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Lucky Edge Labeling of H-Super Subdivision of Graphs

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Abstract. A function $f:V\rightarrow N$ is said to be lucky edge labeling if there exists a function $f^*: E \rightarrow N$ such that $f^*(uv) = f(u) + f(v)$and the edge set E(G) has a proper edge coloring of G. That is $f^*(e_i)\neq f^*(e_j)$,whenever e_i and e_j are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {2, 3, . . . , k} is the lucky number of G and is denoted by η(G). A graph which admits lucky edge labeling is the lucky edge labeled graph. In this paper, we show that the H- super subdivision of path,cycle and corona of C_n graphs are lucky edge labeled graphs and we obtain their lucky numbers of these graphs.

Keywords: Lucky edge labeling, Lucky edge labeled graph, Lucky number, H- super subdivision , path graph, cycle graph, corona of C_n graphs.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Graph labeling is an interesting area of research in graph theory introduced by Rosa in 1967[12]. It is defined as an assignment of integers to the vertices or edges or both subject to certain conditions. For a graph G (V,E), an edge labeling is a function from E to a set of labels. A graph in which such a function is defined is called an edge-labeled graph. The concept of lucky edge labeling was introduced by Nellai Murugan and Mariya Irudhaya Aspin Chitra [9]. Some results on lucky edge labeling of path graph, cycle graph, corona of cycle and path graphs, bi-star, wheel graph, fan graph, ladder graph, shell graph, Triangular graph, and planar grid graph were discussed in [1,9,10,11]. Sethuraman and Selvaraju [13] introduced the concept of super subdivision of graph. The Lucky edge labeling of super subdivision of some graphs (path graphs, planar grid graph, star and wheel graphs) were discussed in [3,4,5]. The H- super subdivision of graph was introduced by Esakkiammal et.al. in 2016 [3]. In this paper we prove that the *H*- Super subdivision of path, cycle, corona of cycle graphs are lucky edge labeled graphs and the lucky numbers of these graphs are obtained. One may refer to Harary [8] for all terminologies and notations and [6] for graph labeling.

In this paper we use the following definitions.

Definition 1.1. [9] Let G be a simple graph with vertex set *V(G)* and the edge set *E(G)*. Vertex set *V(G)* is labeled arbitrarily by positive integers and let $f^*(e)$ denotes the edge label such that it is the sum of labels of vertices incident with *e*. The labeling is said to be lucky edge labeling if the edge set *E(G)* has a proper edge coloring of G.That is $f^*(e_i)\neq f^*(e_j)$,whenever e_i and e_j are adjacent edges. The least integer *k* for which a graph G has a lucky edge labeling from the set $\{2, 3, . . .k\}$ is the lucky number of G and is denoted by $\eta(G)$.

Definition 1.2. [3] Let G be a (*p,q*) graph. A graph obtained from *G* by replacing each edge e_i by a *H*-graph in such a way that the ends of e_i are merged with a pendent vertex in P_2 and a pendent vertex P_2' is called *H*-super subdivision of G and it is denoted by *HSS(G)*,where the *H*-graph is a tree on 6 vertices in which exactly two vertices of degree 3.

Definition 1.3. [6] A path graph is a sequence of vertices and edges, beginning and ending with vertices such that each edge is incident with the vertices preceding and following it. Edges and vertices appear only once in a path. A path graph of length *n* has *n*+1 vertices and this graph is denoted by P_n.

Definition 1.4. [6] A closed path is called a cycle and a cycle of length n is denoted by C_n.

Definition 1.5. [6] The corona of cycle graph C_n is obtained from C_n by attaching a pendent vertex to each vertex of C_n it is denoted by C_n^+.

2. Main results

Lucky Edge labeling of H- Super Subdivision of a path graph P_n

Algorithm 2.1.

Procedure: Lucky edge labeling of $HSS(P_n)$,, n≥1

Input: $HSS(P_n)$ graph

V← $\{v_i, v_{i(i+1)}^{(1)}, v_{(i+1)i}^{(1)}, v_{i(i+1)}^{(2)}, v_{(i+1)i}^{(2)}$ / $1\leq i \leq n\} \cup \{v_{n+1}\}$

For i= 1to *n*+1 do

{

If i≡ 1(mod 4) or i≡ 2(mod 4)

v_i← 2;

else

```
                    v_i ← 1;
            end if
        }
end for
For  i= 1 to n do
        {
            If  i≡ 1(mod 4) do
                    {
                    v^(1)_i(i+1) ← 1; v^(1)_(i+1)i →1;
                    }
            Else if  i≡ 2(mod 4) do
                    {
                    v^(1)_i(i+1) ← 2; v^(1)_(i+1)i →1;
                    }
                  Else if  i≡ 3(mod 4) do
                    {
                    v^(1)_i(i+1) ← 2; v^(1)_(i+1)i →2;
                    }
                  else
                    {
                    v^(1)_i(i+1) ← 1; v^(1)_(i+1)i →2;
                    }
                  end if
               end if
          end if
        }
end for
For i = 1 to n do
   {
        v^(2)_i(i+1) ← 3; , v^(2)_(i+1)i →3;
   }
end for
end procedure
```

Output: The vertex labeled *HSS* (P_n), $n \geq 1$.

Complexity of the Algorithm : Clearly this algorithm runs in linear time.

Theorem 2.1. *The H- Super subdivision of a path is a lucky edge labeled graph and the lucky number is* $\eta(HSS(P_n)) = 5$.

Proof: Let *HSS (P_n)*, be the H- Super subdivision of a path graph whose vertex set is V= $\{v_i, v^{(1)}_{i(i+1)}, v^{(1)}_{(i+1)i}, v^{(2)}_{i(i+1)}, v^{(2)}_{(i+1)i}$ / 1≤ i ≤ n }∪{ $v_{n+1}\}$ and the edge set E =$\{v_i v^{(1)}_{i(i+1)}, v^{(1)}_{i(i+1)} v^{(2)}_{i(i+1)}, v^{(1)}_{i(i+1)} v^{(1)}_{(i+1)i}, v^{(1)}_{(i+1)i} v^{(2)}_{(i+1)i}, v^{(1)}_{(i+1)i} v_{(i+1)}$/1≤ i ≤ n} where n≥ 1.

The vertices of *HSS (P_n)* are labeled by defining a function $f : V(HSS(P_n)) \rightarrow N$ as given in Algorithm 2.1.

Define the induced function $f^*: E(HSS(P_n)) \rightarrow N$ such that $f^*(uv) = f(u) + f(v)$,for every $uv \in$ E.

Now the edges labels are calculated as follows:

For $1 \leq i \leq n$,

Case(i): i≡ 1(mod 4)

$f^*(v_i v^{(1)}_{i(i+1)}) = 3$; $f^*(v^{(1)}_{i(i+i)} v^{(2)}_{i(i+1)}) = 4$; $f^*(v^{(1)}_{i(i+1)} v^{(1)}_{(i+1)i}) = 2$; $f^*(v^{(1)}_{(i+1)i} v^{(2)}_{(i+1)i}) = 4$; $f^*(v^{(1)}_{(i+1)i} v_{(i+1)}) = 3$;

Case(ii): i≡ 2(mod 4)

$f^*(v_i v^{(1)}_{i(i+1)}) = 4$; $f^*(v^{(1)}_{(i+1)i} v_{(i+1)}) = 2$; $f^*(v^{(1)}_{i(i+i)} v^{(2)}_{i(i+1)}) = 5$; $f^*(v^{(1)}_{(i+1)i} v^{(2)}_{(i+1)i}) = 4$; $f^*(v^{(1)}_{i(i+1)} v^{(1)}_{(i+1)i}) = 3$;

Case(iii): i≡ 3(mod 4)

$f^*(v_i v^{(1)}_{i(i+1)}) = 3$; $f^*(v_{(i+1)} v^{(1)}_{(i+1)i}) = 3$; $f^*(v^{(1)}_{i(i+1)} v^{(2)}_{i(i+1)}) = 5$;
$f^*(v^{(1)}_{(i+1)i} v^{(2)}_{(i+1)i}) = 5$;
$f^*(v^{(1)}_{i(i+1)} v^{(1)}_{(i+1)i}) = 4$;

Case(iv): i≡ 0(mod 4)

$f^*(v_i v^{(1)}_{i(i+1)}) = 2$; $f^*(v_{(i+1)} v^{(1)}_{(i+1)i}) = 4$; $f^*(v^{(1)}_{i(i+1)} v^{(2)}_{i(i+1)}) = 4$;
$f^*(v^{(1)}_{(i+1)i} v^{(2)}_{(i+1)i}) = 5$;
$f^*(v^{(1)}_{i(i+1)} v^{(1)}_{(i+1)i}) = 3$;

From all the above cases, we get $f^*(E(HSS(P_n))$ ={ 2,3,4,5} and all the adjacent edges are properly colored. Thus *HSS(P_n)* admits lucky edge labeling and the lucky number is $\eta(HSS(P_n)) = 5$.

Hence *HSS(P_n)* is a lucky edge labeled graph.

Example 2.1.

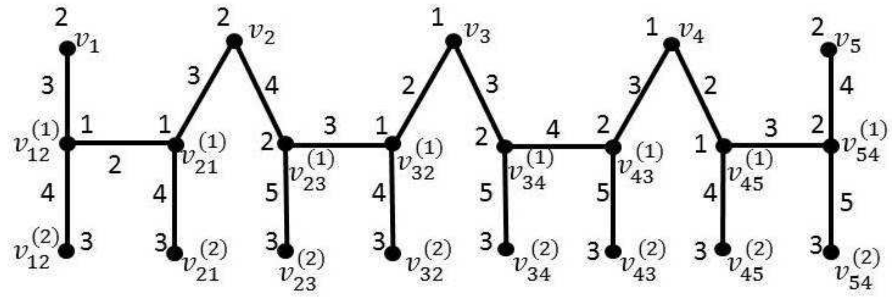

Figure 1: Lucky edge labeling of *HSS(P_4)*

Lucky Edge Labeling of H- Super Subdivision of Cycle graph C_n, n≥ 3
Algorithm 2.2
Procedure: Lucky edge labeling of *HSS(C_n)* , n≥ 3
Input: *HSS(C_n)* graph
V= $\{v_i, v_{i(i+1)}^{(1)}, v_{(i+1)i}^{(1)}, v_{i(i+1)}^{(2)}, v_{(i+1)i}^{(2)}$ / 1≤ i ≤ n-1} $\cup\{v_n, v_{n1}^{(1)}, v_{n1}^{(2)}, v_{1n}^{(1)}, v_{1n}^{(2)}\}$
$v_{n1}^{(1)} \leftarrow 1; v_{n1}^{(2)} \leftarrow 1; v_{1n}^{(1)} \leftarrow 2; v_{1n}^{(2)} \leftarrow 2;$
For i = 1 to n do
$v_i \leftarrow 3;$
end for
For i = 1 to n-1 do
{
$v_{i(i+1)}^{(1)} \leftarrow 1; , v_{(i+1)i}^{(1)} \rightarrow 2;$
$v_{i(i+1)}^{(2)} \leftarrow 1; , v_{(i+1)i}^{(2)} \rightarrow 2:$
}
end for
end procedure
Output: The vertex labeled *HSS(C_n)* , n≥ 3
Complexity of the Algorithm : Clearly this algorithm runs in linear time.

Theorem 2.2. *The H- super subdivision of a cycle is a lucky edge labeled graph and the lucky number is η(HSS(C_n)) = 5 , (n≥ 3).*
Proof: Let *HSS(C_n)*, n≥ 3 be the *H*- super subdivision of a cycle graph, whose vertex set is
V= $\{v_i, v_{i(i+1)}^{(1)}, v_{(i+1)i}^{(1)}, v_{i(i+1)}^{(2)}, v_{(i+1)i}^{(2)}$ / 1≤ i ≤ n-1} $\cup\{v_n, v_{n1}^{(1)}, v_{n1}^{(2)}, v_{1n}^{(1)}, v_{1n}^{(2)}\}$ and the edge set

$E = \{v_i v^{(1)}_{i(i+1)}, v^{(1)}_{i(i+1)} v^{(2)}_{i(i+1)}, v^{(1)}_{i(i+1)} v^{(1)}_{(i+1)i}, v^{(1)}_{(i+1)i} v^{(2)}_{(i+1)i}, v^{(1)}_{(i+1)i} v_{(i+1)} / 1 \leq i \leq n-1\} \cup \{v_n v^{(1)}_{n1}, v^{(1)}_{n1} v^{(2)}_{n1}, v^{(1)}_{n1} v^{(1)}_{1n}, v^{(1)}_{1n} v^{(2)}_{1n}, v^{(1)}_{1n} v_1\}.$

The vertices of *HSS*(C_n) are labeled by defining a function $f: V(HSS(C_n)) \rightarrow N$ as given in Algorithm 2.2. Thus the vertices of *HSS*(C_n) are labeled.

Define the induced function $f^* : E(HSS(C_n)) \rightarrow N$ such that $f^*(uv) = f(u) + f(v)$ for every $uv \in E$.

Now the edge labels are calculated as follows.

For $1 \leq i \leq n-1$

$f^*(v_i v^{(1)}_{i(i+1)}) = 4$; $f^*(v^{(1)}_{i(i+1)} v^{(2)}_{i(i+1)}) = 2$; $f^*(v^{(1)}_{i(i+1)} v^{(1)}_{(i+1)i}) = 3$;

$f^*(v^{(1)}_{(i+1)i} v^{(2)}_{(i+1)i}) = 4$; $f^*(v^{(1)}_{(i+1)i} v_{(i+1)}) = 5$;

For the remaining edges that is when i=n

$f^*(v_n v^{(1)}_{n1}) = 4$; $f^*(v^{(1)}_{n1} v^{(2)}_{n1}) = 2$; $f^*(v^{(1)}_{n1} v^{(1)}_{1n}) = 3$;

$f^*(v^{(1)}_{1n} v^{(2)}_{1n}) = 4$; $f^*(v^{(1)}_{1n} v_1) = 5$;

Thus $f^*(E(HSS(C_n)) = \{2,3,4,5\}$ and all the adjacent edges are properly colored. It is clear that *HSS*(C_n), $n \geq 3$ admits lucky edge labeling and the lucky number is $\eta(HSS(C_n)) = 5$. Hence *HSS*(C_n), $n \geq 3$ is a lucky edge labeled graph.

Example 2.2.

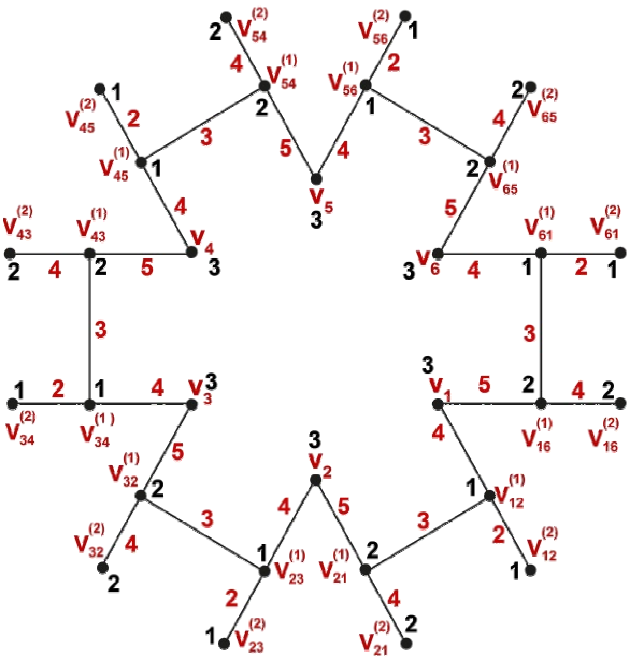

Figure 2: Lucky edge labeling of HSS(C_6)

Lucky Edge Labeling of *H*- Super Subdivision of Corona graph C_n^+, n≥ 3

Algorithm 2.3.

Procedure : Lucky edge labeling of $HSS(C_n^+)$, n≥ 3

Input: $HSS(C_n^+)$ graph

V= $\{v_i, v_{i(i+1)}^{(1)}, v_{(i+1)i}^{(1)}, v_{i(i+1)}^{(2)}, v_{(i+1)i}^{(2)}$ / 1≤ i ≤ n-1}

$\cup \{v_n, v_{n1}^{(1)}, v_{n1}^{(2)}, v_{1n}^{(1)}, v_{1n}^{(2)}\} \cup \{(vu)_i^{(1)}(vu)_i^{(2)}(uv)_i^{(1)}(uv)_i^{(2)}u_i$/ 1≤ i ≤ n}

$v_{n1}^{(1)} \leftarrow 1; v_{n1}^{(2)} \leftarrow 1; v_{1n}^{(1)} \leftarrow 2; v_{1n}^{(2)} \leftarrow 2;$

For i = 1 to n do

{

$v_i \leftarrow 3;\ u_i \leftarrow 2;\ (vu)_i^{(1)} \leftarrow 3;\ (vu)_i^{(2)} \leftarrow 2; (uv)_i^{(1)} \leftarrow 1;\ (uv)_i^{(2)} \leftarrow 1;$

}

end for

For i = 1 to (n-1) do

{

$v_{i(i+1)}^{(1)} \leftarrow 1; v_{i(i+1)}^{(2)} \leftarrow 1; v_{(i+1)i}^{(1)} \leftarrow 2;\ v_{(i+1)i}^{(2)} \leftarrow 2;$

}

end for

end procedure

Output: The vertex labeled HSS(C_n^+), n≥ 3

Complexity of the Algorithm : Clearly this algorithm runs in linear time.

Theorem 2.3. *The H- super subdivision of a corona of cycle graph is a lucky edge labeled graph and the lucky number is η(HSS(C_n^+)) = 6, (n≥ 3).*

Proof: Let *HSS(C_n^+)*,n ≥ 3 be the *H*- super subdivision of a corona graph, whose vertex set is

V= $\{v_i, v_{i(i+1)}^{(1)}, v_{(i+1)i}^{(1)}, v_{i(i+1)}^{(2)}, v_{(i+1)i}^{(2)}$ / 1≤ i ≤ n-1} $\cup \{v_n, v_{n1}^{(1)}, v_{n1}^{(2)}, v_{1n}^{(1)}, v_{1n}^{(2)}\}$

$\cup \{(vu)_i^{(1)}(vu)_i^{(2)}(uv)_i^{(1)}(uv)_i^{(2)}u_i$/ 1≤ i ≤ n}

and the edge set

E=$\{v_i v_{i(i+1)}^{(1)}, v_{i(i+1)}^{(1)} v_{i(i+1)}^{(2)}, v_{i(i+1)}^{(1)} v_{(i+1)i}^{(1)}, v_{(i+1)i}^{(1)} v_{(i+1)i}^{(2)}, v_{(i+1)i}^{(1)} v_{(i+1)}$/1≤ i ≤ n-1}

$\cup \{v_n v_{n1}^{(1)}, v_{n1}^{(1)} v_{n1}^{(2)}, v_{n1}^{(1)} v_{1n}^{(1)}, v_{1n}^{(1)} v_{1n}^{(2)}, v_{1n}^{(1)} v_1\} \cup$

$\{v_i(vu)_i^{(1)}, (vu)_i^{(1)}(vu)_i^{(2)}, (uv)_i^{(1)}(uv)_i^{(2)}, u_i(uv)_i^{(1)}(vu)_i^{(1)}(uv)_i^{(1)}/1 \le i \le n\}$.

The vertices of *HSS(C_n^+)* are labeled by defining a function *f:V(HSS(C_n^+))→N* as given in Algorithm 2.3. Thus the vertices of *HSS(C_n^+)* are labeled.

Define the induced function f^* *: E(HSS(C_n^+))→ N* such that *f*(uv) = f(u) + f(v)*for every *uv* ∈*E.*

Now the edge labels are calculated as follows.

Case(i): For 1≤ i ≤ n-1

f*($v_i v_{i(i+1)}^{(1)}$) =4; f*($v_{i(i+1)}^{(1)} v_{i(i+1)}^{(2)}$)=2; f*($v_{i(i+1)}^{(1)} v_{(i+1)i}^{(1)}$)=3;

f*($v_{(i+1)i}^{(1)} v_{(i+1)i}^{(2)}$)=4; f*($v_{(i+1)i}^{(1)} v_{(i+1)}$) =5;

Case(ii): For 1≤ i ≤ n

f*($v_i (vu)_i^{(1)}$) =6; f*($(vu)_i^{(1)} (vu)_i^{(2)}$) =5; f*($(vu)_i^{(1)} (uv)_i^{(1)}$) =4;

f*($(uv)_i^{(1)} (uv)_i^{(2)}$) =2; f*($(uv)_i^{(1)} u_i$) =3;

Case(iii): For the remaining edges

f*($v_n v_{n1}^{(1)}$) =4; f*($v_{n1}^{(1)} v_{n1}^{(2)}$) =2; f*($v_{n1}^{(1)} v_{1n}^{(1)}$) =3;

f*($v_{1n}^{(1)} v_{1n}^{(2)}$) =4; f*($v_{1n}^{(1)} v_1$) =5;

From all the above cases *f*(E(HSS(C_n^+))* ={ 2,3,4,5,6} and all the adjacent edges are properly colored. Thus the graph *HSS(C_n^+)*,n≥ 3 admits lucky edge labeling and the lucky number is *η(HSS(C_n^+)) = 6*.

Hence *HSS(C_n^+)*, n≥ 3 is a lucky edge labeled graph.

Example 2.3.

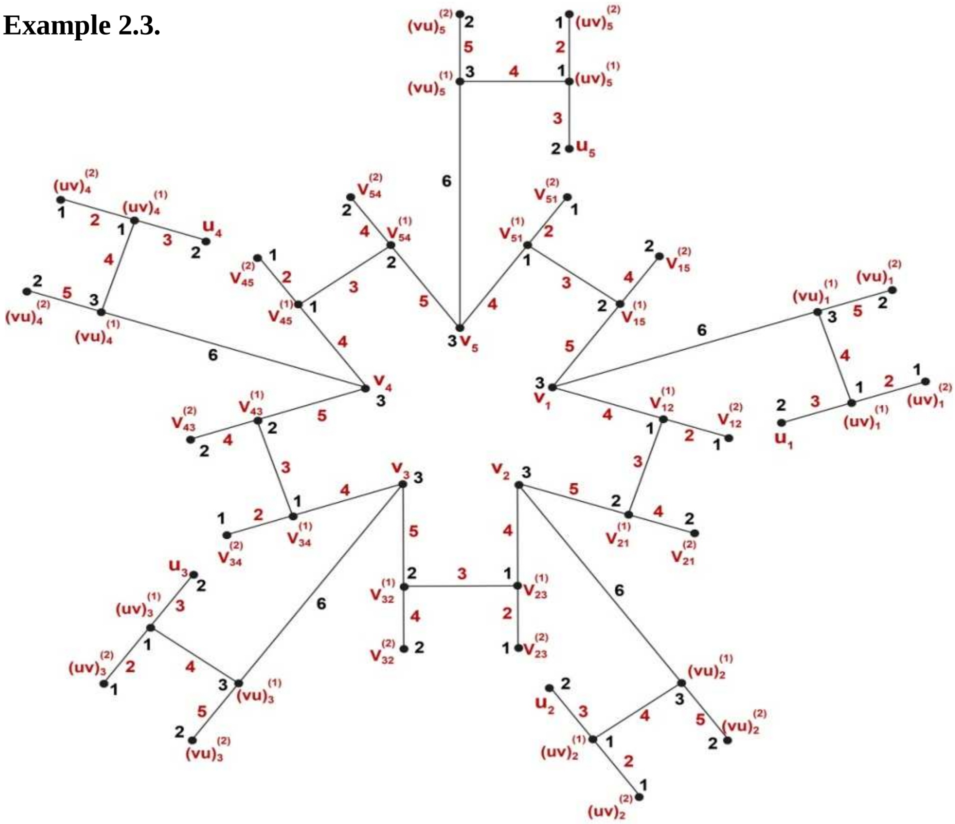

Figure 3: Lucky Edge Labeling of HSS(C_5^+)

3. Conclusion

In this paper, we have proved that the *H*- Super subdivision of path, cycle and corona of cycle graph are lucky edge labeled graphs and their lucky numbers are obtained.

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