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The Sum Connectivity Revan Index of Silicate and Hexagonal Networks

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Abstract. In Chemical Graph Theory, the connectivity indices are applied to measure the chemical characteristics of chemical compounds. In this paper, we introduce the sum connectivity Revan index of a molecular graph. Also we compute the sum connectivity Revan index for certain networks of chemical importance like silicate networks and hexagonal networks. The first and second Revan indices of silicate networks and hexagonal networks are determined.

Keywords: Sum connectivity Revan index, silicate networks, hexagonal networks.

AMS Mathematics Subject Classification (2010): 05C05, 05C07, 05C12, 05C35

1. Introduction

We consider only finite, simple and connected graph G with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. Let $\Delta(G)(\delta(G))$ denote the maximum (minimum) degree among the vertices of G. We refer [1] for undefined notations and terminologies.

Chemical graph theory is a branch of Mathematical chemistry which has an important effect on the development of the chemical sciences. Numerous topological indices have been considered in theoretical chemistry, and have found some applications in *QSPR*/*QSAR* study, see [2].

The Revan vertex degree of a vertex v of a graph G is defined as

$$r_G(v) = \Delta(G) + \delta(G) - d_G(v).$$

The Revan edge connecting the Revan vertices u and v will be denoted by uv. In [3], Kulli introduced the first and second Revan indices of a graph G. These indices are defined as

$$R_1(G) = \sum_{uv\in E(G)} \left[r_G(u) + r_G(v) \right], \qquad R_2(G) = \sum_{uv\in E(G)} r_G(u) r_G(v).$$

In [4], the product connectivity Revan index of a graph G is defined as

$$PR(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)r_G(v)}}.$$

We propose the sum connectivity Revan index of a graph as follows:

The sum connectivity Revan index of a molecular graph G is defined as

$$SR(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{r_G(u)+r_G(v)}}. \tag{1}$$

Recently several topological indices were studied, for example, in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,16,17,18].

In this paper, the sum connectivity Revan index of certain important chemical structures like silicate networks and hexagonal networks are computed. Also the first and second Revan indices of these networks are determined. For networks see [19] and references cited therein.

2. Results for silicate networks

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network in symbolized by SL_n where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional siliciate network is shown in Figure 1.

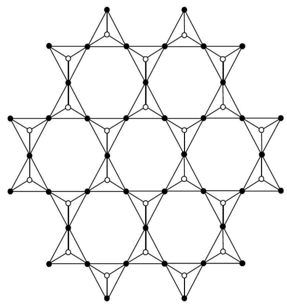

Figure 1: A 2-dimensional silicate network

In the following theorem, we compute the exact formulas of $R_1(SL_n)$, $R_2(SL_n)$ for silicate networks.

Theorem 1. Let SL_n be the silicate networks. Then

(1) $R_1(SL_n) = 270n^2 + 54n$.

(2) $R_2(SL_n) = 486n^2 + 216n$.

Proof: Let G be the graph of silicate network SL_n with $15n^2+3n$ vertices and $36n^2$ edges. From Figure 1, it is easy to see that the vertices of SL_n are either of degree 3 or 6. In SL_n, by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

$E_{33} = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 6n$.

$E_{36} = \{uv\in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_{36}| = 18n^2+6n$.

$E_{66} = \{uv\in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_{36}| = 18n^2 - 12n$.

Thus there are three types of Revan edges based on the degree of end revan vertices of each revan edge as follows:

We have $\Delta(G) + \delta(G) = 9$, $r_G(u) = 9 - d_G(u)$.

$RE_{66} = \{uv \in E(G) \mid r_G(u) = r_G(v) = 6\}$, $|RE_{66}| = 6n$.

$RE_{63} = \{uv \in E(G) \mid r_G(u) = 6, r_G(v) = 3\}$, $|RE_{63}| = 18n^2+6n$.

$RE_{33} = \{uv \in E(G) \mid r_G(u) = r_G(v) = 3\}$, $|RE_{33}| = 18n^2 - 12n$.

(1) To compute $R_1(SL_n)$, we see that

$$\begin{aligned} R_1(G) &= \sum_{uv \in E(G)} \left[r_G(u) + r_G(v) \right] \\ &= \sum_{RE_{66}} \left[r_G(u) + r_G(v) \right] + \sum_{RE_{63}} \left[r_G(u) + r_G(v) \right] + \sum_{RE_{33}} \left[r_G(u) + r_G(v) \right] \\ &= 6n \times 12 + (18n^2+6n)9 + (18n^2 - 12n)6 \\ &= 270n^2 + 54n. \end{aligned}$$

(2) To compute $R_2(SL_n)$, we see that

$$\begin{aligned} R_2(G) &= \sum_{uv \in E(G)} r_G(u) r_G(v) \\ &= \sum_{RE_{66}} r_G(u) r_G(v) + \sum_{RE_{63}} r_G(u) r_G(v) + \sum_{RE_{33}} r_G(u) r_G(v) \\ &= 6n \times 36 + (18n^2 + 6n)18 + (18n^2 - 12n) \\ &= 486n^2 + 216n. \end{aligned}$$

In the following theorem, we compute the sum connectivity Revan index of SL_n.

Theorem 2. Let SL_n be the silicate networks. Then

$$SR(SL_n) = \left(6 + 3\sqrt{6}\right) n^2 + \left(\sqrt{3} + 2 - 2\sqrt{6}\right) n.$$

Proof: Let $G=SLn$. From equation (1), and by cardinalities of the revan edge partition of silicate network SL_n, we have

$$\begin{aligned} SR(G) &= \sum_{uv \in E(G)} \frac{1}{\sqrt{r_G(u) + r_G(v)}} \\ &= \sum_{RE_{66}} \frac{1}{\sqrt{r_G(u) + r_G(v)}} + \sum_{RE_{63}} \frac{1}{\sqrt{r_G(u) + r_G(v)}} + \sum_{RE_{33}} \frac{1}{\sqrt{r_G(u) + r_G(v)}} \\ &= \left(\frac{1}{\sqrt{6+6}} \right) 6n + \left(\frac{1}{\sqrt{6+3}} \right) \left(18n^2 + 6n\right) + \left(\frac{1}{\sqrt{3+3}} \right) \left(18n^2 - 12n\right) \\ &= \left(6 + 3\sqrt{6}\right) n^2 + \left(\sqrt{3} + 2 - 2\sqrt{6}\right) n. \end{aligned}$$

3. Results for hexagonal networks

Hexagonal network is symbolized by HX_n where n is the number of vertices in each side of hexagon. A hexagonal network of dimension six is shown in Figure 2.

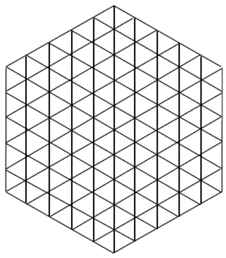

Figure 2: 6-dimensional hexagonal network

In the following theorem, we compute the exact formulas of $R_1(HX_{\text{n}})$, $R_2(HX_n)$ for hexagonal networks.

Theorem 3. Let HX_n be the hexagonal networks. Then

(1) $R_1(HX_n) = 54n^2 - 42n - 6$.

(2) $R_2(HX_n) = 81n^2 + 33n - 72$.

Proof: Let H be the graph of hexagonal network HX_n with $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges. From Figure 2, it is easy to see that the vertices of HX_n are either of degree 3, 4 or 6. Thus $\Delta(H) = 6$, $\delta(H) = 3$. In H, by algebraic method, there are five types of edges as follows:

$E_{34} = \{uv\in E(H)|\ d_H(u) = 3,\ d_H(v) = 4\}\ |E_{34}| = 12.$

$E_{36} = \{uv\in E(H)|\ d_H(u) = 3,\ d_H(v) = 6\}\ |E_{36}| = 6.$

$E_{44} = \{uv\in E(H)|\ d_H(u) = d_H(v) = 4\}\ |E_{44}| = 6n - 18.$

$E_{46}= \{uv\in E(H)|\ d_H(u) = 4,\ d_H(v) = 6\}\ |E_{46}| = 12n - 24.$

$E_{66}= \{uv\in E(H)|\ d_H(u) = d_H(v) = 6\}\ |E_{66}| = 6\}\ |E_{66}| = 9n^2 - 33n + 30$

We have $r_H(u) = \Delta(H) + \delta(H) - d_H(u) = 9 - d_H(u)$.

$RE_{65} = \{uv\in E(H)\ |\ r_G(u) = 6,\ r_G(v) = 5\},\ |RE_{65}| = 12.$

$RE_{63} = \{uv\in E(H)\ |\ r_G(u) = 6,\ r_G(v) = 3\},\ |RE_{63}| = 6.$

$RE_{55} = \{uv\in E(H)\ |\ r_G(u) = 5,\ r_G(v) = 5\},\ |RE_{55}| = 6n - 18.$

$RE_{53} = \{uv\in E(H)\ |\ r_G(u) = 5,\ r_G(v) = 3\},\ |RE_{53}| = 12n - 24.$

$RE_{33} = \{uv\in E(H)\ |\ r_G(u) = r_G(v) = 3\},\ |RE_{33}| = 9n^2 - 33n + 30$

(1) To compute $R_1(HX_n)$, we see that

$$
\begin{aligned}
R_1(H) &= \sum_{uv\in E(H)} \left[r_G(u) + r_G(v)\right] \\
&= \sum_{RE_{65}} \left[r_G(u) + r_G(v)\right] + \sum_{RE_{63}} \left[r_G(u) + r_G(v)\right] + \sum_{RE_{55}} \left[r_G(u) + r_G(v)\right] \\
&\quad + \sum_{RE_{53}} \left[r_G(u) + r_G(v)\right] + \sum_{RE_{33}} \left[r_G(u) + r_G(v)\right] \\
&= (6+5)12 + (6+3)6 + (5+5)(6n - 18) + (5+3)(12n - 24) \\
&\quad + (3+3)(9n^2 - 33n + 30) \\
&= 54n^2 - 42n - 6\ .
\end{aligned}
$$

(2) To compute $R_2(HX_n)$, we see that

$$R_2(H)=\sum_{uv\in E(H)} r_G(u)r_G(v)$$

$$=\sum_{RE_{65}} r_G(u)r_G(v)+\sum_{RE_{63}} r_G(u)r_G(v)+\sum_{RE_{55}} r_G(u)r_G(v)+\sum_{RE_{53}} r_G(u)r_G(v)+\sum_{RE_{33}} r_G(u)r_G(v)=$$

$(6\times5)12+(6\times3)6+(5\times5)(6n-18)+(5\times3)(12n-24)+(3\times3)(9n^2-33n+30)$

$= 81n^2 + 33n - 72.$

In the next theorem, we compute the sum connectivity Revan index of HX_n.

Theorem 4. Let HX_n be the hexagonal networks. Then

$$SR(HX_n)=\frac{9}{\sqrt{6}}n^2+\left(\frac{6}{\sqrt{10}}+\frac{6}{\sqrt{2}}-\frac{33}{\sqrt{6}}\right)n+\left(\frac{12}{\sqrt{11}}+2-\frac{18}{\sqrt{10}}+\frac{30}{\sqrt{6}}\right)$$

Proof: Let $H = HX_n$. From equation (2) and by cardinalities of the revan edge partition of hexagonal network HX_n, we have

$$SR(HX_n)=\sum_{uv\in E(H)}\frac{1}{\sqrt{r_H(u)+r_H(v)}}$$

$$=\sum_{RE_{65}}\frac{1}{\sqrt{r_H(u)+r_H(v)}}+\sum_{RE_{63}}\frac{1}{\sqrt{r_H(u)+r_H(v)}}+\sum_{RE_{55}}\frac{1}{\sqrt{r_H(u)+r_H(v)}}$$

$$+\sum_{RE_{53}}\frac{1}{\sqrt{r_H(u)+r_H(v)}}+\sum_{RE_{33}}\frac{1}{\sqrt{r_H(u)+r_H(v)}}$$

$$=\left(\frac{1}{\sqrt{6+5}}\right)12+\left(\frac{1}{\sqrt{6+3}}\right)6+\left(\frac{1}{\sqrt{5+5}}\right)(6n+8)$$

$$+\left(\frac{1}{\sqrt{5+3}}\right)(12n-24)+\left(\frac{1}{\sqrt{3+3}}\right)(9n^2-33n+30)$$

$$=\frac{9}{\sqrt{6}}n^2+\left(\frac{6}{\sqrt{10}}+\frac{6}{\sqrt{2}}-\frac{33}{\sqrt{6}}\right)n+\left(\frac{12}{\sqrt{11}}+2-\frac{18}{\sqrt{10}}+\frac{30}{\sqrt{6}}\right)$$

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