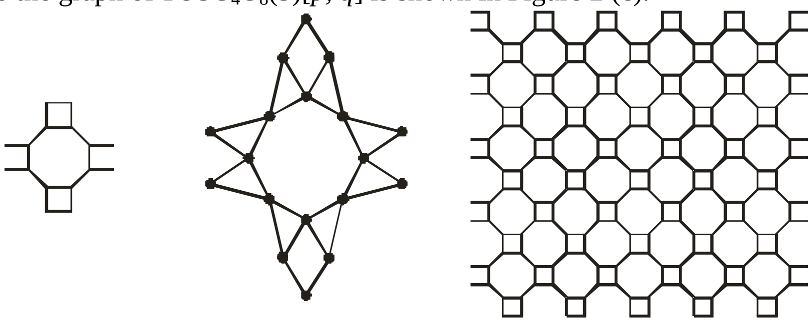
Corollary 2.2. The edge version of the hyper-Zagreb index of $TUC_4C_6C_8[p, q]$ nanotube is given by

$$HM_e\left(TUC_4C_6C_8\left[p,q\right]\right)=1152pq-432p.$$

Proof: Put $a=2$ in equation (1), we get the desired result.

3. Results for $TUSC_4C_8(S)[p, q]$ nanotube

We consider the graph of 2-dimensional lattice of $TUSC_4C_8(S)[p, q]$ nanotube with p columns and q rows. The graph of 2-dimensional lattice of $TUSC_4C_8(S)[1,1]$ nanotube is shown in Figure 2(a). The line graph of $TUSC_4C_8(S)[1,1]$ nanotube is shown in Figure 2(b). Also the graph of $TUSC_4C_8(S)[p, q]$ is shown in Figure 2 (c).



(a) (b) (c)

Figure 2:

Let G be the graph of 2-dimensional lattice of $TUSC_4C_8(S)[p,q]$ nanotube. By calculation, we obtain that the line graph of $TUSC_4C_8(S)[p, q]$ has $24pq + 4p$ edges. In $L(TUSC_4C_8(S)[p, q])$, there are three types of edges based on the degree of the vertices of each edge. Thus by calculation, we obtain that edge partitions of $L(TUSC_4C_8(S)[p, q])$ based on the sum of degrees of the end vertices of each edge as given in Table 2.

$d_{L(G)}(e), d_{L(G)}(f) \backslash ef \in E(L(G))$	(2, 3)	(3, 4)	(4,4)
Number of edges	$4p$	$8p$	$24pq - 8p$

Table 2: Edge partition of $L(G)$

In the following theorem, we compute the exact value of the edge version of *F*-index of $TUSC_4C_8(S)[p,q]$ nanotube.

Theorem 3. Let G be the graph of $TUSC_4C_8(S)[p, q]$ nanotube. Then

$$F_e\left(TUSC_4C_8(S)[p,q]\right) = 768pq - 4p.$$

Proof: We have $F_e(G) = \sum_{ef\in E(L(G))}\left[d_{L(G)}(e)^2 + d_{L(G)}(f)^2\right].$

By using Table 2, we have

$$\begin{aligned} F_e\left(TUSC_4C_8(S)[p,q]\right) &= \left(2^2+3^2\right)4p + \left(3^2+4^2\right)8p + \left(4^2+4^2\right)(24pq-8p) \\ &= 768pq - 4p. \end{aligned}$$

In the next theorem, we compute the exact value of the edge version of the general sum connectivity index of $TUSC_4C_8(S)[p,q]$ nanotube.

Theorem 4. Let G be the graph of $TUSC_4C_8(S)[p, q]$ nanotube. Then

$$M_{1e}^a\left(TUSC_4C_8(S)[p,q]\right) = 24\times 8^a pq + \left(4\times 5^a + 8\times 7^a - 8\times 8^a\right)p. \quad (2)$$

Proof: We have $M_{1e}^a(G) = \sum_{ef\in E(L(G))}\left[d_{L(G)}(e) + d_{L(G)}(f)\right]^a.$

By using Table 2, we have

$$\begin{aligned} M_{1e}^a\left(TUSC_4C_8(S)[p,q]\right) &= (2+3)^a 4p + (3+4)^a 8p + (4+4)^a (24pq-8p) \\ &= 24\times 8^a pq + \left(4\times 5^a + 8\times 7^a - 8\times 8^a\right)p. \end{aligned}$$

We obtain the following results by using Theorem 4.

Corollary 4.1. The edge version of the sum connectivity index of $TUSC_4C_8(S)[p,q]$ nanotube is given by

$$X_e\left(TUSC_4C_8(S)[p,q]\right) = 3\sqrt{8}pq + \left(\frac{4}{\sqrt{5}} + \frac{8}{\sqrt{7}} - \sqrt{8}\right)p.$$

Proof: Put $a = -\frac{1}{2}$ in equation (2), we get the desired result.

Corollary 4.2. The edge version of the hyper-Zagreb index of $TUSC_4C_8(S)[p, q]$ nanotube is given by

$$HM_e\left(TUSC_4C_8(S)[p,q]\right) = 1536pq - 20p.$$

Proof: Put $a = 2$ in equation (2), we get the desired result.

5. Conclusion

Chemical Graph Theory is an important tool for studying molecular structures. At present, the study of topological indices is one of the active research fields in chemical graph theory. In this paper, we have obtained some theoretical results on "Edge version of F-index, general sum connectivity index of certain nanotubes". These formulas make it possible to correlate chemical structure of nanotubes with an information about their physical features.

REFERENCES

1. I.Gutman and O.E.Polansky, *Mathematical Concepts in Organic Chemistry,* Springer, Berlin (1986).
2. R.Todeschini and V.Consonni, *Molecular Descriptors for Chemo-informatics*, Wiley-VCH, Weinheim, (2009).
3. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
4. I.Gutman and N.Trinajstić, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.,* 17 (1972) 535-538.
5. B.Furtula and I.Gutman, A forgotten topological index, *J. Math. Chem.,* 53 (2015) 1184-1190.
6. G.H.Shirdel, H.Rezapour and A.M.Sayadi, The hyper-Zagreb index of graph operations, *Iranian J. Math. Chem.,* 4(2) (2013) 213-220.
7. B.Zhou and N.Trinajstić, On a novel connectivity index, *J. Math. Chem.,* 46 (2009) 1252-1270.
8. B.Zhou and N.Trinajstić, On general sum connectivity index, *J. Math. Chem.,* 47 (2010) 210-218.
9. V.R.Kulli, Computation of general topological indices for titania nanotubes, *International Journal of Mathematical Archive,* 7(12) (2016) 33-38.
10. V.R.Kulli, *F*-index and reformulated Zagreb index of certain nanostructures, *International Research Journal of Pure Algebra*, 7(1) (2017) 489-495.
11. V.R.Kulli, On *K* indices of graphs, *International Journal of Fuzzy Mathematical Archive*, 10(2) (2016) 105-109.
12. V.R.Kulli, On *K* coindices of graphs, *Journal of Computer and Mathematical Sciences*, 7(3) (2016) 107-112.
13. V.R.Kulli, *K*-Banhatti indices of graphs, *Journal of Computer and Mathematical Sciences,* 7(4) (2016) 213-218.
14. V.R.Kulli, On *K* hyper-Banhatti indices and coindices of graphs, *Journal of Computer and Mathematical Sciences,* 6(5) (2016) 300-304.
15. V.R.Kulli, Computation of some topological indices of certain networks, *International Journal of Mathematical Archive,* 8(2) (2017) 99-106.
16. V.R.Kulli, New K Banhatti topological indices, *International Journal of Fuzzy Mathematical Archive*, 12(1) (2017) 29-37.
DOI:http://dx.doi.org/10.22457/ijfma.v12n1a4
17. V.R.Kulli, New arithmetic-geometric indices, *Annals of Pure and Applied Mathematics*, 13(2) (2017) 165-172.
DOI:http://dx.doi.org/10.22457/apam.13n2a2
18. V.R.Kulli, A new Banhatti geometric-arithmetic index, *International Journal of Mathematical Archive,* 7(4) (2016) 213-218.
19. V.R.Kulli, The Gourava indices and coindices of graphs, *Annals of Pure and Applied Mathematics*, 14(1) (2017) 33-38.

DOI: http://dx.doi.org/10.22457/apam.v14n1a4

20. V.R.Kulli, The product connectivity Gourava index, *Journal of Computer and Mathematical Sciences*, 8(6) (2017) 235-242.
21. V.R.Kulli, On the sum connectivity Gourava index, *International Journal of Mathematical Archive*, 8(6) (2017) 211-217.
22. V.R.Kulli, The sum connectivity Revan index of silicate and hexagonal networks, *Annals of Pure and Applied Mathematics*, 14(3) (2017) 401-406.
DOI: http://dx.doi.org/10.22457/apam.v14n3a6
23. V.R.Kulli, B.Chaluvaraju and H.S.Boregowda, Some degree based connectivity indices of Kulli cycle windmill graphs, *South Asian Journal of Mathematics,* 6(6) (2016) 263-268.
24. V.R.Kulli, B.Chaluvaraju and H.S.Boregowda, Connectivity Banhatti indices for certain families of benzenoid systems, *J. of Ultra Chemistry*, 13(4) (2017) 81-87.
DOI: http://dx.doi.org/10.22147/juc/130402
25. I.Gutman, V.R.Kulli, B.Chaluvaraju and H.S.Boregowda, On Banhatti and Zagreb indices, *Journal of the International Mathematical Virtual Institute*, 7 (2017) 53-67.
26. V.R.Kulli, Edge version of multiplicative atom bond connectivity index of certain nanotubes and nanotori, submitted.
27. V.R.Kulli, Edge version of multiplicative connectivity indices of some nanotubes and nanotori, submitted.
28. M.N.Husin, R.Hasni, M.Imran and H.Kamarulhaili, The edge version of geometric-arithmetic index of nanotubes and nanotori, *Optoelectron Adv. Mater. Rapid Comm*. 9 (9-10) (2015) 1292-1300.