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On the *k*-Metro Domination Number of Paths

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Abstract. A dominating set D of a graph $G = G(V, E)$ is called metro dominating set of G. If for every pair of vertices u, v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$, The k-metro domination number of a path $\gamma_{\beta_k}(P_n)$, is the order of a smallest k-dominating set of P_n which resolves as a metric set. In this paper, we calculate the k-metro domination number of paths.

Keywords: Domination, metric dimension, metro domination.

AMS Mathematics Subject Classification (2010): 05C56, 05C38

1. Introduction

Let $G(V, E)$ be a graph. A subset of vertices $D \subseteq V$ is called a dominating set (γ- set) if every vertex in $V - D$ adjacent to at least one vertex in D [4].

The minimum cardinality of a dominating set is called the domination number of the graph G and is denoted by $\gamma(G)$.[4].

The metric dimension of a graph G is denoted by $\beta(G)$ is defined as the cardinality of a minimal subset $S \subseteq V$ having the property that for each pair of vertices u, v in G there exists a vertex w in S such that $d(u,w) \neq d(v,w)$The coordinate of each vertex v of V (G) with respect of each landmark u_i belong to S is defined as usual with ith component of v as $d(u, v_i)$ for each i and is of dimension $\beta(G)$[2].

Metro domination number introduced by Sooryanarayana and Raghunath [5]. Fink and Jacobson [6, 7] in 1985 introduced the concept of multiple domination. A subset D of V (G) is k-dominating in G if every vertex of $V - D$ has at least k neighbor's in D. The cardinality of minimum k- dominating set is called k- domination number $\boldsymbol{\gamma_k(G)}$.
A dominating set D of a graph G(V;E) is called metro dominating set of G if for each pair of vertices u; v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$.

2. Our results

Theorem 2.1. For $n \geq 11$, $\quad \gamma_{\beta_2}(P_n) = \left\lceil \frac{n}{5} \right\rceil$.

Proof: Let $v_1, v_2, v_3, \dots \dots \dots \dots . . v_n$ be the vertices of the path P_n. Let D be the minimum 2-dominating set of P_n. Let $W = V - D$, Now each $v_i \in W$ is either adjacent to any of the vertex D or atmost at distance two from atleast one of the vertex of D. Any vertex $v_k \in D$, will dominates at most 5 vertices including itself. Since the metric dimension of the Path $\beta(P_n) = 1$, as in [5], D itself serve as a metric set.

Thus $\gamma_{\beta_2}(P_n) \geq \left\lceil \frac{n}{5} \right\rceil$ (i)

To prove $\gamma_{\beta_2}(P_n) \leq \left\lceil \frac{n}{5} \right\rceil$,

We define a set D as follows

Case 1: For $n = 5l + 6,\ l \geq 1,$

$D = \left\{V_{5k+3};\ 0 \leq k \leq \left\lfloor \frac{n-2}{5} \right\rfloor\right\} \cup v_n$

Case 2: For $n = 5l + 7,\ l \geq 1,$

$$D = \left\{V_{5k+3};\ 0 \leq k \leq \left\lfloor \frac{n-3}{5} \right\rfloor\right\} \cup v_n$$

Case 3: For $n = 5l + 8,\ l \geq 1,$

$D = \left\{V_{5k+3};\ 0 \leq k \leq \left\lfloor \frac{n-4}{5} \right\rfloor\right\} \cup v_n$

Case 4: For $n = 5l + 9,\ l \geq 1,$

$D = \left\{V_{5k+3};\ 0 \leq k \leq \left\lfloor \frac{n}{5} \right\rfloor\right\} \cup v_n$

Case 5: For $n = 5l + 10,\ l \geq 1,$

$$D = \left\{V_{5k+3};\ 0 \leq k \leq \left\lfloor \frac{n-1}{5} \right\rfloor\right\} \cup v_n$$

We note that D is a 2-dominating set for P_n, and also D will serves as a metric set of P_n .

Thus $\gamma_{\beta_2}(P_n) \leq \left\lceil \frac{n}{5} \right\rceil$ (ii)

From (i) and (ii)

$\gamma_{\beta_2}(P_n) = \left\lceil \frac{n}{5} \right\rceil$.

Case 1: For $n = 5l + 6,\qquad l \geq 1,$

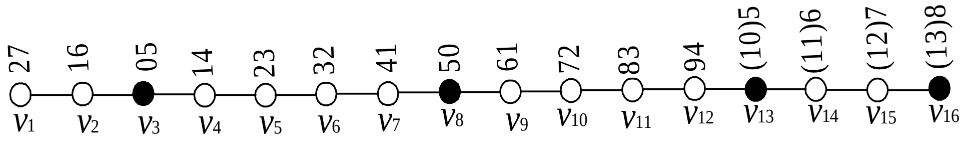

Figure 1: $\gamma_{\beta_2}(P_{16}) = 4$

Case 2: For $n = 5l + 7, \quad l \geq 1,$

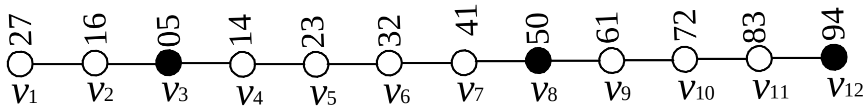

Figure 2: $\gamma_{\beta_2}(P_{12}) = 3$

Case 3: For $n = 5l + 8, \quad l \geq 1,$

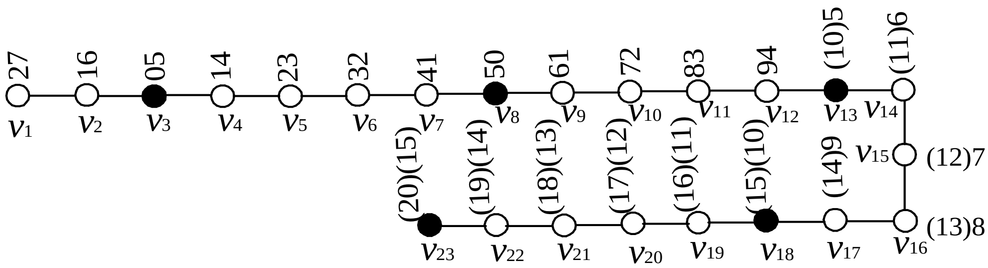

Figure 3: $\gamma_{\beta_2}(P_{23}) = 5$

Case 4: For $n = 5l + 9, \quad l \geq 1,$

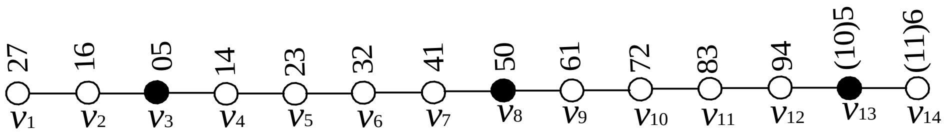

Figure 4: $\gamma_{\beta_2}(P_{14}) = 3$

Case 5: For $n = 5l + 10, \quad l \geq 1,$

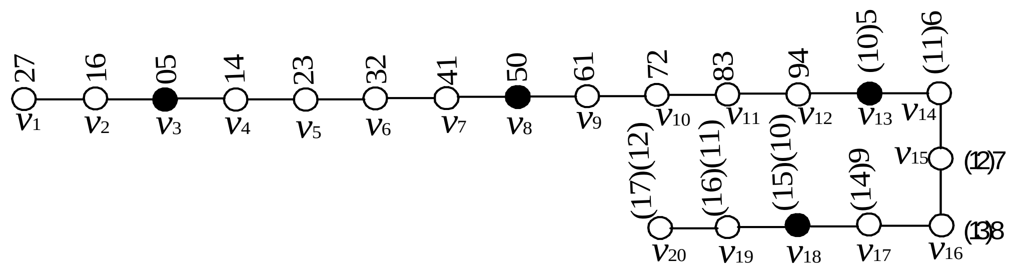

Figure 5: $\gamma_{\beta_2}(P_{20}) = 4$

Theorem 2.2. For $n \geq 8$, $\gamma_{\beta_3}(P_n) = \left\lceil \frac{n}{7} \right\rceil$.

Proof: Let $v_1, v_2, v_3, \dots\dots\dots\dots.. v_n$ be the vertices of the path P_n. Let D be the minimum 3-dominating set of P_n. Let $W = V - D$, Now each $v_i \in W$ is either adjacent to any of the vertex D are atleast at the distance three from atleast one of the vertex of D. Any vertex $v_k \in D$, will dominates at most 7 vertices including itself, D also serves as a metric set.

Thus $\gamma_{\beta_3}(P_n) \geq \left\lceil \frac{n}{7} \right\rceil$ (i)

To prove $\gamma_{\beta_3}(P_n) \leq \left\lceil \frac{n}{7} \right\rceil$,

We define a set D as follows

Case 1: $n = 7l + 1, 7l + 2, 7l + 3, \quad l \geq 1$

$D = \left\{ V_{7k-3};\ 1 \leq k \leq \left\lfloor \frac{n}{7} \right\rfloor \right\} \cup v_n$

Case 2: $n = 7l + 4, 7l + 5, 7l + 6, 7l + 7, \quad l \geq 1$

$D = \left\{ V_{7k-3};\ 1 \leq k \leq \left\lfloor \frac{n+3}{7} \right\rfloor \right\}$.

We note that D is a 3-dominating set for P_n, and also D will serves as a metric set of P_n .

Thus $\gamma_{\beta_3}(P_n) \leq \left\lceil \frac{n}{7} \right\rceil$ (ii)

From (i) and (ii)

$\gamma_{\beta_3}(P_n) = \left\lceil \frac{n}{7} \right\rceil$.

Case 1: $n = 7l + 1, 7l + 2, 7l + 3, \qquad l \geq 1,$

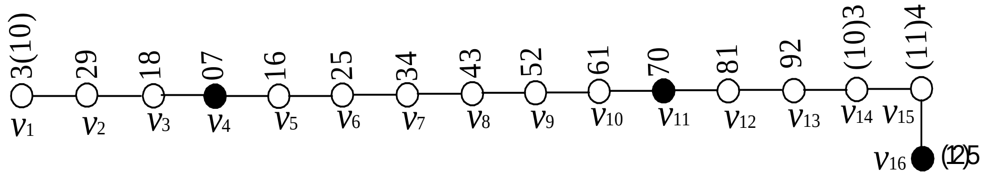

Figure 6: $\gamma_{\beta_3}(P_{16}) = 3$

Case 2: $n = 7l + 4, 7l + 5, 7l + 6, 7l + 7, \qquad l \geq 1,$

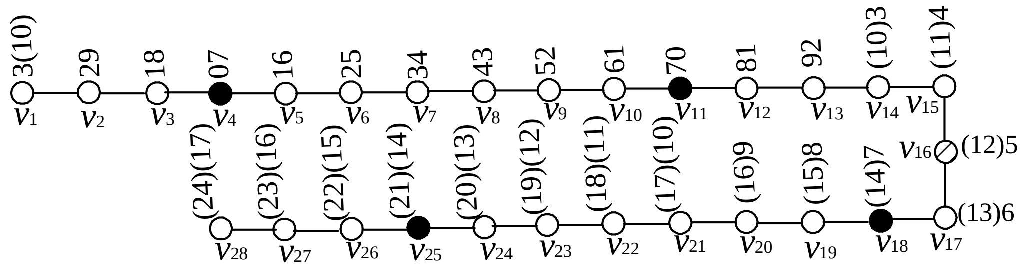

Figure 7: $\gamma_{\beta_3}(P_{28}) = 4$

Theorem 2.3. For $n \geq 10$, $\gamma_{\beta_4}(P_n) = \left\lceil \frac{n}{9} \right\rceil$.

Proof: Let $v_1, v_2, v_3, \dots\dots\dots\dots.. v_n$ be the vertices of the path P_n. Let D be the minimum 4-dominating set of P_n. Let $W = V - D$, Now each $v_i \in W$ is either adjacent to any of the vertex D are atleast at the distance four from atleast one of the vertex of D. Any vertex $v_k \in D$, will dominates at most 9 vertices including itself D also serves as a metric set.

Thus $\gamma_{\beta_4}(P_n) \geq \left\lceil \frac{n}{9} \right\rceil$ (i)

To prove $\gamma_{\beta_4}(P_n) \leq \left\lceil \frac{n}{9} \right\rceil$,

We define a set D as follows

Case 1: $n = 9l + 1, 9l + 2, 9l + 3, 9l + 4, 9l + 5, \quad l \geq 1$

$D = \left\{ V_{9k-4};\ 1 \leq k \leq \left\lfloor \frac{n}{9} \right\rfloor \right\} \cup v_n$

Case 2: $n = 9l + 6, 9l + 7, 9l + 8, 9l + 9, \quad l \geq 1$

$D = \left\{ V_{9k-4};\ 1 \leq k \leq \left\lfloor \frac{n+3}{9} \right\rfloor \right\}$.

We note that D is a 4-dominating set for P_n, and also D will serves as a metric set of P_n .

Thus $\gamma_{\beta_4}(P_n) \leq \left\lceil \frac{n}{9} \right\rceil$ (ii)

From (i) and (ii)

$\gamma_{\beta_4}(P_n) = \left\lceil \frac{n}{9} \right\rceil$.

Case 1: $n = 9l + 1, 9l + 2, 9l + 3, 9l + 4, 9l + 5$, $\quad l \geq 1$,

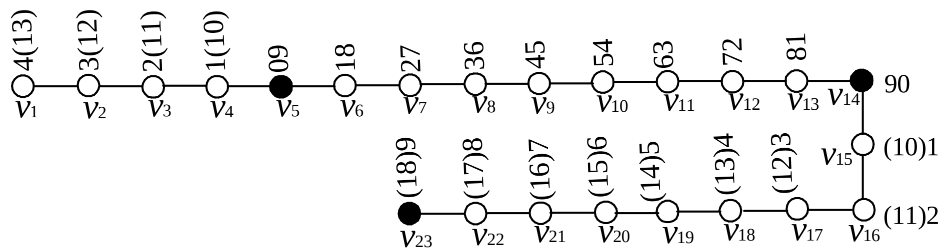

Figure 8: $\gamma_{\beta_4}(P_{23}) = 3$

Case 2: $n = 9l + 6, 9l + 7, 9l + 8, 9l + 9$, $\quad l \geq 1$,

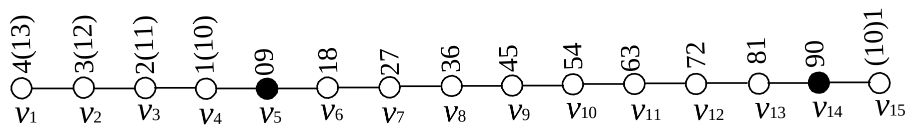

Figure 9: $\gamma_{\beta_4}(P_{15}) = 2$

Theorem 2.4. For $n \geq 12$, $\gamma_{\beta_5}(P_n) = \left\lceil \frac{n}{11} \right\rceil$.

Proof: Let $v_1, v_2, v_3, \ldots\ldots\ldots\ldots v_n$ be the vertices of the path P_n. Let D be the minimum 5-dominating set of P_n. Let $W = V - D$, Now each $v_i \in W$ is either adjacent to any of the vertex D are atleast at the distance five from atleast one of the vertex of D. Any vertex $v_k \in D$, will dominates at most 11 vertices including v_k, D also serves as a metric set of P_n.

Thus $\gamma_{\beta_5}(P_n) \geq \left\lceil \frac{n}{11} \right\rceil$ (i)

To prove $\gamma_{\beta_5}(P_n) \leq \left\lceil \frac{n}{11} \right\rceil$,

We define a set D as follows

Case 1:

$n = 11l + 1, 11l + 2, 11l + 3, 11l + 4, 11l + 5, 11l + 6, 11l + 7$, $l \geq 1$,

$D = \left\{V_{11k-5};\ 1 \leq k \leq \left\lfloor \frac{n}{11} \right\rfloor\right\} \cup v_n$

Case 2: $n = 11l + 8, 11l + 9, 11l + 10, 11l + 11$, $l \geq 1$

$D = \left\{V_{11k-5};\ 1 \leq k \leq \left\lfloor \frac{n+3}{11} \right\rfloor\right\}$.

We note that D is a 5-dominating set for P_n, and also D will serves as a metric set of P_n.

Thus $\gamma_{\beta_5}(P_n) \leq \left\lceil \frac{n}{11} \right\rceil$ (ii)

From (i) and (ii)

$\gamma_{\beta_5}(P_n) = \left\lceil \frac{n}{11} \right\rceil$.

Case 1:

$n = 11l + 1, 11l + 2, 11l + 3, 11l + 4, 11l + 5, 11l + 6, 11l + 7$, $l \geq 1$,

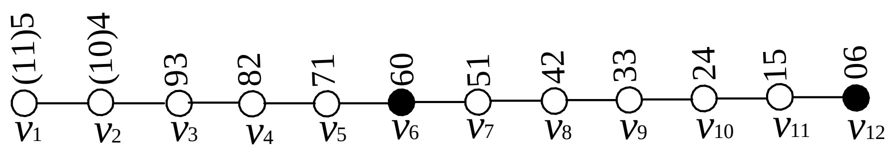

Figure 10: $\gamma_{\beta_5}(P_{12}) = 2$

Case 2: $n = 11l + 8, 11l + 9, 11l + 10, 11l + 11$, $l \geq 1$,

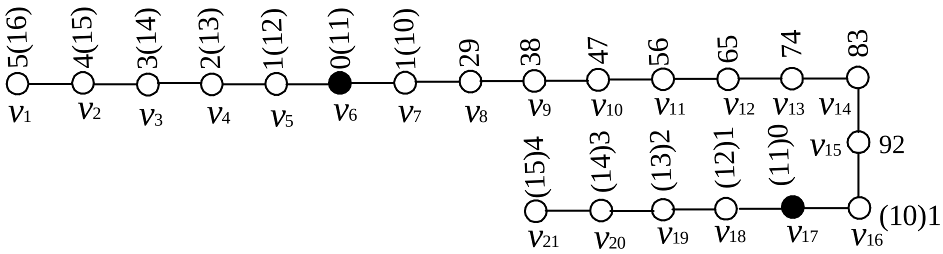

Figure 11: $\gamma_{\beta_5}(P_{21}) = 2$

Theorem 2.5. For $n \geq 2l + 6, \quad l \geq 1,$

$$\gamma_{\beta_k}(P_n) = \left\lceil \frac{n}{2m+3} \right\rceil, k \geq 2,\ m \geq 1.$$

Proof follows from generalization of theorem1, theorem2, theorem3 and theorem4.

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