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k–Super Harmonic Mean Labeling of Certain Snake Graphs

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Abstract. Harmonic mean labeling was introduced by Sandhya et al. We extend this notion to k-super harmonic mean labeling. In this paper, we investigate k-Super harmonic mean labeling of some snake graphs.

Keywords: Super harmonic mean labeling, Super harmonic mean graph, k-Super harmonic mean labeling, k-Super harmonic mean graph.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3].

A graph labelingis an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by Gallian (2016) can be found in [1]. Somasundaram and Ponraj [12] have introduced the notion of mean labeling of graphs. Ponraj and Ramya introduced Super mean labeling of graphs in [5]. Somasundaram and Sandhya introduced the concept of Harmonic mean labeling in [6] and studied their behavior in [7,8,9]. Sandhya and David Raj introduced Super harmonic labeling in [9]. k-super harmonic mean labeling was introduced by Tamilselvi and Revathi in [14]. In this paper, we investigate k-Super harmonic mean labeling of some snake related graphs.

2. Preliminaries

Definition 2.1. Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3 \dots, p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then f is called *Super harmonic mean labeling* if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, \dots, p+q\}$. A graph that admits a *Super harmonic mean labeling is called Super harmonic mean graph.*

Definition 2.2. Let G be a (p, q) graph and f: V (G) → {k, k + 1, k + 2, … ,p + q + k − 1} be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then f is called *k-Super harmonic mean labeling* if f (V) ∪ {$f^*(e)$: e∈$E(G)$} = {k, k + 1, k + 2, … , p + q + k − 1}. A graph that admits a *k-Super harmonic mean labeling* is called *k-Super harmonic mean graph.*

Definition 2.3. A *Triangular snake* T_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and $u_{i+1}$to a new vertex v_i for $1 \le i \le n-1$.

Definition 2.4. An *Alternate Triangular snake* $A(T_n)$ is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} alternatively to a new vertex v_i.

Definition 2.5. A *Quadrilateral snake* Q_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and $u_{i+1}$to new vertices v_i and w_i for $1 \le i \le n-1$ respectively and then joining $v_i$and w_i .

Definition 2.6. An *Alternate Quadrilateral snake*$A(Q_n)$is obtained from a path $u_1, u_2, \dots, u_n$by joiningu_i and u_{i+1} alternatively to new vertices $v_i$and $w_i$respectively and then joining $v_i$and w_i .

3. Main results

Theorem 3.1. A Triangular snake T_n $(n \ge 2)$ is k - super harmonic mean graph for all $k \ge 2$.

Proof: Let $V(T_n) = \{u_i\,; 1 \le i \le n, v_i\,; 1 \le i \le n-1\}$

$E(T_n) = \{u_i u_{i+1}\,; 1 \le i \le n-1, u_i v_i\,; 1 \le i \le n-1, u_{i+1} v_i\,; 1 \le i \le n-1\}$

be denoted as in the following figure

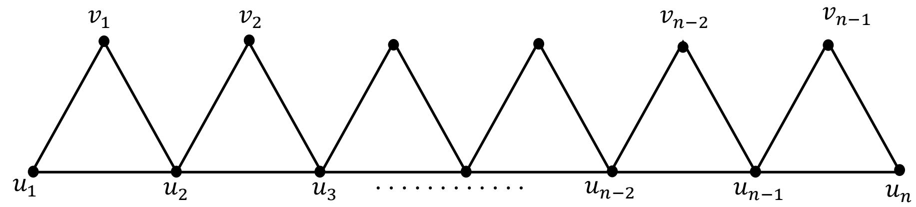

Figure 1:

First we label the vertices as follows

Define a function $f: V(T_n) \to \{k, k+1, k+2, \dots k+p+q-1\}$ by

$f(u_1) = k+3$

$f(u_i) = k+5i-5$, $for\ 2 \le i \le n$

$f(v_1) = k$

$f(v_i) = k+5i-2$, $for\ 2 \le i \le n-1$.

Then the induced edge labels are

$f^*(u_1u_2) = k+4$

$f^*(u_iu_{i+1}) = k+5i-3$, $for\ 2 \le i \le n-1$

$f^*(u_iv_i) = k+5i-4$, $for\ 1 \le i \le n-1$

$f^*(v_1u_2) = k + 2$

$f^*(v_iu_{i+1}) = k + 5i - 1, \; for\ 2 \le i \le n - 1.$

Thus $f(V) \cup \{f^*(e): e\epsilon E(G)\} = \{k, k + 1, k + 2, \ldots, k + p + q - 1\}$.

Hence T_n is a k -Super harmonic mean graph for all $k \ge 2$.

Example 3.1.

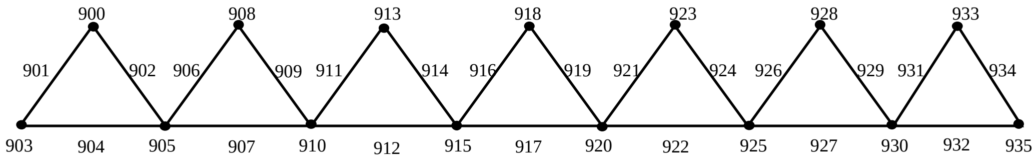

Figure 2: 900-Super harmonic mean labeling of T_8

Theorem 3.2. An alternate Triangular snake $A(T_n)$ $(n \ge 3)$ is k - super harmonic mean graph for all $k \ge 2$, if the triangle starts from the first vertex of $A(T_n)$.

Proof: Consider a path $u_1, u_2, \ldots, u_n$.

To construct alternate triangular snake joinu_i and u_{i+1} alternatively with a new vertex v_i.

The ordinary labeling is

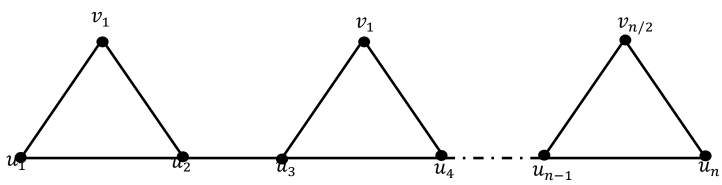

Figure 3:

Here we consider two cases

Case (i): If n is even

Define a function $f: V(A(T_n)) \to \{k, k + 1, k + 2, \ldots k + p + q - 1\}$ by

$f(u_1) = k + 3$

$f(u_{2i-1}) = k + 7(i - 1), \; for\ 2 \le i \le n/2$

$f(u_{2i}) = k + 7i - 2, \; for\ 1 \le i \le n/2$

$f(v_1) = k$

$f(v_i) = k + 7i - 4, \; for\ 2 \le i \le n/2.$

Then the induced edge labels are

$f^*(u_1u_2) = k + 4$

$f^*(u_{2i}u_{2i+1}) = k + 7i - 1, \; for\ 1 \le i \le \frac{n}{2} - 1$

$f^*(u_{2i-1}u_{2i}) \quad = k + 7i - 5, \; for\ 2 \le i \le n/2$

$f^*(u_{2i-1}v_i) \quad = k + 7i - 6, \; for\ 1 \le i \le n/2$

$f^*(u_2v_1) = k + 2$

$f^*(u_{2i}v_i) = k + 7i - 3, \; for\ 2 \le i \le n/2.$

Case (ii) : If n is odd

Define a function $f: V(A(T_n)) \to \{k, k + 1, k + 2, \ldots k + p + q - 1\}$ by

$f(u_1) = k + 3$
$f(u_{2i-1}) = k + 7(i-1)\,,\ \ for\ 2 \le i \le \frac{n+1}{2}$
$f(u_{2i}) = k + 7i - 2\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$
$f(v_1) = k$
$f(v_i) = k + 7i - 4\,,\ \ for\ 2 \le i \le \frac{n-1}{2}$.

Then the induced edge labels are
$f^*(u_1u_2) = k + 4$
$f^*(u_{2i}u_{2i+1}) = k + 7i - 1\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$
$f^*(u_{2i-1}u_{2i}) \quad = k + 7i - 5\,,\ \ for\ 2 \le i \le \frac{n-1}{2}$
$f^*(u_{2i-1}v_i) \quad = k + 7i - 6\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$
$f^*(u_2v_1) = k + 2$
$f^*(u_{2i}v_i) = k + 7i - 3\,,\ \ for\ 2 \le i \le \frac{n-1}{2}$

Thus $f(V) \cup \{f^*(e): e\epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+p+q-1\}$.
Hence $A(T_n)$, is a k -Super harmonic mean graph for all $k \ge 2$, if the triangle starts from the first vertex of $A(T_n)$.

Example 3.2.

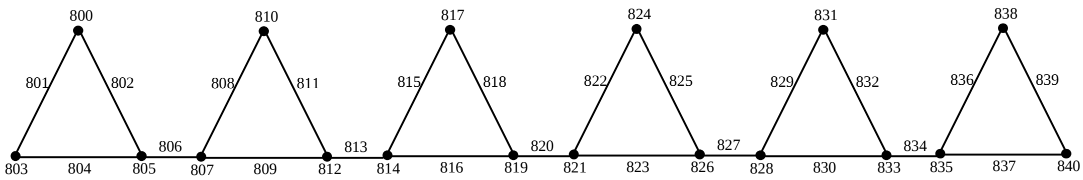

Figure 4: $800 - Super\ harmonic\ mean\ labeling\ of\ A(T_{12})$

Theorem 3.3. An Alternate Triangular snake $A(T_n)$ $(n \ge 3)$ is k - super harmonic mean graph for all $k \ge 1$, if the triangle starts from the second vertex of $A(T_n)$.
Proof: The ordinary labeling is

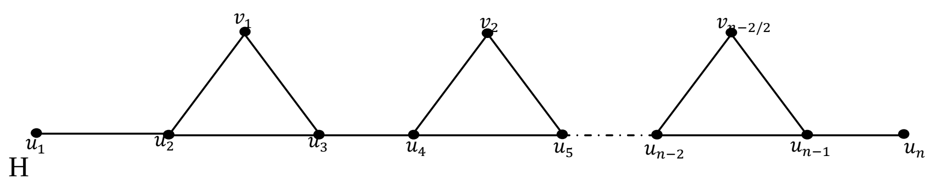

Figure 5:

Here we consider two cases
Case (i): If n is even
Define a function $f: V(A(T_n)\,) \to \{k\,, k+1\,,\ k+2\,, \ldots k+p+q-1\}$ by
$f(u_{2i-1}) = k + 7(i-1)\,,\ \ for\ 1 \le i \le n/2$
$f(u_{2i}) = k + 7i - 5\,, \qquad for\ 1 \le i \le n/2$

$f(v_i) = k + 7i - 2\,, \qquad for\ 1 \le i \le \frac{n-2}{2}.$

Then the induced edge labels are

$f^*(u_{2i-1}u_{2i}) = k + 7i - 6\,,\ \ for\ 1 \le i \le n/2$

$f^*(u_{2i}u_{2i+1}) \quad = k + 7i - 3\,,\ \ for\ 1 \le i \le \frac{n-2}{2}$

$f^*(u_{2i}v_i) = k + 7i - 4\,,\ \ for\ 1 \le i \le \frac{n-2}{2}.$

$f^*(u_{2i+1}v_i) = k + 7i - 1\,,\ \ for\ 1 \le i \le \frac{n-2}{2}.$

Case (ii) : If n is odd

Define a function $f: V(A(T_n)) \to \{k\,, k+1\,,\ k+2\,, \ldots k+p+q-1\}$ by

$f(u_{2i-1}) = k + 7(i-1)\,,\ \ for\ 1 \le i \le \frac{n+1}{2}$

$f(u_{2i}) = k + 7i - 5\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$

$f(v_i) = k + 7i - 2\,,\ \ for\ 1 \le i \le \frac{n-1}{2}.$

Then the induced edge labels are

$f^*(u_{2i-1}u_{2i}) = k + 7i - 6\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$

$f^*(u_{2i}u_{2i+1}) \quad = k + 7i - 3\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$

$f^*(u_{2i}v_i) = k + 7i - 4\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$

$f^*(u_{2i+1}v_i) = k + 7i - 1\,,\ \ for\ 1 \le i \le \frac{n-1}{2}$

Thus $f(V) \cup \{f^*(e): e\epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+p+q-1\}$.

Hence $A(T_n)$ is a k -Super harmonic mean graph for all $k \ge 1$, if the triangle starts from the second vertex of $A(T_n)$.

Example 3.3.

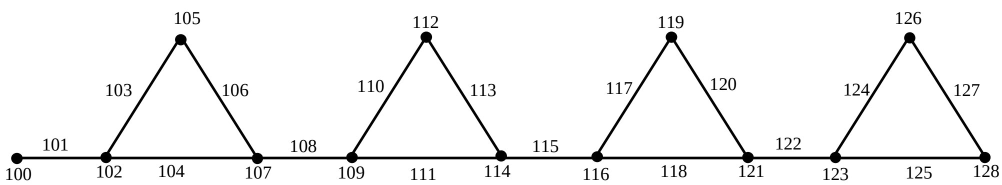

Figure 6: $100 - Super\ harmonic\ mean\ labeling\ of\ A(T_9)$

Theorem 3.4. A Quadrilateral snake Q_n $(n \ge 2)$ is k - super harmonic mean graph for all $k \ge 2$.

Proof: Let $V(Q_n) = \{u_i\,; 1 \le i \le n\,,\ \ v_{i\,,}w_{i\,,}\,; 1 \le i \le n-1\,\}$

$E(Q_n) \ = \{u_iu_{i+1}\,, u_iv_i\,, v_iw_i\,, w_iu_{i+1}\,; 1 \le i \le n-1\}$

be denoted as in the following figure

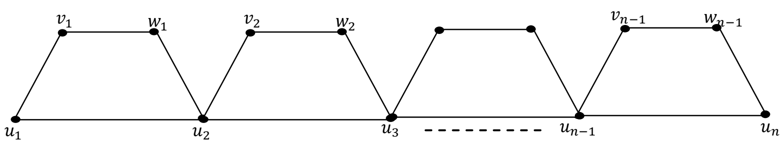

Figure7:

First we label the vertices as follows

Define a function $f: V(Q_n) \to \{k, k+1, k+2, \dots k+p+q-1\}$ by

$$f(u_1) = \begin{cases} k+5 & if k = 2 \\ k+4 & if\ k \geq 3 \end{cases}$$
$$f(u_i) = k + 7i - 7, \quad for\ 2 \leq i \leq n$$
$$f(v_1) = k$$
$$f(v_i) = k + 7i - 5, \quad for\ 2 \leq i \leq n-1$$
$$f(w_1) = k + 3$$
$$f(w_i) = k + 7i - 2, \quad for\ 2 \leq i \leq n-1.$$

Then the induced edge labels are

$$f^*(u_1u_2) = k + 6$$
$$f^*(u_iu_{i+1}) = k + 7i - 4, \quad for\ 2 \leq i \leq n-1$$
$$f^*(u_1v_1) = k + 2$$
$$f^*(u_iv_i) = k + 7i - 6, \quad for\ 2 \leq i \leq n-1$$
$$f^*(w_1u_2) = \begin{cases} k+4 & if k = 2 \\ k+5 & if\ k \geq 3 \end{cases}$$
$$f^*(w_iu_{i+1}) = k + 7i - 1, \quad for\ 2 \leq i \leq n-1$$
$$f^*(v_1w_1) = k + 1$$
$$f^*(v_iw_i) = k + 7i - 3, \quad for\ 2 \leq i \leq n-1.$$

Thus $f(V) \cup \{f^*(e): e \epsilon E(G)\} = \{k, k+1, k+2, \dots, k+p+q-1\}$.

Hence Q_n is a k -Super harmonic mean graph for all $k \geq 2$.

Example 3.4.

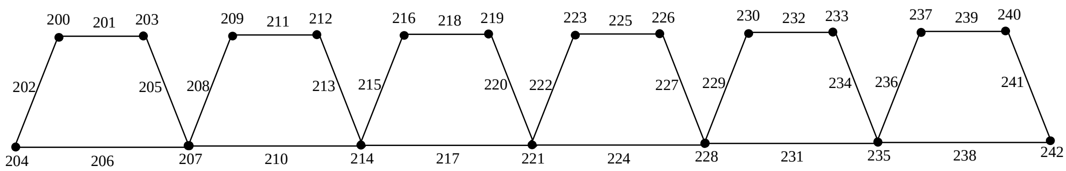

Figure 8: $200 - super\ harmonic\ mean\ labeling\ of\ Q_7$

Theorem 3.5. An alternate Quadrilateral snake $A(Q_n)$ $(n \geq 3)$ is k - super harmonic mean graph for all $k \geq 2$, if the quadrilateral starts from the first vertex of $A(Q_n)$.

Proof: Consider a path $u_1, u_2, \dots, u_n$.

To construct alternate quadrilateral snake join u_i and u_{i+1} alternatively to new vertices v_i, w_i respectively and then joining v_i and w_i

The ordinary labeling is

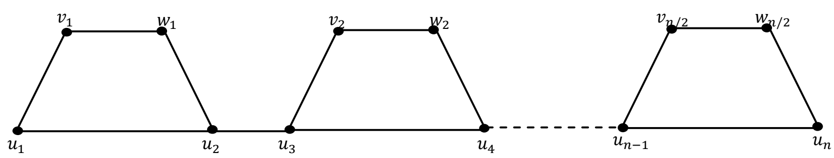

Figure 9:

Here we consider two cases

Case (i): If n is even

Define a function $f: V(A(Q_n)) \rightarrow \{k, k+1, k+2, \dots k+p+q-1\}$ by

$$f(u_1) = \begin{cases} k+5 & if k = 2 \\ k+4 & if\ k \geq 3 \end{cases}$$

$f(u_{2i}) = k + 9i - 2, \ for\ 1 \leq i \leq n/2$

$f(u_{2i-1}) = k + 9i - 9, \ for\ 2 \leq i \leq n/2$

$f(v_1) = k$

$f(v_i) = k + 9i - 7, \ for\ 2 \leq i \leq n/2$

$f(w_1) = k + 3$

$f(w_i) = k + 9i - 4, \ for\ 2 \leq i \leq n/2.$

Then the induced edge labels are

$f^*(u_1u_2) = k + 6$

$f^*(u_{2i}u_{2i+1}) = k + 9i - 1, \ for\ 1 \leq i \leq \frac{n}{2} - 1$

$f^*(u_{2i-1}u_{2i}) = k + 9i - 6, \ for\ 2 \leq i \leq \frac{n}{2}$

$f^*(u_1v_1) = k + 2$

$f^*(u_{2i-1}v_i) = k + 9i - 6, \ for\ 2 \leq i \leq n/2$

$$f^*(w_1u_2) = \begin{cases} k+4 & if k = 2 \\ k+5 & if\ k \geq 3 \end{cases}$$

$f^*(w_iu_{2i}) = k + 9i - 3, \ for\ 2 \leq i \leq n/2$

$f^*(v_1w_1) = k + 1$

$f^*(v_iw_i) = k + 9i - 5, \ for\ 2 \leq i \leq n - 1.$

Case (ii) : If n is odd

$$f(u_1) = \begin{cases} k+5 & if k = 2 \\ k+4 & if\ k \geq 3 \end{cases}$$

$f(u_{2i}) = k + 9i - 2, \ for\ 1 \leq i \leq \frac{n-1}{2}$

$f(u_{2i-1}) = k + 9i - 9, \ for\ 2 \leq i \leq \frac{n+1}{2}$

$f(v_1) = k$

$f(v_i) = k + 9i - 7, \ for\ 2 \leq i \leq \frac{n-1}{2}$

$f(w_1) = k+3$

$f(w_i) = k + 9i - 4, \ for\ 2 \leq i \leq \frac{n-1}{2}$

Then the induced edge labels are

$f^*(u_1u_2) = k + 6$

$f^*(u_{2i}u_{2i+1}) = k + 9i - 1, \ for\ 1 \leq i \leq \frac{n-1}{2}$

$f^*(u_{2i-1}u_{2i}) = k + 9i - 6, \ for\ 2 \leq i \leq \frac{n-1}{2}$

$f^*(u_1v_1) = k + 2$

$f^*(u_{2i-1}v_i) = k + 9i - 8, \ for\ 2 \leq i \leq \frac{n-1}{2}$

$$f^*(w_1u_2) = \begin{cases} k+4 & if k = 2 \\ k+5 & if\ k \geq 3 \end{cases}$$

$f^*(w_iu_{2i}) = k + 9i - 3, \ for\ 2 \leq i \leq \frac{n-1}{2}$

$f^*(v_1w_1) = k + 1$

$f^*(v_iw_i) = k + 9i - 5\ ,\ \ for\ 2 \le i \le \frac{n-1}{2}$.

Thus $f(V) \cup \{f^*(e): e\epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+p+q-1\}$.

Hence A(Q_n) is a k -Super harmonic mean graph for all $k \ge 2$, if the quadrilateral starts from the first vertex of $A(Q_n)$

Example 3.5.

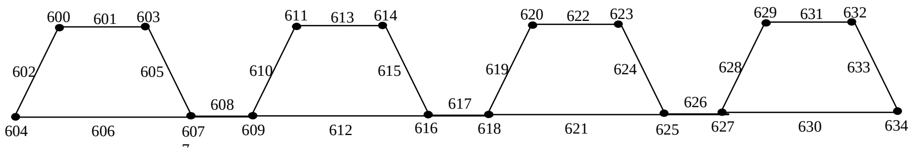

Figure 10: 600-Super harmonic mean labeling of A(Q_8)

Theorem 3.6. An alternate Quadrilateral snake $A(Q_n)$ $(n \ge 3)$ is k - super harmonic mean graph for all $k \ge 2$, if the quadrilateral starts from the second vertex of $A(Q_n)$.

Proof: The ordinary labeling is

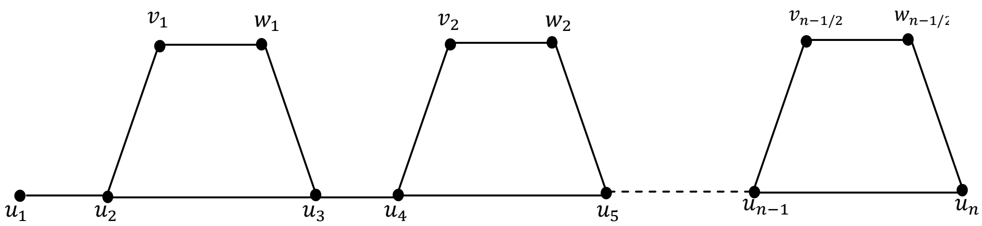

Figure 11:

Here we consider two cases

Case (i): If n is even

Define a function $f: V(AQ_n)\ ) \rightarrow \{k\ , k+1\ ,\ k+2\ , \ldots k+p+q-1\}$ by

$f(u_{2i})\ \ = k + 9i - 7\ ,\ \ for\ 1 \le i \le n/2$

$f(u_{2i-1}) = k + 9i - 9\ ,\ \ for\ 1 \le i \le n/2$

$f(v_1) = \begin{cases} k+5 & if\ k = 2 \\ k+4 & if\ k \ge 3 \end{cases}$

$f(v_i) = k + 9i - 5\ ,\ \ for\ 2 \le i \le \frac{n}{2} - 1$

$f(w_i) = k + 9i - 2\ ,\ \ for\ 1 \le i \le \frac{n}{2} - 1$.

Then the induced edge labels are

$f^*(u_2u_3) = \begin{cases} k+4 & if\ k = 2 \\ k+5 & if\ k \ge 3 \end{cases}$

$f^*(u_{2i}u_{2i+1}) = k + 9i - 4\ ,\ \ for\ 2 \le i \le \frac{n}{2} - 1$

$f^*(u_{2i-1}u_{2i}) = k + 9i - 8\ ,\ \ for\ 1 \le i \le \frac{n}{2}$

$f^*(u_{2i}v_i)\ \ \ = k + 9i - 6\ ,\ \ for\ 1 \le i \le \frac{n}{2} - 1$

$f^*(v_i w_i) = k + 9i - 3\ ,\ \ for\ 1 \le i \le \frac{n}{2} - 1.$

$f^*(w_i u_{2i+1}) = k + 9i - 1\ ,\ \ for\ 1 \le i \le \frac{n}{2} - 1$

Case (ii) : If n is odd

Define a function $f: V(AQ_n)\ ) \to \{k\ , k+1\ ,\ k+2\ , \dots k+p+q-1\}$ by

$f(u_{2i}) \quad = k + 9i - 7\ ,\ \ for\ 1 \le i \le \frac{n-1}{2}$

$f(u_{2i-1}) = k + 9i - 9\ ,\ \ for\ 1 \le i \le \frac{n+1}{2}$

$f(v_1) = \begin{cases} k+5 & if\ k = 2 \\ k+4 & if\ k \ge 3 \end{cases}$

$f(v_i) = k + 9i - 5\ ,\ \ for\ 2 \le i \le \frac{n-1}{2}$

$f(w_i) = k + 9i - 2\ ,\ \ for\ 1 \le i \le \frac{n-1}{2}.$

Then the induced edge labels are

$f^*(u_2 u_3) = \begin{cases} k+4 & if\ k = 2 \\ k+5 & if\ k \ge 3 \end{cases}$

$f^*(u_{2i} u_{2i+1}) = k + 9i - 4\ ,\ \ for\ 2 \le i \le \frac{n-1}{2}$

$f^*(u_{2i-1} u_{2i}) = k + 9i - 8\ ,\ \ for\ 1 \le i \le \frac{n-1}{2}$

$f^*(u_{2i} v_i) \quad = k + 9i - 6\ ,\ \ for\ 1 \le i \le \frac{n-1}{2}$

$f^*(v_i w_i) = k + 9i - 3\ ,\ \ for\ 1 \le i \le \frac{n-1}{2}.$

$f^*(w_i u_{2i+1}) = k + 9i - 1\ ,\ \ for\ 1 \le i \le \frac{n-1}{2}$

Thus $f(V) \cup \{f^*(e): e \epsilon E(G)\} = \{k, k+1, k+2, \dots, k+p+q-1\}$.

Hence A(Q_n) is a k -Super harmonic mean graph for all $k \ge 2$, , if the quadrilateral starts from the second vertex of $A(Q_n)$

Example 3.6.

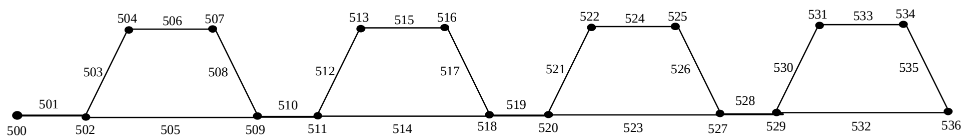

Figure 12: 500-Super harmonic mean labeling of A(Q_9)

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