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Metro Domination of Square Path

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Abstract. Let G=(V,E) be a graph. A set S $\subseteq V$ is called resolving set if for every u,v $\in V$ *there exist* $w \in V$, such that d(u,w) $\neq$ d(v,w). The resolving set with minimum cardinality is called metric basis and its cardinality is called metric dimension and it is denoted by β(G). A set D $\subseteq$ V is called dominating set if every vertex not in 'D' is adjacent to at least one vertex in D. The dominating set with minimum cardinality is called domination number of G and it is denoted by γ(G). A set D which is both resolving set as well as dominating set is called metro dominating set. The minimum cardinality of a metro dominating set is called metro domination number of G and it is denoted by γ_β (G). In this paper, we determine metro domination number of square path.

Keywords: Power graph, metric dimension, landmark, distance matrix.

AMS Mathematics Subject Classification (2010): 05C56

1. Introduction

A set of vertices S resolves a graph G if every vertex of G is uniquely determined by its vector of distances to the vertices in S. This work undertakes a general study of resolving sets in square path of graphs. All the graphs considered are simple, finite and connected. Given a graph G=(V,E) and u,v $\in$ V, d_G(u,v) (or simply d(u,v)) denoted the distance between u and v in G, i.e the length of a shortest u-v path.

In 1976, Harary and Melter [4] introduce the notion of metric dimension. The vertex set and edge set of a graph G are denoted by V(G) and E(G).The distance between vertices u,v $\in$ V(G) is denoted by d_G(v,w) or d(v,w) if the graph G is clear from the context. A vertex x $\in$ V(G) resolves a pair of vertices v,w $\in$ V(G) if d(v,x) $\neq$ g(w,v). A set of vertices S $\subseteq$V(G) resolves G, and S is a resolving set S of G with minimum cardinality is a metric basis of G, and its cardinality is the metric dimension of G, denoted by β(G).

2. Some known result on metric dimension

In this section we mention some of the known result on metric dimension due to various authors, which we use in the subsequent sections.

Theorem 2.1. (Harary and Melter[4]) The metric dimension of a non-trivial complete graph of order n is n-1.

Theorem 2.2. (Khuller, Raghavachari, Rosenfeld [8]) The metric dimension of a graph G is 1 if and only if G is a path.

Theorem 2.3. (Sooryanarayana and Geetha[10]) For any non trivial graph G on $n \geq 2$ vertices, $\beta_k(G)=n-1$ if and only if $diam(G) \leq K$, where $K \geq 1$ is any integer.

Theorem 2.4 (Harary and Melter[4]) The metric dimension of a complete bipartite graph Km,n is m+n-2.

3. **Locating number**

A subset D of V (G) is called a dominating set, if every vertex is V-D is adjacent to at least one vertex in D. The minimum cardinality of a dominating set is called the domination number of the graph G 1976. The metric dimension of a graph G(V,E) is the cardinality of a minimal subset S of V such that for each pair of vertices u, v of V there is a vertex w in S such that the length of the shortest path from w to u is different from the length of a shortest path from w to v.

The metric dimension of a graph G is also called as a locating number of G and studied its dominating property independently by Slater [10]. A dominating set D is called a locating dominating set or simply LD-set if for each pair of vertices u, v ϵ V-D, $ND(u) \neq ND(v)$, where $ND(u)=N(u) \cap D$. The minimum cardinality of an LD-set of the graph G is called the locating domination number of G denoted by $\gamma_L(G)$.

For example: The set of darkened vertices of the graph G of Figure 1, serves as a minimal locating dominating set, so $\gamma_L(G) = 3$.

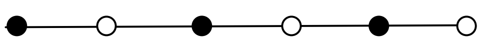

Figure 1: The graph G and its LD-set.

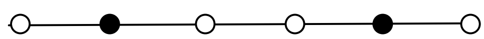

Figure 2: The graph G and its MD-set

4. Domination number of square of graphs

A set $S \subseteq V(G)$ is a dominating set of G if every vertex of V(G) – S is adjacent to at least one vertex of S. The cardinality of the smallest dominating set of G is called the domination number of G, denoted by $\gamma(G)$. A dominating set S is a minimal dominating set if no proper subset $S^{|} \subseteq S$ is dominating set. Given any graph G, its square graph G^2 is a graph with vertex set V(G) and two vertices are adjacent whenever they are at distance 1 or 2 in G. A set $S \subseteq V(G)$ is a 2-distance dominating set of G if $d_G(u,S)=1$ or 2 for every vertex of V(G)-S. The cardinality of the smallest 2-distance dominating set of G is a dominating set of G^2, so $\gamma^2(G)= \gamma(G^2)$.

5. Some result

In this section, we mention some of the known result on metric dimension due to various authors, which we use in the subsequent sections.

Theorem 5.1. [9] Let S be a dominating set of G^2. Then S is a minimal dominating set of G^2 if and only if each vertex u ∈ S satisfies at least one of the following conditions:

(a) There exists a vertex $v \in V(G)-S$ for which $N_2(v) \cap S=\{u\}$
(b) $d(u,w) \geq 2$ for every vertex $w \in S-\{u\}$

Theorem 5.2. [9] If G is a graph with no isolated vertices and S is a minimal dominating set of G, then V(G)-S is a dominating set of G.

Proof: Let S be a γ-set of G. S is a minimal dominating set of G. By theorem 5.2, V(G)-S is a dominating set of G, so $|S| \leq |V(G)-S|$, so $|S| \leq \frac{n}{2}$.

Theorem 5.3. [9] For every $n \geq 1$, $\gamma(p^2_n) = \lceil n/5 \rceil$.

Theorem 5.4. [9] If diam(G) ≤ 3, then $\gamma^2(G) \leq \gamma(G^2)$.

Corollary 5.5. $\gamma(p_n) = \lceil n/3 \rceil$.

6. Metro domination number

We now define a metro dominating set, which can be served as a better alternating t for the locating dominating set as, A dominating set D of V(G) having the property that for each pair of vertices u,v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$ is called the metro dominating set of G or simply an MD-set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and it is denoted by γ_β(G). For example: The set of darkened vertices of the graph G, of figure 2, is a minimal metro dominating set and hence γ_β (G) = 2.

We recall the following result which we use in next sections.

Theorem 6.1. (Sooryanarayan and Raghunath [7]) The metro domination number of a graph G is $\lceil n/5 \rceil$ if and only if G is a cycle.

Theorem 6.2. (Sooryanarayan and Raghunath [7]) let G be a graph on n vertices. Then γ_β (G) = n-1 if and only if G is K_n or $K_{1,n-1}$, for $n \geq 1$.

Theorem 6.3. (Sooryanarayan and Raghunath [7]) If γ_β(G) = 2, then G cannot have K_4 as a sub graph of G.

Theorem 6.4. (Sooryanarayan and Raghunath [7]) For any integer n, $\gamma_\beta(p_n) = \lceil n/3 \rceil$.

Remarks 6.5. For any connected graph G, $\gamma_\beta(G) \geq \max\{ \gamma(G), \beta(G) \}$.

7. Power graph

Let G=(V,E) connected a graph. K^{th} power of is denoted by G^k . whose vertex is same as that of G and two vertices (u,v) in G^k are adjacent K. If and only if $d(u,v) \leq K$ in G.

For Example 1

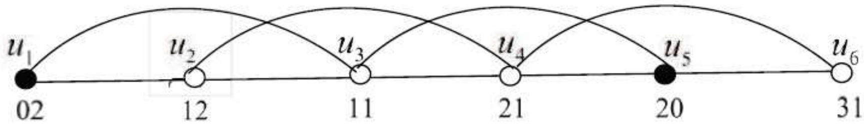

Figure 3: $\gamma_\beta(p_6^2)$ = 2

Theorem 7.1. For any integer n≥3 $\gamma_\beta(P_n^2) = \begin{cases} 2 & if \quad 3 \le n \le 7 \\ 3 & if \;\; 8 \le n \le 10 \\ \lceil n/5 \rceil & if \qquad n \ge 11 \end{cases}$

Proof: By theorem 2.2 and the remark 6.5 $\gamma_\beta(P_n^2)$≥2 for all n. For n=3,4 choose S={u_1,u_2} then S is dominating set as well as metric basis , hence $\Upsilon_\beta(P_n^2)$ =2. For n=5,6 choose S = {u_1,u_5} then S is both metric basis as well as dominating set, hence for n=5,6, $\Upsilon_\beta(P_n^2)$ = 2. For n = 7 choose S = {u_2,u_6} then S is both metric basis as well as dominating set, hence $\Upsilon_\beta(P_n^2)$ = 2. For n = 8,9,10 choose S = {u_3,u_7,u_8} then S is metric basis as well as dominating set, hence $\Upsilon_\beta(P_n^2)$ = 3.

Lemma 7.2. [11] Let G = p_n^k where n ≥ K+1. Then diam(G) ≤ K

Lemma 7.3. Let G = p_n^2 , n > 3. Then diam(G) = 2

Proof: Let P_n be the graph with v(p_n) = {v_0, v_1 ,……v_{n-1}}. Take w = {v_0, v_1} according to lemma 7.2, w is resolving set and diam(G) ≤ 2. Since G is not a path , then according to theorem 2.2 we get diam(G) = 2.

Lemma 7.4. [9] For n ≥ 5, γ (p_n^2) = ⌈n/5⌉.

Lemma 7.5: For any integer n, $\gamma_\beta(P_n^2) = \lceil n/5 \rceil$, n ≥ 11.

Proof: Let D be a dominating set P_n^2 , Let u_1, u_2 ....u_n be vertices of P_n , such that u_i is adjacent to u_{i+1}. For i = 1, 2, 3,…,n-1. But we know that $\gamma(p_n^2)$ = ⌈n/5⌉ by the lemma 7.4 and $\beta(p_n^2)$ = 2 by the lemma 7.3. Since a metro dominating set D is also a dominating set then we show that

$$\gamma_\beta(p_n^2) \ge \lceil n/5 \rceil \tag{1}$$

To prove the reverse inequality we find a metro dominating set of cardinality ⌈n/5⌉.

(i) D= {u_{5k-4} : k ≥ 1}, n ≡ 1(mod5)
(ii) D= {u_{5k-3} : k ≥ 1}, n ≡ 2(mod5)
(iii) D= {u_{5k-2} : k ≥ 1}, n ≡ 3(mod5)
(iv) D= {u_{5k-1} : k ≥ 1}, n ≡ 4(mod5)
(v) D= {u_{5k} : k ≥ 1}, n ≡ 0(mod5)

Choose D in the above cases, then |D| =⌈n/5⌉ proved and D is metro domination set, in fact for every $u_j \in$ V-D, by the choice of D, at least one of u_{j-2} , u_{j-1} ,u_{j+1} or u_{j+2} must be in D and which dominates u_j, by the lemma 7.5, D is resolving set. Hence

$$\gamma_\beta(p_n^2) \leq \lceil n/5 \rceil \qquad (2)$$

Therefore from (1) and (2), $\gamma_\beta(p_n^2)$ = ⌈n/5⌉

Case 1. For n ≡ 0 (mod5)

Choose D= $\{u_{5k} : k \geq 1\}$, n ≡ 0(mod5), 1 ≤ k ≤ n-4, then $|D|= \lceil n/5 \rceil$ proved and D is a dominating set , in fact for every $u_j \in$ V-D, by the choice of D, at least one of u_{j-2}, u_{j-1}, u_{j+1} or u_{j+2} must be in 'D' and which dominates u_j, by the lemma 7.5, D is resolving set.

∴ Hence $\gamma_\beta(p_n^2) \leq \lceil n/5 \rceil$, n ≡ 0 (mod 5).

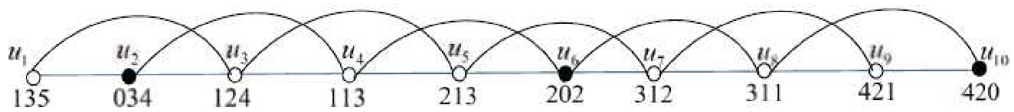

Figure 4: $\gamma_\beta(p_{10}^2)$ = 3

Case2. For n ≡ 1 (mod 5)

Choose D= $\{u_{5k-4} : k \geq 1\}$, n ≡ 0(mod5), 1 ≤ k ≤ n-3, then |D|= ⌈n/5⌉ proved and D is a dominating set , in fact for every $u_j \in$ V-D, by the choice of D, at least one of u_{j-2}, u_{j-1}, u_{j+1} or u_{j+2} must be in 'D' and which dominates u_j, by the lemma 7.5, D is resolving set.

∴ Hence $\gamma_\beta(p_n^2) \leq \lceil n/5 \rceil$, n ≡ 1 (mod 5)

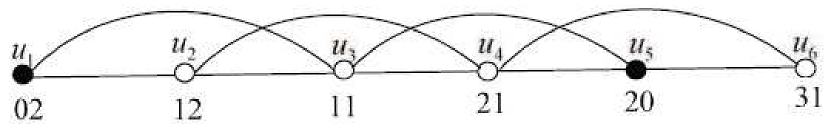

Figure 5: $\gamma_\beta(p_6^2)$ = 2

Case 3. For n ≡ 2 (mod 5)

Choose D= $\{u_{5k-3} : k \geq 1\}$, n ≡ 2(mod5), 1 ≤ k ≤ n-2, then |D|= ⌈n/5⌉ proved and D is a dominating set , in fact for every $u_j \in$ V-D, by the choice of D, at least one of u_{j-2}, u_{j-1}, u_{j+1} or u_{j+2} must be in 'D' and which dominates u_j, by the lemma 7.5, D is resolving set.

∴ Hence $\gamma_\beta(p_n^2) \leq \lceil n/5 \rceil$, n ≡ 2 (mod 5)

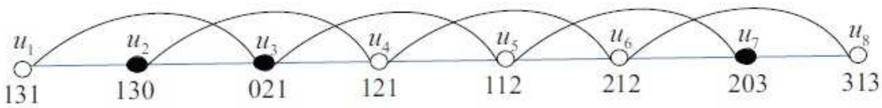

Figure 6: $\gamma_\beta(p_8^2)$ = 3

Case 4. For n ≡ 3 mod 5

Choose D= {u_{5k-2} :k≥1}, n ≡ 3 (mod5), 1 ≤ k ≤ n-1, then |D|= ⌈n/5⌉ proved and D is a dominating set, in fact for every u_j ∈ V-D, by the choice of D, at least one of u_{j-2}, u_{j-1}, u_{j+1} or u_{j+2} must be in 'D' and which dominates u_j, by the lemma 7.5, D is resolving set.

∴ Hence $\gamma_\beta(p_n^2) \leq \lceil n/5 \rceil$, n ≡ 3 (mod 5)

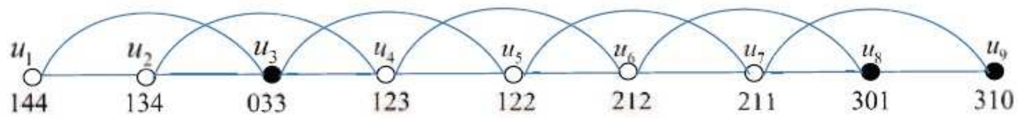

Figure 7: $\gamma_\beta(p_9^2)$ = 3

Case 5. n ≡ 4 mod 5

Choose D= {u_{5k-1} :k≥1}, n ≡ 4 (mod5), 1 ≤ k ≤ n, then |D|= ⌈n/5⌉ proved and D is a dominating set , in fact for every u_j ∈ V-D, by the choice of D, at least one of u_{j-2}, u_{j-2}, u_{j+1} or u_{j+2} must be in 'D' and which dominates u_j, by the lemma 7.5, D is resolving set.

∴ Hence $\gamma_\beta(p_n^2) \leq \lceil n/5 \rceil$, n ≡ 4 (mod 5)

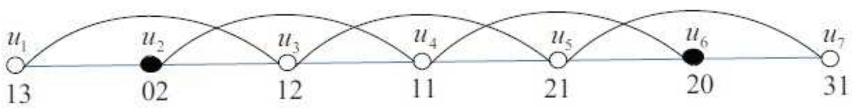

Figure 8: $\gamma_\beta(p_7^2)$ = 2

Conclusion: In this we have computed certain interesting variation of Metro domination of graphs related to square path. In this concept consists of domination number as well as metric dimension. In this concept strongly better than the locating number.

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