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Some Graph Labeling on Inflated Graph of Triangular Snake and Alternate Triangular Snake Graph

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Abstract. In this paper, we prove the existence of difference cordial labeling and pell labeling for inflated triangular snake and alternate triangular snake graph.

Keywords: Graph labeling, difference cordial labeling, pell labeling, triangular snake and alternate triangular snake graph,

AMS Mathematics Subject Classification (2010): 05C78

1.Introduction

In 1987, Cahit introduced the concept of cordial labeling [1]. In [7] Ponraj, Shatish Narayanan and Kala introduced the notions of difference cordial labeling for finite, undirected and simple graph. Let G = (V, E) be a (p, q) graph, and f be a map from V (G) to {1, 2, ..., p} for each edge uv assign the label

$$f^*(uv)=\begin{cases} 1, |f(u) - f(v)| = 1 \\ 0, |f(u) - f(v)| \neq 1 \end{cases}$$

f is called a difference cordial labeling if f is one to one map and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ denotes the number of edges labeled with '1' while $e_f(0)$ denotes the number of edges not labeled with '1'. A graph which admits a difference cordial labeling is called a difference cordial graph. Ponraj et al. shown that every graph is a sub graph of a difference cordial graph and any r − regular graph with $r \geq 4$ is not difference cordial graph, every path and cycle are difference cordial graphs, the star graph $K_{1,n}$ is difference cordial if and only if $n \leq 5$, the graph K_n is difference cordial only when $n \leq 4$ while the bipartite graph $K_{m,n}$ is not difference cordial if $m \geq 4$ and $n \geq 4$, the bistar $B_{m,n}$ is not difference cordial when $m + n \geq 9$ but the wheel W_n, the fan F_n, the gear G_n, the helm H_n and all webs are difference cordial graphs for all n [4]. The authors investigated the difference cordial labeling behavior of $G \odot P_n$, $G \odot mK_1$ (m = 1, 2, 3) where G is either unicyclic or a tree and $G_1 \odot G_2$ are some more standard graphs. Some graphs obtained from triangular snake and quadrilateral snake were investigated with respect to the difference cordial labeling behavior. Also the behavior of subdivision of some snake graphs is investigated in [8]., [9] Shiama defined a new labeling called Pell labeling. Let G be a graph with vertex set V and edge set E and let f be function from V to {0,1,2,…….p-1}. Define f^*: E→ N such that for any u,v ∈E, $f^*(uv) = f(u)+2f(v)$.

In [2], Dunbar and Haynes introduced the concept of the Inflation or Inflated graph G_1 of a graph G without isolated vertices it is obtained as follows: each vertex x_i of degree $d(x_i)$ of G is replaced by a clique $X_i \cong K_{d(xi)}$ and each edge x_ix_j of G is replaced by an edge uv in such a way that $u \in X_i$,$v \in X_j$, and two different edges of G are replaced by non adjacent edges of G_1.

2. Preliminaries

Definition 2.1. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a 3-cycle C_3.

Definition 2.2. An alternate triangular snake $A(T_n)$ is obtained from a path $u_1u_2 \ldots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.

3. Main results

In this section, we provide a procedure for obtaining difference cordial labeling and pell labeling for inflated triangular snake graph and alternate triangular snake graph.

Algorithm 3.1:

Input (triangular snake graph)
Procedure (Structure of Inflation of triangular snake graph)
$V \leftarrow \{v_1,v_2,\ldots,v_{6n-6}\}$
$E \leftarrow \{e_1,e_2,\ldots,e_{10n-14}\}$
for i= 1 to 2n-3
  $v_i\, v_{i+1} \leftarrow e_i$;
end for
for i= 1 to 2n-2
  $v_{2n-2+i}\, v_{4n-4+i} \leftarrow e_{7n-23+i}$;
end for
for i= 1 to n-1
  $v_{4n-5+2i}\, v_{4n-4+2i} \leftarrow e_{9n-13+i}$;
  end for
for i= 1 to n-2
  $v_{2i}\, v_{2n-2+2i} \leftarrow e_{2n+5i-6}$; $v_{2i+1}\, v_{2n-1+2i} \leftarrow e_{2n+5i-3}$;
  $v_{2i}\, v_{2n+2i-1} \leftarrow e_{2n+5i-5}$; $v_{2i+1}\, v_{2n+2i-2} \leftarrow e_{2n+5i-4}$; $v_{2n+2i-2}\, v_{2n+2i-1} \leftarrow e_{2n+5i-2}$;
end for
  $v_1v_{2n-1} \leftarrow e_{2n-2}$; $v_{2n-2}v_{4n-4} \leftarrow e_{7n-11}$;
end procedure
Output (Inflation of triangular snake graph)

Algorithm 3.2:

Input (Inflation of triangular snake graph)
Procedure (difference cordial labeling for Inflation of triangular snake graph)
$V \leftarrow \{v_1,v_2,\ldots,v_{6n-6}\}$
$E \leftarrow \{e_1,e_2,\ldots,e_{10n-14}\}$
  $f(v_1) \leftarrow 1$; $f(v_{2n-1}) \leftarrow 4n-4$; $f(v_{2n-2}) \leftarrow 4n-6$; $f(v_{4n-4}) \leftarrow 4n-5$;

for i= 1 to n-2;
f(v_{2i}) ←4i-2; f(v_{2i+1}) ←4i+1; f($v_{2n+2i-2}$) ←4i-1; f($v_{2n+2i-1}$) ←4i;
end for
for i= 1 to n-1;
f(v_{2i}) ←4i-2; f(v_{2i+1}) ←4i+1; f($v_{4n-4+2i}$) ←4n-4+2i-1;
f($v_{4n-4+2i-1}$) ←4n-4+2i;
end for
end procedure
Output (Labeled Inflation of triangular snake graph)

Theorem 3.3. *The Inflation of triangular snake $I(T_n)$ is difference cordial.*
Proof: Let T_n be a triangular snake graph, the Inflation of triangular snake graph has 6n-6 vertices and 10n-14 edges. To label the vertices, define a map f : V → {0,1} using algorithm 3.2. The number of vertices labeled by '1' and '0' is **3n-3**. Hence the number of vertices labeled by '1' and '0' are differ by atmost 1.

The 10n-14 edges are labeled by defining a induced map f^* : E → {0,1}such that $f^*(uv)=\begin{cases} 1; if\ f(u) = f(v) \\ 0; if\ f(u) \neq f(v) \end{cases}$. The edge labels are as follows:

1. for i= 1 to n-1, $f^*(v_{4n-4+2i-1}\, v_{4n-4+2i})=1$;
2. for i= 1 to n-2, $f^*(v_{2i}\, v_{2n-1+2i}) = f^*(v_{2i+1}\, v_{2n-1+2i-1}) =0$;
   $f^*(v_{2n-1+2i-1}\, v_{2n-1+2i})=1$;
3. for i= 1 to 2n-2, $f^*(v_{2n-2+i}\, v_{4n+i-4})=0$;
4. for i= 1 to 2n-4, $f^*(v_{i+1}\, v_{2n-1+i})=1$;
5. for i= 1 to 2n-3, $f^*(v_i\, v_{i+1})=\begin{cases} 1\,, i \equiv 1(mod\ 2) \\ 0, i \equiv 0(mod\ 2) \end{cases}$;
6. $f^*(v_{2n-2}\, v_{4n-4})=1$; $f^*(v_1\, v_{2n-1})=0$;

Now, the number of edges labeled '0' and '1' is **5n-7**. Thus the number of edges labeled '0' and the number of edges labeled '1'differ by atmost 1.

Hence Inflation of triangular snake graph is a difference cordial graph.

Algorithm 3.4:
Input (Inflation of triangular snake graph)
Procedure (pell labeling for Inflation of triangular snake graph)
V ←{v_1,v_2,…..,v_{6n-6} }
E←{e_1,e_2,…..,e_{10n-14}}
f(v_{2n-1}) ←2; f(v_{4n-3}) ←1; f(v_{4n-4}) ←6n-7; f(v_{6n-6}) ←6n-8;
for i= 1 to 2n-2;
f(v_i) ←3i-3;
end for
for i= 1 to 2n-4;
f(v_{2n+i-1}) ←3i+2; f(v_{4n+i-3}) ←3i+1;
end for
end procedure

Output (Labeled Inflation of triangular snake graph)

Theorem 3.5. *The inflation of triangular snake $I(T_n)$ is a pell graph.*

Proof: Let T_n be a triangular snake graph, from the algorithm 3.1 triangular snake graph become a Inflation of triangular snake and it has 6n-6 vertices and 10n-14 edges. To label the vertices, using algorithm 3.4, define a map $f : V \rightarrow \{0,1,2,..,p-1\}$.

In order to get the labels for all 10n-14 edges, define a induced map f^*: $E \rightarrow \{N\}$ such that for any $uv \in E$, $f^*(uv)=f(u)+2f(v)$, Thus the edge labels are as follows:

1. for i= 1 to n-1, $f^*(v_{2i-1}\, v_{2i})=18i-12$; $f^*(v_{4n-4+2i-1}\, v_{4n-4+2i})=18i-9$;
2. for i= 1 to n-2,
   $f^*(v_{2i+1}\, v_{2i})=18i-6$; $f^*(v_{2i}\, v_{2n-2+2i}) =18i-5$; $f^*(v_{2i}\, v_{2n-1+2i}) =18i+1$;
   $f^*(v_{2i+1}\, v_{2n-1+2i}) =18i+4$; $f^*(v_{2n-2+2i}\, v_{2n-1+2i})=18i+3$;
   $f^*(v_{2n-2+2i}\, v_{2i+1}) = 18i-1$;
3. for i= 1 to 2n-2, $f^*(v_{4n+i-3}\, v_{2n+i-1})=9i+5$;
4. $f^*(v_{2n-1}\, v_{4n-3})=4$; $f^*(v_{2n-2}\, v_{4n-4})=18n-23$; $f^*(v_{6n-6}v_{4n-4})=18n-22$;
5. $f^*(v_{2n-1}\, v_1)=2$; $f^*(v_{6n-5}v_{6n-6})=18n-27$;

The resultant edge labels are distinct.
Hence inflation of triangular snake graph is a pell graph.

Algorithm 3.6:
Input (Alternate triangular snake graph)
Procedure (Structure of Inflation of alternate triangular snake $I(A(T_n))$
$V \leftarrow \{v_1,v_2,\ldots..,v_{4n-4}\}$
$E \leftarrow \{e_1,e_2,\ldots..,e_{(11n-10)/2}\}$
  for i= 1 to 2n-3;
    $v_i\, v_{i+1} \leftarrow e_i$;
  end for
  for i= 1 to n-2;
    $v_{2i}v_{2n-1+i} \leftarrow e_{2n+2i-3}$; $v_{2i+1}\, v_{2n-1+i} \leftarrow e_{2n+2i-2}$;
  end for
  for i= 1 to n;
    $v_{2n+i-2}\, v_{3n+i-2} \leftarrow e_{4n+i-5}$;
  end for
  for i= 1 to $\frac{n}{2}$;
    $v_{3n+2i-3}\, v_{3n+2i-2} \leftarrow e_{5n+i-5}$;
  end for
    $v_{2n-2}\, v_{3n-2} \leftarrow e_{4n-5}$;
end procedure
Output (Inflation of Alternate triangular snake graph).

Algorithm 3.7:
Input (Inflation of alternate triangular snake graph $I(A(T_n)$,even n)

Procedure (difference cordial labeling for Inflation of Alternate triangular snake graph)

$V \leftarrow \{v_1, v_2, \ldots, v_{4n-4}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{(11n-10)/2}\}$

$f(v_1) \leftarrow 2$; $f(v_2) \leftarrow 6$; $f(v_3) \leftarrow 7$; $f(v_{2n-2}) \leftarrow 4n-2$; $f(v_{2n-1}) \leftarrow 1$; $f(v_{2n}) \leftarrow 5$;

$f(v_{3n-2}) \leftarrow 4n-3$; $f(v_{3n-1}) \leftarrow 4$; $f(v_{3n}) \leftarrow 3$;

for i= 1 to $\frac{n-2}{2}$;

$f(v_{4i}) \leftarrow 8i$; $f(v_{4i+1}) \leftarrow 8i+1$; $f(v_{2n+2i-1}) \leftarrow 8i+2$; $f(v_{3n+2i-1}) \leftarrow 8i-4$; $f(v_{3n+2i}) \leftarrow 8i+3$;

end for

for i= 1 to $\frac{n-4}{2}$;

$f(v_{4i+2}) \leftarrow 8i+5$; $f(v_{4i+3}) \leftarrow \begin{cases} 8i+6, i \equiv 1 (mod\ 2) \\ 8i+7, i \equiv 0 (mod\ 2) \end{cases}$;

$f(v_{2n+2i}) \leftarrow \begin{cases} 8i+7, i \equiv 1 (mod\ 2) \\ 8i+6, i \equiv 0 (mod\ 2) \end{cases}$;

end for

end procedure

Output (Labeled Inflation of Alternate triangular snake graph)

Theorem 3.8. *For even n, the inflation of alternate triangular snake $I(A(T_n))$ is difference cordial.*

Proof: Let $A(T_n)$ be a alternate triangular snake graph, the Inflation of alternate triangular snake has 4n-4 vertices and (11n-10)/2 edges. To label the vertices, define a map $f : V \rightarrow \{0,1\}$ using algorithm 3.7. Thus the number of vertices labeled by '1' and '0' is **2n-2**. Hence the number of vertices labeled by '1' and '0' are differ by atmost 1.

In order to get the labels for all (11n-10)/2 edges, define the induced map $f^* : E \rightarrow \{0,1\}$ such that $f^*(uv) = \begin{cases} 1; if\ f(u) = f(v) \\ 0; if\ f(u) \neq f(v) \end{cases}$. Thus the edge labels are as follows;

1. for i= 1 to n-3,

   $f^*(v_{2i+2} v_{2i+3}) = \begin{cases} 1, i \equiv 1,2,3 (mod\ 4) \\ 0, i \equiv 0 (mod\ 4) \end{cases}$;

   $f^*(v_{2i+2} v_{2n+i}) = \begin{cases} 0, i \equiv 1,2,3 (mod\ 4) \\ 1, i \equiv 0 (mod\ 4) \end{cases}$; $f^*(v_{2i+3} v_{2n+i}) = 1$;
2. for i= 1 to n-2,

   $f^*(v_{2i+1} v_{2i+2}) = \begin{cases} 1, i \equiv 1 (mod\ 4) \\ 0, i \equiv 0,2,3 (mod\ 4) \end{cases}$;
3. for i= 1 to n, $f^*(v_{2n-2+i} v_{3n-2+i}) = 0$;
4. for i= 1 to $\frac{n}{2}$, $f^*(v_{3n+2i-3} v_{3n+2i-2}) = 1$;
5. $f^*(v_1 v_2) = f^*(v_3 v_{2n}) = 0$;

   $f^*(v_2 v_3) = f^*(v_2 v_{2n}) = f^*(v_{2n-2} v_{3n-2}) = f^*(v_1 v_{2n-1}) = 1$;

Now, the number of edges labeled ‘1’ is $\frac{\mathbf{11n-8}}{4}$ and the number of edges labeled ‘0’ is $\frac{\mathbf{11n-12}}{4}$. Thus the number of edges labeled ‘0’ and the number of edges labeled ‘1’differ by atmost 1.

Hence inflation of alternate triangular snake graph is a difference cordial graph.

Algorithm 3.9:
Input (Inflation of alternate triangular snake graph)
Procedure (pell labeling for Inflation of alternate triangular snake graph)
$V \leftarrow \{v_1,v_2,\ldots,v_{4n-4}\}$
$E \leftarrow \{e_1,e_2,\ldots,e_{((11n-10)/2)}\}$
$f(v_1) \leftarrow 0$; $f(v_{2n-2}) \leftarrow 4n-5$; $f(v_{2n-1}) \leftarrow 1$;
$f(v_{3n-2}) \leftarrow 4n-4$; $f(v_{3n-1}) \leftarrow 2$; $f(v_{4n-2}) \leftarrow 4n-3$;
for i= 1 to n-2;
$f(v_{2i}) \leftarrow 4i-1$; $f(v_{2i+1}) \leftarrow 4i+1$;
$f(v_{2n-1+i}) \leftarrow 4i$; $f(v_{3n-1+i}) \leftarrow 4i+2$;
end for
end procedure
Output (Labeled Inflation of alternate triangular snake graph)

Theorem 3.10: *The Inflation of alternate triangular snake $I(A(T_n))$ is a Pell graph.*
Proof: Let $A(T_n)$ is an alternate triangular snake graph, from the algorithm 3.6 alternate triangular snake graph become a Inflation of alternate triangular snake and it has 4n-4 vertices and (11n-10)/2 edges. To label the vertices, using algorithm 3.9, define a map $f : V \rightarrow \{0,1,2,\ldots\ldots.p-1\}$.

In order to get the labels for all (11n-10)/2 edges, define a induced map $f^*: E\rightarrow\{N\}$such that for any $uv\in E$, $f^*(uv)=f(u)+2f(v)$, The edge labels are as follows:

$f^*(v_1 v_2)=6$; $f^*(v_1 v_{2n-1})=2$; $f^*(v_{2n-1} v_{3n-1})=5$; $f^*(v_{3n} v_{3n-1})=10$;
$f^*(v_{2n-3} v_{2n-2})=12n-17$; $f^*(v_{2n-2} v_{3n-2})=12n-13$;
$f^*(v_{3n-2} v_{4n-2})=12n-10$; $f^*(v_{4n-3} v_{4n-2})=12n-12$;

1. for i= 1 to n-2,
   $f^*(v_{2i} v_{2i+1}) =12i+1$; $f^*(v_{2i+1} v_{2i+2})=12i+7$; $f^*(v_{2i} v_{2n-1+i}) = 12i-1$;
   $f^*(v_{2n-1+i} v_{2i+1})=12i+2$; $f^*(v_{2n-1+i} v_{3n-1+i}) = 12i+4$;
2. for i= 1 to $\frac{n-2}{2}$, $f^*(v_{3n+2i} v_{3n+2i-1})=24i+10$;

Now, the edge labels are distinct.
Hence Inflation of alternate triangular snake graph is a pell graph.

4. Illustration

We now give the difference cordial labeling for inflated triangular snake $I(T_5)$ in fig 4.1 and Pell labeling for the alternate triangular snake graph $I(A(T_6))$ in fig 4.2.

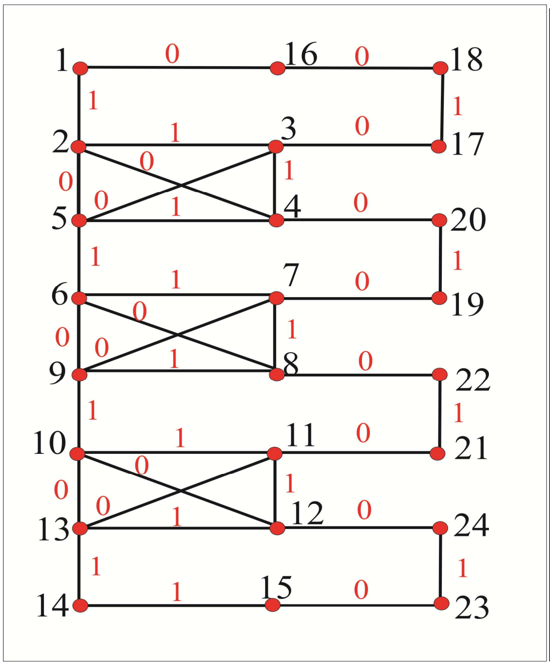

Figure 4.1:

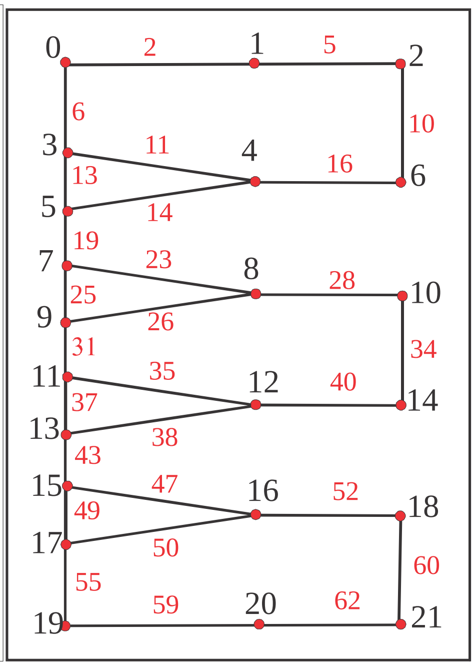

Figure 4.2:

4. Conclusion

In this paper, we proved that the Inflated triangular snake graph and inflated alternate triangular snake Graph admits difference cordial labeling and pell labeling .

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REFERENCES:

1. I.Cahit, Cordial graph: A weaker version of graceful and harmonious graphs, *Ars Combinatorial*, 23 (1987) 201-207.
2. J.E.Dunbar and T.W Haynes, Domination in inflated graphs, *Congr. Number* 118 (1996) 143-154.
3. O.Favaron. Irredundance in inflated graphs, *J. Graph Theory*, 28(2) (1998) 97-104.
4. J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics* 18(2013) #Ds6.
5. F.Harrary, Graph Theory, *Addison-Wesely*, *Reading*, *Massachusetts*, 1969.
6. Indra Rajasingh, Bharati Rajan and V.Annamma, On total vertex irregularity strength of triangle related graphs, *Annals of Pure and Applied Mathematics*, 1(2) (2012) 108-116.
7. R.Ponraj and S.Sathish Narayanan and R. Kala, Difference cordial labeling graphs, *Global J. Math. Sciences: Theory and Practical*, 3 (2013) 192- 201.
8. R.Ponraj and S.Sathish Narayanan, Difference cordial labeling of subdivision of snake graphs, U*niversal Journal of Applied Mathematics*, 2(1) (2014) 40-45.

9. J.Shiama, Pell labeling for some graphs, *Asian Journal of Current Engineering and Maths*, 2(4) (2013).
10. Sharon Philomena. V and K. Thirusangu, Square and cube difference labeling of cycle cactus, special tree and a new key graphs, *Annals of Pure and Applied Mathematics*, 8(2) (2014) 115-121.
11. K.Thirusangu, P.P.Ulaganathan and P. Vijaya Kumar, Some cordial labeling of duplicate graph of ladder graph, *Annals of Pure and Applied Mathematics*, 8(2) (2014) 43-50.