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Bounds of Location-2-Domination Number for Products of Graphs

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Abstract. In this paper Location-2-Domination set and their properties are being studied. A subset $S \subseteq V$ is Location-2-Dominating set of G if S is 2-Dominating set of G and for any two vertices $u, v \in V - S$ such that $N(u) \cap S \neq N(v) \cap S$,its denoted by $R_2^D(G)$. Based on this definition the bounds of the Location-2-domination number for direct product, Cartesian product and semi-strong product of graphs namely $P_n \square C_m$, $C_n \square S_m$, $P_n \times W_m$, $C_n \times S_m$ $P_n \bowtie P_m$, $C_n \bowtie P_m$, $C_n \bowtie C_m$ have been found.

Keywords: 2-Domination, Location Domination, Product of Graphs

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1. Introduction

Throughout this paper let us follow the terminology and notation of Harary [11]. Cockayne and Hedetniemi [7] introduce the concept dominating set. A subset S of vertices from V is called a dominating set for G if every vertex of G is either a member of S or adjacent to a member of S. A dominating set of G is called a minimum dominating set if G has no dominating set of smaller cardinality. The cardinality of minimum dominating set of G is called the dominating number for G and it is denoted by $\gamma(G)$ [6].

Harary and Haynes [5] introduced the concepts of double domination in graphs. A dominating set S of G is called double dominating set if every vertex in V-S is adjacent to at least two vertices in S. Given a dominating set S for graph G, for each u in V-S let S(u) denote the set of vertices in S which are adjacent to u. The set S is called locating dominating set, if for any two vertices u and w in V-S one has S(u) not equal to S(w) and the minimum cardinality of Location Domination set is denoted by RD(G) [7]. The Cartesian product $G \square H$ of graphs G and H is the graph with vertex set $V(G) \times V(H)$ and edge set is $(u,a)(v,b) \in E(G \square H)$ if and only if $a = b$ and

$uv \in E(G)$ or $u = v$ and $ab \in E(H)$ [3]. The direct product $G \times H$ of graphs G and H is the graph with vertex set $V(G) \times V(H)$ and edge set is $(u,a)(v,b) \in E(G \times H)$ if and only if $uv \in E(G)$ and $ab \in E(H)$ [14]. The Semi-Strong Product of two graphs G and H is the graph G⋈H with vertices V(G⋈H)=V(G)×V(H) and edges E(G⋈H)={(a,x)(b,y)} if and only if (a,b)ϵ E(G) and x=y or (a,b)ϵE(G) and (x,y)∈E(H)}[12].

2. Preliminaries

2.1 Location-2-domination

Definition 2.1.1. [8] A subset $S \subseteq V$ is Location – 2 -Dominating set of G if S is 2 Dominating set of G and if for any two vertices $u, v \in V - S$ such that

$$N(u) \cap S \neq N(v) \cap S .$$

The minimum cardinality of Location-2-Dominating is denoted by $R_2^D(G) = |S|$

2.2. Location-2-domination for simple graphs

Theorem 2.2.1. [9] In Location-2-Domination for any graph the vertex $\{v\}$ is an pendent vertex then $\{v\} \in \mathrm{R}_2^D(G)$ only.

Theorem 2.2.2. [8] Location-2-Domination number of a Path P_n is

$$R_2^D(P_n) = \begin{cases} \frac{n-1}{2} + 1, & n \text{ is odd} \\ \frac{n}{2} + 1 & n \text{ is even} \end{cases}$$

Theorem 2.2.3. [8] Location-2-Domination for any cycle C_n, for $n \neq 4$ is

$$R_2^D(G) = \begin{cases} \frac{n}{2} & n \text{ is even} \\ \frac{n-1}{2} + 1 & n \text{ is odd} \end{cases}$$

2.3 Location-2-domination for Cartesian product of graphs

Theorem 2.3.1. [10**]** For any graph $G = (P_n \square S_m)$,

$$R_2^D(\mathrm{G}) = \begin{cases} R_2^D(\mathrm{P_n}) + \frac{\mathrm{m(n-1)}}{2} & n \text{ is odd} \\ \frac{n}{2}(m+1) & n \text{ is even} \end{cases}$$

Theorem 2.3.2. [10**]** Location-2-Domination for any graph $G = (P_n \square P_m)$ is

$$R_2^D(G)=\begin{cases}\dfrac{nm}{2} & n \text{ is even, } m \text{ is either even or odd}\\ \dfrac{nm-1}{2} & n \text{ is odd}, m \text{ is odd}\end{cases}, m\neq 2, n\neq 2$$

Theorem 2.3.3. [10**]** Location-2-domination for any graph $G=(C_n \square C_m)$ is

$$R_2^D(G)=\begin{cases}\dfrac{nm}{2} & n \text{ is even, } m \text{ is either even or odd}\\ \dfrac{nm-1}{2} & n \text{ is odd}, m \text{ is odd}\end{cases}$$

2.4. Location-2-domination for direct product of graphs

Theorem 2.4.1. [10**]** For Graphs P_n $(n\neq 3)$ and S_m, $R_2^D(\mathrm{P_n}\times \mathrm{S_m})=nm,\ n,m=1,2,3,...$

Theorem 2.4.2. [10**]** Location -2-Domination for P_n and P_m, $m\neq 3$,

$$R_2^D\left(P_n\times P_m\right)=\begin{cases}\dfrac{nm}{2}+2 & n, m \text{ is even}\\ \dfrac{nm}{2}+2 & \text{either } n \text{ is odd}, m \text{ is even } (or)\, n \text{ is even } m \text{ is odd}\\ \dfrac{n(m+1)}{2} & n,m \text{ is odd but } n<m\\ \dfrac{m(n+1)}{2} & n,m \text{ is odd but } n>m\end{cases}$$

Theorem 2.4.3. [10**]** For $n,m\geq 5$,

$R_2^D(C_n\times C_m)=\dfrac{nm}{2}$, n,m *is even,*

$R_2^D(C_n\times C_m)=\dfrac{(n-1)m}{2}$; n *is odd* m *is even,*

$R_2^D(C_n\times C_m)=\dfrac{n(m-1)}{2}$; n *is even* m *is odd,*

$R_2^D(C_n\times C_m)=\dfrac{n(m-1)}{2}$; n,m *is odd but* $n>m$,

$R_2^D(C_n\times C_m)=\dfrac{m(n-1)}{2}$; n,m *is odd but* $n<m$.

3. Location -2-domination of products of graph

3.1. Location -2-domination (Cartesian product) of $C_n \square P_m$, $C_n \square S_m$

Theorem 3.1.1. For any graph P_m and C_n $G = (C_n \square P_m)$ we have

$$R_2^D(G) = \begin{cases} \frac{nm}{2} & n \text{ is even}, \text{m is either even } (or) \text{ odd} \\ \frac{nm}{2}+1 & n \text{ is odd } m \text{ is even} \\ \frac{nm+1}{2} & n \text{ is odd}, m \text{ is odd} \end{cases}$$

Proof: Consider path of m vertices and Cycle of n vertices. The Vertex set of P_m and C_n are $\{1,2,...,m\}$ and $\{1,2,....,n\}$ respectively. Clearly $|G| = nm$ in which $2n$ vertices are degree 3 and $(m-2)n$ vertices are degree 4 and let S - Set denote Location-2-Domination of G.

Case(i): Suppose n is even and m is either even or odd, in this situation $|V(G)| = nm$ even number of vertices. In G fix any vertex from C_1 and form open path through vertices of P_1, continue the open path starts with C_2 through P_2, continue the same process till C_n through P_m, each time the process of continuation of open path from C_1 to C_n done only by either towards right or left direction only not alternatively. Finally the collection of vertices from C_1 to C_n through P_1 to P_m forms a cycle of length even with nm vertices. So by the Theorem: 2.2.3, $|S| = \frac{nm}{2}$, i.e. $R_2^D(G) = \frac{nm}{2}$.

Case (ii): Suppose n is odd, but m is even , in this situation $|V(G)| = nm$ is even, in G fix any vertex from C_1 and form an open path through P_1, continue the open path starts with C_2 through P_2, continue the same process till C_n through P_m, each time in the process of continuation open path from C_1 to C_n done only by either towards right or left direction only not for alternatively. Finally the collection of vertices from C_1 to C_n through P_1 to P_m forms a path of length even with nm vertices. So by the Theorem: 2.2.2 $|S| = \frac{nm}{2}+1$, i.e. $R_2^D(G) = \frac{nm}{2}+1$

Case(iii): Suppose n is odd and m is odd .Clearly $|G| = nm$ odd number of vertices, From G, let us consider $|S| = |S_1| + |S_2|$ where $|S_1|$ denote the Location-2-Domination for $\{C_1, C_3, ..., C_m\}$ and $|S_2|$ denote the Location-2-Domination for $\{C_2, C_4, ..., C_{m-1}\}$, but

$|C_1|=|C_2|=...=|C_{m-1}|=|C_m|=n.$ And for $S_1=\{C_1,C_3,...,C_m\}$ the vertex set of S_1 are $\{C_{11},C_{12},...,C_{1n},C_{31},C_{32},...,C_{3n},...,C_{m1},C_{m2},...,C_{mn}\}$, By the Theorem: 2.2.3 Location -2-Domination for Cycle of length odd is $\frac{n-1}{2}+1=\frac{n+1}{2}$. Therefore $|C_i|=\frac{n+1}{2}, i=1,3,...,\mathrm{m}$. Clearly S_1-set contains $\frac{m+1}{2}$ times of cycle with odd length. Therefore $|S_1|=\left(\frac{m+1}{2}\right)\left(\frac{n+1}{2}\right)$ and $S_2=\{C_2,C_4,...,C_{m-1}\}$ the vertex set of S_2 are $\{C_{21},C_{22},...,C_{2n},C_{41},C_{42},...,C_{4n},...,C_{(m-1)1},C_{(m-1)2},...,C_{(m-1)n}\}$, now collect the vertex from C_2 as $N(V-S_1)-S_1$ in C_1. This gives $\frac{n-1}{2}$ vertices in C_2. Continuing the same process for $\{C_4,C_6,...,C_{m-1}\}$, i.e. collect the vertex for C_{i+1} as $N(V-S_i)-S_i$ from C_i for $i=1,2,...,m-2$.

Clearly S_2 -set contains $\frac{m-1}{2}$ times of C_{i+1} , $i=1,2,...,m-2$. i.e. $|S_2|=\left(\frac{m-1}{2}\right)\left(\frac{n-1}{2}\right)$

$|S|=\left(\frac{m+1}{2}\right)\left(\frac{n+1}{2}\right)+\left(\frac{m-1}{2}\right)\left(\frac{n-1}{2}\right)=\frac{nm+1}{2}$. Therefore, $R_2^D(G)=|S|=\frac{nm+1}{2}$.

Theorem 3.1.2. For graphs P_n and S_m, $R_2^D\left(C_n\square S_m\right)=\begin{cases}\frac{n(m+1)}{2} & n \textit{ is even}\\ \frac{n+1}{2}+\frac{m(\mathrm{n}-1)}{2} & n \textit{ is odd}\end{cases}$

Proof: Consider the vertex set of G namely $\{v_{ij}\}$ for $1\le i\le n$, $1\le j\le m+1$. Clearly $|G|=nm.$ Let $S-$set denote Location-2-Dominating set, by observing G, $d_G\left(v_{ij}\right)=m+1$ for $i=1,n.$ and $1\le j\le m+1$ also $d_G\left(v_{ij}\right)=4$ for $2\le i\le n-1$, $2\le j\le m+1$. i.e. v_{i1}, $i=1,2,...,n$ is adjacent with v_{ij}, $2\le j\le m+1$.

Case (i): Suppose n is even and m is either even or odd. Clearly $|G|=nm$ has even number of vertices, in this sense now collect $S-$set possibly by $\{v_{i1}\}$, $i=1,3,5,...,n-1$ and $\{v_{ij}\}$ for $i=2,4,...,n$, $2\le j\le m+1$, or $\{v_{i1}\}$, $i=2,4,6,...,n$ and $\{v_{ij}\}$ for $i=1,3,...,n-1$, $2\le j\le m+1$, this gives $\frac{n}{2}$ times a single vertex and $\frac{n}{2}$ times m vertices or $\{v_{ij}\}$ for $i=1,3,5,...,n-1$, $1\le j\le m+1$, this gives $\frac{n}{2}$ times $m+1$ vertices.

i.e. $\left|S\right| = \frac{n}{2} + \frac{nm}{2} = \frac{n(m+1)}{2}$ therefore . $R_2^D(G) = \frac{n(m+1)}{2}$.

Suppose, $\left\{v_{i1}\right\} \in S,\ i = 1,3,5,...,n-1$ and $\left\{v_{ij}\right\} \notin S$ for $i = 2,4,...,n,\ 2 \le j \le m+1$ or some $\left\{v_{ij}\right\} \notin S$ for $i = 2,4,...,n,\ 2 \le j \le m+1.$ Clearly this contradicts the definition of Location-2-Domination or minimum cardinality of $S-$set or $\left\{v_{i1}\right\} \notin S,\ i = 1,3,5,...,n-1$ and $\left\{v_{ij}\right\} \in S$ for $i = 2,4,...,n.\ 2 \le j \le m+1$, in this situation $\left\{v_{ij}\right\},\ i = 1,3,5,...,n\text{-}1,$ $1 \le j \le m+1$ needs additional vertex, clearly it also contradicts the minimum cardinality of $S-$set.

Case (ii): Suppose n is odd, m is either even or odd. Clearly $\left|V(G)\right| = nm$ gives even number of vertices, in this sense now collect S – set possibly by $\left\{v_{i1}\right\},\ i = 1,3,5,...,n$ and $\left\{v_{ij}\right\}$ for $i = 2,4,...,n-1.\ 2 \le j \le m+1$, this gives $\frac{n+1}{2}$ times a single vertex and $\frac{n-1}{2}$ times $m+1$ vertices.

That is, $\left|S\right| = \frac{n+1}{2} + \frac{(n-1)m}{2}$ and therefore $R_2^D(G) = \frac{n+1}{2} + \frac{m(n-1)}{2}$.

Suppose, $\left\{v_{i1}\right\} \in S,\ i = 2,4,...,n-1$ and $\left\{v_{ij}\right\} \in S$ for $i = 1,3,...,n,\ 2 \le j \le m+1$ this gives $\left|S\right| = \frac{n-1}{2} + \frac{(n+1)m}{2}$ contradicts minimum cardinality of $S-$set or some $\left\{v_{ij}\right\} \notin S$ for $i = 2,4,...,n,\ 2 \le j \le m+1$, clearly it contradicts the definition of Location-2-Domination.

3.2. Location-2-domination (Direct product) of $P_n \times W_m$, $P_n \times C_m$

Theorem 3.2.1. For any Graphs P_n, $n \ne 2$ and W_m, $m \ne 5$ we have

$$R_2^D(G) = \begin{cases} \frac{nm}{2} & n \text{ is even} \\ \frac{m(n+1)}{2} & n \text{ is odd} \end{cases}$$

Proof: Label the vertices of G as $\left\{v_{ij}\right\}, 1 \le i \le n, 1 \le j \le m$, clearly $\left|G\right| = nm$ from G $d_G\left(v_{11}\right) = d_G\left(v_{n1}\right) = m-1$, $d_G\left(v_{1j}\right) = d_G\left(v_{nj}\right) = 3, 2 \le \mathrm{j} \le \mathrm{m}$ and $d_G\left(v_{ij}\right) = 6,$ $2 \le i \le n-1$, $2 \le j \le m.$ Now labels of G are partitioned into n different sets namely $U_i, 1 \le \mathrm{i} \le \mathrm{n}$ are $\left\{v_{ij}\right\}, 1 \le i \le n, 1 \le j \le m$ respectively. But there is no adjacency from v_{ij} to v_{ji} for $1 \le i \le n, 1 \le j \le m.$ Clearly u_i is adjacent to u_{i-1} and u_{i+1} for $i = 1,2,...,n.$

Case (i): Suppose n is even, then the collection of the sets U_i, $i=1,3,...,n-1$ or $i=2,4,...,n$ will have $\frac{n}{2}$ times m vertices i.e. $|S|=\frac{nm}{2}$ therefore $R_2^D(G)=\frac{nm}{2}$.

Case (ii): Suppose n is odd by based on Theorem 2.2.2, the collection of the sets U_i, $i=2,4,...,n-1$ will have $\frac{n-1}{2}$ times m vertices i.e. $|S|=\frac{(n-1)m}{2}$ therefore $R_2^D(G)=\frac{(n-1)m}{2}$. Suppose if we collect the sets U_i, $i=1,3,...,n$ this contradicts the minimum cardinality.

Result 3.2.1. $R_2^D(P_2\times W_m)=m.$

Result 3.2.2. $R_2^D(P_n\times W_5)=\begin{cases} 3n, & n \text{ is even} \\ 5\left(\frac{n+1}{2}\right)+\frac{n-1}{2}, & n \text{ is odd} \end{cases}$

Theorem 3.2.2. For Graphs C_n and S_m, $R_2^D(C_n\times S_m)=nm$ $n,m=1,2,3,...$

Proof: The vertex set of G are $\{v_{ij}\}, 1\le i\le n, 1\le j\le m+1$. Let $S-$set denote Location-2-Dominating set of $G.$ Clearly by observation of G $d_G(v_{i1})=2m,$ $1\le i\le n$ and $d_G(v_{ij})=2,$ $1\le i\le n$, $2\le j\le m+1$. Now collect the $S-$set possibly by either $v_{ij}, 1\le i\le n, 2\le j\le m+1$ or v_{i1}, $1\le i\le n$ and $v_{ij}, 1\le i\le n, 3\le j\le m+1$ i.e. leaving anyone of the same base vertex of $i=1,2,...$ or $j=1,2,...$ clearly this n times m vertices. i.e. $|S|=nm.$ Suppose $v_{i1}\in S,$ $i=1,2,3,...,n$ and $v_{ij}\notin S, 1\le i\le n, 2\le j\le m+1$ then this contradicts the definition of Location-2-Domination. Therefore $R_D^2(G)=|S|=nm.$

3.3 Location-2-domination (semi-strong product) of $P_n \bowtie P_m$, $C_n \bowtie P_m$, $C_n \bowtie C_m$

Theorem 3.3.1. For any graphs $P_n, n\ne 3$ and P_m, $m\ne 2$, G=$P_n \bowtie P_m$ is

$$R_2^D(G)=\begin{cases} \frac{nm}{2}, & n \text{ is even}, \ m \text{ is even or odd} \\ \frac{nm}{2}+2, & n \text{ is odd}, m \text{ is even} \\ \frac{n(m+1)}{2}, & n,m \text{ is odd } n<m \\ \frac{m(n+1)}{2}, & n,m \text{ is odd } m<n \\ \frac{n(m+1)}{2}, & n,m \text{ is odd } n=m \end{cases}$$

Proof: Label the vertices of G as $\{v_{ij}\}, 1 \le i \le n$ and $1 \le j \le m$. Let $S-$set be the Location-2-Domination set of G, then clearly $d_G(v_{ij}) \ge 2$, $i = 1, 2, .., n$ $j = 1, 2, ..., m$.

Case (i): Suppose n is even and m is even or odd, now let us collect the S–set possibly by $\{v_{ij}\}$ $i = 1, 3, ..., n-1$, $j = 1, 2, ..., m$ or $\{v_{ij}\}$ $i = 2, 4, ..., \mathrm{n}$, $j = 1, 2, 3, ..., m$ this gives $\frac{nm}{2}$ vertices i.e. $|S| = \frac{nm}{2}$ $R_2^D(G) = \frac{nm}{2}$.

Case (ii): Suppose n is odd, m is even in this sense let us collect the S-set possibly by $\{v_{ij}\}$ $i = 1, 2, 3, ..., n$, $j = 1, 3, ..., m-1$ and v_{1m}, v_{nm} and this gives $\frac{nm}{2} + 2$ vertices that is $|S| = \frac{nm}{2} + 2$ therefore $R_2^D(G) = \frac{nm}{2} + 2$. Suppose the vertices v_{1m}, v_{nm} does not belong to $S-$set or $\{v_{ij}\}$ $i = 1, 3, ..., n-1$, $j = 1, 2, 3, ..., m$ then this is a contradiction to minimum cardinality.

Case (iii): Suppose n, m is odd but $n < m$, in this case let us collect the S–set possibly by $\{v_{ij}\}$ $i = 1, 2, 3, ..., n$, $j = 1, 3, ..., m-1$ and this gives n times $\frac{m+1}{2}$ vertices that is $|S| = \frac{n(m+1)}{2}$ therefore $R_2^D(G) = \frac{n(m+1)}{2}$. Then the collection $\{v_{ij}\}$ $i = 1, 3, ..., n-1$, $j = 1, 2, 3, ..., m$ is not a minimum cardinality set.

Case (iv): Similar to the case (iii).

Case (v): suppose n, m is odd but $n = m$ in this case let us collect the S–set possibly by $\{v_{ij}\}$ $i = 1, 2, 3, ..., n$, $j = 1, 3, ..., m$ or $\{v_{ij}\}$ $i = 1, 3, ..., n$, $j = 1, 2, 3, ..., m$ then this gives $\frac{n(m+1)}{2}$ vertices that is $|S| = \frac{n(m+1)}{2}$ and hence $R_2^D(G) = \frac{n(m+1)}{2}$.

Result 3.3.1. $R_2^D(P_n \times P_2) = n$

Result 3.3.2. $R_2^D(P_3 \times P_m) = 2m$

Observation 3.3.1. The semi-strong product of $C_n \times P_m$ is not equal to $P_n \times C_m$

Theorem 3.3.2. For any graphs $C_n, \mathrm{n} > 5$ and $P_m, \mathrm{m} \ne 2$, $G = C_n \bowtie P_m$

$$R_2^D(\mathrm{G}) = \begin{cases} 2\left(\frac{m}{2}+1\right)+\frac{m}{2}+m\left(\frac{n-4}{2}\right) & n,m \textit{ is even} \\ (m+1)+m\left(\frac{n-3}{2}\right) & n,m \textit{ is odd} \\ m\left(\frac{n-3}{2}\right)+2(m+2) & n \textit{ is odd } m \textit{ is even} \\ 3\left(\frac{m+1}{2}\right)+m\left(\frac{n-4}{2}\right) & n \textit{ is even } m \textit{ is odd} \end{cases}$$

Proof: $|V(G)| = nm = \{v_{ij}\}, i=1,2,...,n,\ j=1,2,...,m$ .Clearly $d_G(v_{ij}) \geq 2$ for $i=1,2,..,n,$ $j=1,2,...,m$

Case (i): Suppose n,m is even, in this case cardinality of $S-$set contains the vertices are $v_{ij}, i=1,n.\ j=1,3,...,m-1,m$ and $v_{(n-1)j},\ j=1,3,...,m-1$ also v_{ij} for $i=3,5,...,n-3.$ $j=1,2,...,m.$ Clearly this gives 2times $\left(\frac{m}{2}+1\right)$ vertices and $\frac{m}{2}$ times a single vertex. Also $\frac{n-4}{2}$ times m vertices. That is $|S| = 2\left(\frac{m}{2}+1\right)+\frac{m}{2}+m\left(\frac{n-4}{2}\right)$ and therefore $R_2^D(G) = 2\left(\frac{m}{2}+1\right)+\frac{m}{2}+m\left(\frac{n-4}{2}\right).$

Case (ii): Suppose n,m is odd, now the $S-$set contains the vertices $v_{ij}, i=1,n.\ j=1,3,...,m$ and also v_{ij} for $i=3,5,...,n-2,\ j=1,2,...,m.$ Then this gives 2 times $\left(\frac{m+1}{2}\right)$ vertices and $\frac{n-3}{2}$ times m vertices. That is $|S| = (m+1)+m\left(\frac{n-3}{2}\right)$ and therefore $R_2^D(G) = (m+1)+m\left(\frac{n-3}{2}\right)$

Case (iii): Proof is similar to Case (i) and hence $R_2^D(G) = (m+2)+m\left(\frac{n-3}{2}\right)$

Case (iv): Suppose n is even $,m$ is odd, now the $S-$set contains the vertices $v_{ij}, i=1,n-1,n;\ j=1,3,...,m$ and also v_{ij} for $i=3,5,...,n-3;\ j=1,2,...,m.$ Then this gives 3times $\left(\frac{m+1}{2}\right)$ vertices and $\frac{n-4}{2}$ times m vertices. That is $|S| = 3(m+1)+m\left(\frac{n-4}{2}\right)$ therefore $R_2^D(G) = 3(m+1)+m\left(\frac{n-4}{2}\right)$

Result 3.3.3. $R_2^D(C_2 \times C_m) = m\,, m \neq 2,3$

Result 3.3.4. $R_2^D(C_3 \times C_m) = 2(m-1)$

Result 3.3.5. $R_2^D(C_4 \times C_m) = 2m$

Theorem 3.3.3. For Graphs $C_n, n \neq 2,3,4.$ and C_m, G=$C_n \bowtie C_m$is

$$R_2^D(\mathrm{G}) = \begin{cases} \dfrac{nm}{2} & n, m \text{ is even} \\ n\left(\dfrac{m-1}{2}\right) & n \text{ is even or odd}, m \text{ is odd} \\ m\left(\dfrac{n-1}{2}\right) & n \text{ is odd}, m \text{ is even} \end{cases}$$

Proof: Let the vertices of G be $\{v_{ij}\}, i = 1,2,...,n\,,\ j = 1,2,...,m$ .Let $S-$ denotes Location-2-Dominating set.

Case (i): Proof is followed by Theorem 3.5 Case (i)

Case (ii): Suppose n is odd, m is odd and let us collect the $S-$set possibly by S $\{v_{ij}\}\, i = 1,3,...,n-2\,,\ j = 1,2,3,...,m.$ Clearly this gives $\dfrac{m-1}{2}$ times n vertices,that is $|S| = n\left(\dfrac{m-1}{2}\right)$ and therefore $R_2^D(G) = n\left(\dfrac{m-1}{2}\right)$.

Case (iii): suppose n *is odd*, m is even and let us collect the S–set possibly by $\{v_{ij}\}\, i = 1,2,3,...,n\,,\ j = 1,3,...,m-1.$ Clearly this gives $\dfrac{n-1}{2}$ times m vertices that is $|S| = m\left(\dfrac{n-1}{2}\right)$ and hence $R_2^D(G) = m\left(\dfrac{n-1}{2}\right)$. Suppose if anyone the vertex collection as $\{v_{ij}\}\, i = 1,3,...,n-2, n-1\,,\ j = 1,2,3,...,m$ then this is once againa contradiction to minimum cardinality.

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