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Gem-Separation Axioms in Topological Space

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Abstract. In this article we introduce a new separation axioms to define Gem-regular space, Gem-normal space, Gem-completely normal space, Gem-perfectly normal space and G^*-T_i-spaces for i = 3, 4, 5 and 6 under the idea of "Gem-set" and study some of its basic properties and relations among them.

Keywords: Gem-set, Gem-regular space, Gem-normal space Gem-completely normal space and Gem-perfectly normal space, and G^*-T_i-spaces.

AMS Mathematics Subject Classification (2010): 46M20

1. Introduction

The concept of ideals in topological spaces are treated in the standard text by Kuratowski [8] and Vaidyanathaswamy [16]. In *'general topology'* Hamlett and Jankovic [2, 3, 4, 17, 18] introduced the application of topological ideal as defined below : An ideal $\mathcal{I}$ on a topological space (X, τ) is a non empty collection of subsets of X having the following properties : (i) A $\in \mathcal{I}$ and B $\subseteq$ A implies B $\in \mathcal{I}$. (ii) A $\in \mathcal{I}$ and B $\in \mathcal{I}$ implies A $\cup$ B $\in \mathcal{I}$. An ideal topological space is a toplogical space (X, τ) with an ideal $\mathcal{I}$ on X and is denoted by (X, τ, $\mathcal{I}$). In addition K. Kuratowski[8] defined the local function for A $\subseteq$ X with respect to $\mathcal{I}$ and τ as below : $A^*(\mathcal{I}, \tau)$ or $A^*(\mathcal{I})$ = { x $\in$ X : A $\cap$ U $\notin \mathcal{I}$ for any U $\in$ τ(x) } where τ(x) = { U $\in \tau$: x $\in$ U }. We simply write A^* instead of $A^*(\mathcal{I})$. Arenas, Dontchev and Puertas [5] introduced some weak separation axioms under the concept of ideal. Swidi and Sada[10] introduced a new type of ideal for a single point x denoted as $\mathcal{I}_x$ and is defined as below : $\mathcal{I}_x$ = { U $\subseteq$ X : x $\in U^c$ }, where U is a non-empty subset of X. Swidi and Nafee [9] introduced a new set in topological space namely "Gem-set" depending on the $\mathcal{I}_x$ and defined a new separation axioms by using the idea of the "Gem-set" namely I^*-T_i-spaces and I^{**}-T_i-spaces for i = 0, 1 and 2. They also defined two mappings namely "I^*-map" and "I^{**}-map" to carry properties of the "Gem-set" from one space to another space and give more properties for new separation axioms. Swidi and Ethary [12] introduced a new class of maps namely "A-map", "AO-map" and "Am-map" under the idea of the Gem-set and studied some of its basic properties and relations as well as the properties of the separation axioms of I^*-T_i-spaces and I^{**}-T_i-spaces for i = 0, 1 and 2 with the functions and their effect upon them are also establised.

Aim of this article is to introduce the separation axioms to define Gem-regular space (G-T_3), Gem-normal space(G-T_4), Gem-compleletely normal space(G-T_5), Gem-perfectly normal space(G-T_6) and G^*-T_i-spaces for i = 3, 4, 5 and 6 and study some of its basic properties. Also we study the relations as well as the properties of G-T_i-spaces and G^*-T_i-spaces for i = 3, 4, 5 and 6 in connection with the functions "I^*-map", "I^{**}-map" "A-map" and "AO-map" and the effect upon them.

Throughout this paper, spaces means topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. Preliminaries

Definition 2.1. Let (X, τ) be a topological space, for A ⊆ X and x ∈ X we define A^{*x} with respect to (X, τ) as follows :
A^{*x} = { y ∈ X : G ∩ A $\notin \mathcal{I}_x$, for every G ∈ τ(y) }, where τ(y) = { G ∈ τ : y ∈ G }. The set A^{*x} is called "Gem-set".

Definition 2.2. Consider the mapping f : (X, τ) → (Y, σ), then f is called

- I^*-map if and only if, for every subset A of X, x ∈ X, $f(A^{*x}) = (f(A))^{*f(x)}$.
- I^{**}-map if and only if, for every subset A of Y, y ∈ Y, $f^{-1}(A^{*y}) = (f^{-1}(A))^{*f^{-1}(y)}$.

Definition 2.3. Consider the mapping f : (X, τ) → (Y, σ), then f is an

- A-map at x ∈ X, if and only if ∀ B ⊆ Y, ∃ A ⊆ X ∋: $f(A^{*x}) \subseteq B^{*f(x)}$.
- A-map on X if and only if it is an A-map at each point on X.
- AO-map if and only if ∀ A ⊆ X, ∃ B ⊆ Y ∋: $B^{*y} \subseteq f(A^{*f^{-1}(y)})$.

3. Gem-separation axioms

In this section we define Gem-regular space, Gem-normal space, Gem-completely normal space, Gem-perfectly normal space and G^*-T_i-spaces for i = 3, 4, 5 and 6 and derive some of its basic properties.

Definition 3.1. A topological space (X, τ) is a

- Gem-regular space or G-T_3-space if and only if for each disjoint pair consisting a point x and a set C in X, there exists subsets A, B of X such that x $\notin B^{*y}$ and C $\nsubseteq A^{*x}$.
- Gem-normal space or G-T_4-space if and only if for each pair C and D of disjoint sets in X, there exists subsets A, B of X such that C $\nsubseteq B^{*y}$ and D $\nsubseteq A^{*x}$.
- Gem-completely normal space or G-T_5-space if and only if for each pair of separated sets C and D in X, there exists subsets A, B of X such that C $\nsubseteq B^{*y}$ and D $\nsubseteq A^{*x}$.
- Gem-perfectly normal space or G-T_6-space if and only if for each pair C and D of disjoint sets in X, there exists a continuous map f : X → [0, 1] such that $C^{*x} \neq f^{-1}(\{1\})$ and $D^{*y} \neq f^{-1}(\{0\})$.
- G^*-T_3-space if and only if for each disjoint pair consisting a point x and a set C in X, there exists subset A of X such that x $\notin A^{*y}$ and C $\nsubseteq A^{*x}$.
- G^*-T_4-space if and only if for each pair C and D of disjoint sets in X, there exists subset A of X such that C $\nsubseteq A^{*y}$ and D $\nsubseteq A^{*x}$.

- G^*-T_5-space if and only if for each pair of separated sets C and D in X, there exists subset A of X such that C $\nsubseteq A^{*y}$ and D $\nsubseteq A^{*x}$.
- G^*-T_6-space if and only if for each pair C and D of disjoint sets in X, there exists a continuous map f : X → [0, 1] such that $C^{*x} \neq f^{-1}(\{1\})$ and $D^{*y} = f^{-1}(\{1\})$**or**$C^{*x} = f^{-1}(\{0\})$ and $D^{*y} \neq f^{-1}(\{0\})$

Theorem 3.2. For a topological space (X, τ) the following properties hold good :

1. Every T_3-space is a G-T_3-space.
2. Every T_4-space is a G-T_4-space.
3. Every T_5-space is a G-T_5-space.
4. Every T_6-space is a G-T_6-space.
5. Every T_3-space is a G^*-T_3-space.
6. Every T_4-space is a G^*-T_4-space.
7. Every T_5-space is a G^*-T_5-space.
8. Every T_6-space is a G^*-T_6-space.

Proof: 1. Let x ∈ X and C be a closed set in X with x ∉ C. Since (X, τ) is a T_3-space. Then there exists disjoint open sets U, V such that x ∈ U and C ⊆ V. Then $U^{*x} \cap V^{*y} = \phi$. Let A = U, B = V. It follows that there exists subsets A, B of X such that x $\notin B^{*y}$ and C $\nsubseteq A^{*x}$. Hence (X, τ) is a G-T_3-space.
2. Let C and D be the disjoint closed sets in X and (X, τ) is a T_4-space. Then there exists disjoint open sets U, V such that C ⊆ U and D ⊆ V. Then $U^{*x} \cap V^{*y} = \phi$. Let A = U, B = V. It follows that there exists subsets A, B of X such that C $\nsubseteq B^{*y}$ and D $\nsubseteq A^{*x}$. Hence (X, τ) is a G-T_4-space.
3. Let C and D be the separated sets in X (i.e $\bar{C}$ ∩ D = C ∩ $\bar{D}$ = ϕ) and (X, τ) is a T_5-space. Then there exists disjoint open sets U, V such that C ⊆ U and D ⊆ V. Then $U^{*x} \cap V^{*y} = hi$. Let A = U, B = V. It follows that there exists subsets A, B of X such that C $\nsubseteq B^{*y}$ and D $\nsubseteq A^{*x}$. Hence (X, τ) is a G-T_5-space.
4. Let C and D be the disjoint closed sets in X and (X, τ) is a T_6-space. Then there exists a continuous map f : X → [0, 1] such that, C = $f^{-1}(\{0\})$ and D = $f^{-1}(\{1\})$. Then $C^{*x} \cap D^{*y} = \phi$. It follows that there exists a continuous map f : X → [0, 1] such that $C^{*x} \neq f^{-1}(\{1\})$ and $D^{*y} \neq f^{-1}(\{0\})$. Hence (X, τ) is a G-T_6-space.
5. Let x ∈ X and C be a closed set in X with x ∉ C. Since (X, τ) is a T_3-space. Then there exists disjoint open sets U, V such that x ∈ U and C ⊆ V. Then $U^{*x} \cap V^{*y} = \phi$. Let U = V = A. It follows that there exists a subset A of X such that x $\notin A^{*y}$ and C $\notin A^{*x}$. Hence (X, τ) is a G^*-T_3-space.
6. Let C and D be the disjoint closed sets in X and (X, τ) is a T_4-space. Then there exists disjoint open sets U, V such that C ⊆ U and D ⊆ V. Then $U^{*x} \cap V^{*y} = \phi$. Let U = V = A. It follows that there existssubsets A, B of X such that C $\nsubseteq A^{*y}$ and D $\nsubseteq A^{*x}$. Hence (X, τ) is a G^*-T_4-space.
7. Let C and D be the separated sets in X and (X, τ) is a T_5-space. Then there exists disjoint open sets U, V such that C ⊆ U and D ⊆ V. Then $U^{*x} \cap V^{*y} = \phi$. Let U = A = V. It follows that there exists subsetA of X such that C $\nsubseteq A^{*y}$ and D $\nsubseteq A^{*x}$. Hence (X, τ) is a G^*-T_5-space.
8. Let C and D be the disjoint closed sets in X and (X, τ) is a T_6-space. Then there exists a continuous map f : X → [0, 1] such that, C = $f^{-1}(\{0\})$ and D = $f^{-1}(\{1\})$. Then

$C^{*x} \cap D^{*y} = \phi$. It follows that there exists a continuous map f : X → [0, 1] such that $C^{*x} \neq f^{-1}(\{1\})$ and $D^{*y} = f^{-1}(\{1\})$ **or** $C^{*x} = f^{-1}(\{0\})$ and $D^{*y} \neq f^{-1}(\{0\})$ Hence (X, τ) is a G^*-T_6-space.

Remark : The converse of the above theorem need not be true.

3.1. G-T_3-space

In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G-T_3-space.

Theorem 3.1.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a G-T_3-space X onto a space Y, then Y is a G-T_3-space.

Proof: Let y_1 and C_2 be a disjoint pair of Y. Since f is one-one and onto, there exists disjoint pair x_1 and C_1 of X such that $f(x_1) = y_1$ and f(C_1) = C_2. Since (X, τ) is G-T_3-space, there exists subsets A and B of X such that $x_1 \notin B^{*x_2}$ and $C_1 \not\subseteq A^{*x_1}$, so that $f(x_1) \notin f(B^{*x_2}) = (f(B))^{*f(x_2)}$ and $f(C_1) \not\subseteq f(A^{*x_1}) = (f(A))^{*f(x_1)}$. Thus $y_1 \notin (f(B))^{*f(x_2)=y_2}$ and $C_2 \not\subseteq (f(A))^{*f(x_1)=y_1}$. Thus Y is a G-$T_3$-space.

Theorem 3.1.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G-T_3-space Y, then X is a G-T_3-space.

Proof: Let x_1 and C_1 be a disjoint pairs of X. Since f is one-one and onto, there exists disjoint pairs y_1 and C_2 of Y such that $f(x_1) = y_1$ and $f(C_1) = C_2$. Since (Y, σ) is G-T_3-space, there exists subsets A, B of Y such that $y_1 \notin B^{*y_2}$ and $C_2 \not\subseteq A^{*y_1}$, so that $f^{-1}(y_1) \notin f^{-1}(B^{*y_2}) = (f^{-1}(B))^{*f^{-1}((y_2))}$ and $f^{-1}(C_2) \not\subseteq f^{-1}(A^{*y_1}) = (f^{-1}(A))^{*f^{-1}(y_1)}$. This implies $x_1 \notin (f^{-1}(B))^{*x_2}$ and $C_1 \not\subseteq (f^{-1}(A))^{*x_1}$. Thus X is a G-$T_3$-space.

Theorem 3.1.3. If f : (X, τ) → (Y, σ) is one-one A-map of a G-T_3-space X onto a space Y, then Y is a G-T_3-space.

Proof: Let y_1 and C_2 be a disjoint pair of Y. Since f is one-one and onto, there exists a disjoint pair x_1 and C_1 of X such that $f(x_1) = y_1$ and $f(C_1) = C_2$. Since (X, τ) is G-T_3-space, there exists subsets A_1, A_2 of X such that $x_1 \notin A_2^{*x_2}$ and $C_1 \not\subseteq A_1^{*x_1}$, so that $f(x_1) \notin f(A_2^{*x_2}) \subseteq B_2^{*f(x_2)}$ and $f(C_1) \not\subseteq f(A_1^{*x_1}) \subseteq B_1^{*f(x_1)}$. This implies $y_1 \notin B_2^{*y_2}$ and $C_2 \not\subseteq B_1^{*y_1}$. Thus Y is a G-$T_3$-space.

Theorem 3.1.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G-T_3-space Y, then X is a G-T_3-space.

Proof: Let x_1 and C_1 be a disjoint pair of X. Since f is one-one and onto, there exists a disjoint pair y_1 and C_2 of Y such that $f(x_1) = y_1$ and $f(C_1) = C_2$. Since (Y, σ) is G-T_3-space, there exists subsets B_1, B_2 of Y such that $y_1 \notin B_2^{*y_2} \subseteq f(A_2^{*f^{-1}(y_2)})$ and $C_2 \not\subseteq B_1^{*y_1} \subseteq f(A_1^{*f^{-1}(y_1)})$, so that $f^{-1}(y_1) \notin f^{-1}(f(A_2^{*f^{-1}(y_2)}))$ and $f^{-1}(C_2) \not\subseteq f^{-1}(f(A_1^{*f^{-1}(y_1)}))$. This implies $x_1 \notin A_2^{*x_2}$ and $C_1 \not\subseteq A_1^{*x_1}$. Thus X is a G-$T_3$-space.

3.2. G-T_4-space

In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G-T_4-space.

Theorem 3.2.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a G-T_4-space X onto a space Y, then Y is a G-T_4-space.

Proof: Let C_2 and D_2 be two disjoint sets in Y. Since f is one-one and onto, there exists disjoint sets C_1 and D_1 of X such that $f(C_1) = C_2$ and f(D_1) = D_2. Since (X, τ) is G-T_4-space, there exists subsets A and B of X such that $C_1 \nsubseteq B^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$, so that $f(C_1) \nsubseteq f(B^{*x_2}) = (f(B))^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A^{*x_1}) = (f(A))^{*f(x_1)}$.
Thus $C_2 \nsubseteq (f(B))^{*f(x_2)=y_2}$ and $D_2 \nsubseteq (f(A))^{*f(x_1)=y_1}$. Thus Y is a G-$T_4$-space.

Theorem 3.2.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G-T_4-space Y, then X is a G-T_4-space.

Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G-T_4-space, there exists subsets A, B of Y such that $C_2 \nsubseteq B^{*y_2}$ and $D_2 \nsubseteq A^{*y_1}$, so that $f^{-1}(C_2) \nsubseteq f^{-1}(B^{*y_2}) = (f^{-1}(B))^{*f^{-1}((y_2))}$ and $f^{-1}(D_2) \nsubseteq f^{-1}(A^{*y_1}) = (f^{-1}(A))^{*f^{-1}(y_1)}$. This implies $C_1 \nsubseteq (f^{-1}(B))^{*x_2}$ and $D_1 \nsubseteq (f^{-1}(A))^{*x_1}$. Thus X is a G-$T_4$-space.

Theorem 3.2.3. If f : (X, τ) → (Y, σ) is one-one A-map of a I^*- T_4-space X onto a space Y, then Y is a G-T_4-space.

Proof: Let C_2 and D_2 be two disjoint sets in Y. Since f is one-one and onto, there exists a disjoint sets C_1 and D_1 of X such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (X, τ) is G-T_4-space, there exists subsets A_1, A_2 of X such that $C_1 \nsubseteq A_2^{*x_2}$ and $D_1 \nsubseteq A_1^{*x_1}$, so that $f(C_1) \nsubseteq f(A_2^{*x_2}) \subseteq B_2^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A_1^{*x_1}) \subseteq B_1^{*f(x_1)}$. This implies $C_2 \nsubseteq B_2^{*y_2}$ and $D_2 \nsubseteq B_1^{*y_1}$. Thus Y is a G-$T_4$-space.

Theorem 3.2.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G-T_4-space Y, then X is a G-T_4-space.

Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists a disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G-T_4-space, there exists subsets B_1, B_2 of Y such that $C_2 \nsubseteq B_2^{*y_2} \subseteq f(A_2^{*f^{-1}(y_2)})$ and $D_2 \nsubseteq B_1^{*y_1} \subseteq f(A_1^{*f^{-1}(y_1)})$, so that $f^{-1}(C_2) \nsubseteq f^{-1}(f(A_2^{*f^{-1}(y_2)}))$ and $f^{-1}(D_2) \nsubseteq f^{-1}(f(A_1^{*f^{-1}(y_1)}))$. This implies $C_1 \nsubseteq A_2^{*x_2}$ and $D_1 \nsubseteq A_1^{*x_1}$. Thus X is a G-$T_4$-space.

3.3. G-T_5-space

In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G-T_5-space.

Theorem 3.3.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a G-T_5-space X onto a space Y, then Y is a G-T_5-space.
Proof: Let C_2 and D_2 be separated sets in Y. Since f is one-one and onto, there exists separated sets C_1 and D_1 of X such that $f(C_1)$ = C_2 and f(D_1) = D_2. Since (X, τ) is G-T_5-space, there exists subsets A and B of X such that $C_1 \nsubseteq B^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$, so that $f(C_1) \nsubseteq f(B^{*x_2}) = (f(B))^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A^{*x_1}) = (f(A))^{*f(x_1)}$. Thus $C_2 \nsubseteq (f(B))^{*f(x_2)=y_2}$ and $D_2 \nsubseteq (f(A))^{*f(x_1)=y_1}$. Thus Y is a G-$T_5$-space.

Theorem 3.3.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G-T_5-space Y, then X is a G-T_5-space.
Proof: Let C_1 and D_1 be separated sets in X. Since f is one-one and onto, there exists separated sets C_2 and D_2 of Y such that $f(C_1)$ = C_2 and $f(D_1)$ = D_2. Since (Y, σ) is G-T_5-space, there exists subsets A, B of Y such that $C_2 \nsubseteq B^{*y_2}$ and $D_2 \nsubseteq A^{*y_1}$, so that $f^{-1}(C_2) \nsubseteq f^{-1}(B^{*y_2})$ = $(f^{-1}(B))^{*f^{-1}((y_2))}$ and $f^{-1}(D_2) \nsubseteq f^{-1}(A^{*y_1})$ = $(f^{-1}(A))^{*f^{-1}(y_1)}$. This implies $C_1 \nsubseteq (f^{-1}(B))^{*x_2}$ and $D_1 \nsubseteq (f^{-1}(A))^{*x_1}$. Thus X is a G-$T_5$-space.

Theorem 3.3.3. If f : (X, τ) → (Y, σ) is one-one A-map of an G-T_5-space X onto a space Y, then Y is a G-T_5-space.
Proof: Let C_2 and D_2 be separated sets in Y. Since f is one-one and onto, there exists separated sets C_1 and D_1 of X such that $f(C_1)$ = C_2 and $f(D_1)$ = D_2. Since (X, τ) is G-T_5-space, there exists subsets A_1, A_2 of X such that $C_1 \nsubseteq A_2^{*x_2}$ and $D_1 \nsubseteq A_1^{*x_1}$, so that $f(C_1) \nsubseteq f(A_2^{*x_2}) \subseteq B_2^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A_1^{*x_1}) \subseteq B_1^{*f(x_1)}$. This implies $C_2 \nsubseteq B_2^{*y_2}$ and $D_2 \nsubseteq B_1^{*y_1}$. Thus Y is a G-$T_5$-space.

Theorem 3.3.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G-T_5-space Y, then X is a G-T_5-space.
Proof: Let C_1 and D_1 be separated sets in X. Since f is one-one and onto, there exists separated sets C_2 and D_2 of Y such that $f(C_1)$ = C_2 and $f(D_1)$ = D_2. Since (Y, σ) is G-T_5-space, there exists subsets B_1, B_2 of Y such that $C_2 \nsubseteq B_2^{*y_2} \subseteq f(A_2^{*f^{-1}(y_2)})$ and $D_2 \nsubseteq B_1^{*y_1} \subseteq f(A_1^{*f^{-1}(y_1)})$, so that $f^{-1}(C_2) \nsubseteq f^{-1}(f(A_2^{*f^{-1}(y_2)}))$ and $f^{-1}(D_2)ot \subseteq f^{-1}(f(A_1^{*f^{-1}(y_1)}))$. This implies $C_1 \nsubseteq A_2^{*x_2}$ and $D_1 \nsubseteq A_1^{*x_1}$. Thus X is a G-$T_5$-space.

3.4. G-T_6-space
In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G-T_6-space.

Theorem 3.4.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a space X onto G-T_6-space Y, then X is a G-T_6-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1)$ = C_2 and $f(D_1)$ = D_2. Since f is an I^*-map, so that f($C_1^{*x_1}$) = $(f(C_1))^{*f(x_1)}$ = $C_2^{*y_1}$ and f($D_1^{*x_2}$) = $(f(D_1))^{*f(x_2)}$ = $D_2^{*y_2}$ Since (Y, σ) is G-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and

$D_2^{*y_2} \neq g^{-1}(\{0\})$. This implies f($C_1^{*x_1}$) $\neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) $\neq g^{-1}(\{0\})$. Now g(f($C_1^{*x_1}$)) $\neq$ ({1}) and g(f($D_1^{*x_2}$)) $\neq$ ({0}). This implies h($C_1^{*x_1}$) $\neq$ ({1}) and h($D_1^{*x_2}$) $\neq$ ({0}). Thus$C_1^{*x_1} \neq h^{-1}(\{1\})$ and $D_1^{*x_2} \neq h^{-1}(\{0\})$ where h = g∘ f : X → [0, 1] is a continuous map. Hence by definition we have (X, τ) is a G-T_6-space.

Theorem 3.4.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G-T_6-space Y, then X is a G-T_6-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and $D_2^{*y_2} \neq g^{-1}(\{0\})$. So that $f^{-1}(\ C_2^{*y_1}\ ) \neq f^{-1}(\ g^{-1}(\{1\})\ )$ and $f^{-1}\ (D_2^{*y_2}) \neq f^{-1}(g^{-1}(\{0\}))$. This implies $f^{-1}(\ C_2^{*y_1}\ ) \neq h^{-1}(\{1\})$ and $f^{-1}\ (D_2^{*y_2}) \neq h^{-1}(\{0\})$. Since f is an I^{**}-map, we have $(f^{-1}(C_2))^{*f^{-1}(y_1)} \neq h^{-1}(\{1\})$ and $(f^{-1}(D_2))^{*f^{-1}(y_2)} \neq h^{-1}(\{0\})$. Thus$C_1^{*x_1} \neq h^{-1}(\{1\})$ and $D_1^{*x_2} \neq h^{-1}(\{0\})$ where h = g∘ f : X → [0, 1] is a continuous map. Hence by definition we have (X, τ) is a G-T_6-space.

Theorem 3.4.3. If f : (X, τ) → (Y, σ) is one-one A-map of a space X onto G-T_6-space Y, then X is a G-T_6-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since f is an A-map, so that f($C_1^{*x_1}$) $\subseteq C_2^{*f(x_1)} = C_2^{*y_1}$ and f($D_1^{*x_2}$) $\subseteq D_2^{*f(x_2)} = D_2^{*y_2}$ Since (Y, σ) is G-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and $D_2^{*y_2} \neq g^{-1}(\{0\})$.This implies f($C_1^{*x_1}$) $\subseteq C_2^{*y_1} \neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) $\subseteq D_2^{*y_2} \neq g^{-1}(\{0\})$. This implies f($C_1^{*x_1}$) $\neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) $\neq g^{-1}(\{0\})$. Now g(f($C_1^{*x_1}$)) $\neq$ ({1}) and g(f($D_1^{*x_2}$)) $\neq$ ({0}). This implies h($C_1^{*x_1}$) $\neq$ ({1}) and h($D_1^{*x_2}$) $\neq$ ({0}). Thus $C_1^{*x_1} \neq h^{-1}(\{1\})$ and $D_1^{*x_2} \neq h^{-1}(\{0\})$ where h = $g \circ f$:X → [0, 1] is a continuous map. Hence by definition we have (X, τ) is a G-T_6-space.

Theorem 3.4.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G-T_6-space Y, then X is a G-T_6-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and $D_2^{*y_2} \neq g^{-1}(\{0\})$. Since f is an AO-map, so that $C_2^{*y_1} \subseteq f(C_1^{*f^{-1}(y_1)}) \neq g^{-1}(\{1\})$ and $D_2^{*y_2} \subseteq f(D_1^{*f^{-1}(y_2)}) \neq g^{-1}(\{0\})$. This implies f($C_1^{*x_1}$) $\neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) $\neq g^{-1}(\{0\})$. Now g(f($C_1^{*x_1}$)) $\neq$ ({1}) and g(f($D_1^{*x_2}$)) $\neq$ ({0}). This implies h($C_1^{*x_1}$) $\neq$ ({1}) and h($D_1^{*x_2}$) $\neq$ ({0}). Thus $C_1^{*x_1} \neq h^{-1}(\{1\})$ and $D_1^{*x_2} \neq h^{-1}(\{0\})$ where h = g ∘ f : X → [0, 1] is a continuous map. Hence by definition we have (X, τ) is a G-T_6-space.

3.5. G^*-T_3-space
In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G^*-T_3-space.

Theorem 3.5.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a G^*-T_3-space X onto a space Y, then Y is a G^*-T_3-space.
Proof: Let y_1 and C_2 be a disjoint pair of Y. Since f is one-one and onto, there exists disjoint pair x_1 and C_1 of X such that $f(x_1) = y_1$ and f(C_1) = C_2. Since (X, τ) is G^*-T_3-space, there exists subset A of X such that $x_1 \notin A^{*x_2}$ and $C_1 \nsubseteq A^{*x_1}$, so that $f(x_1) \notin f(A^{*x_2}) = (f(A))^{*f(x_2)}$ and $f(C_1) \nsubseteq f(A^{*x_1}) = (f(A))^{*f(x_1)}$. Thus $y_1 \notin (f(A))^{*f(x_2)=y_2}$ and $C_2 \nsubseteq (f(A))^{*f(x_1)=y_1}$. Thus Y is a G^*-T_3-space.

Theorem 3.5.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G^*-T_3-space Y, then X is a G^*-T_3-space.
Proof: Let x_1 and C_1 be a disjoint pairs of X. Since f is one-one and onto, there exists disjoint pairs y_1 and C_2 of Y such that $f(x_1) = y_1$ and $f(C_1) = C_2$. Since (Y, σ) is G^*-T_3-space, there exists subset A of Y such that $y_1 \notin A^{*y_2}$ and $C_2 \nsubseteq A^{*y_1}$, so that $f^{-1}(y_1) \notin f^{-1}(A^{*y_2}) = (f^{-1}(A))^{*f^{-1}((y_2))}$ and $f^{-1}(C_2) \nsubseteq f^{-1}(A^{*y_1}) = (f^{-1}(A))^{*f^{-1}(y_1)}$. This impies $x_1 \notin (f^{-1}(A))^{*x_2}$ and $C_1 \nsubseteq (f^{-1}(A))^{*x_1}$. Thus X is a G^*-T_3-space.

Theorem 3.5.3. If f : (X, τ) → (Y, σ) is one-one A-map of an G^*-T_3-space X onto a space Y, then Y is a G^*-T_3-space.
Proof: Let y_1 and C_2 be a disjoint pair of Y. Since f is one-one and onto, there exists a disjoint pair x_1 and C_1 of X such that $f(x_1) = y_1$ and $f(C_1) = C_2$. Since (X, τ) is G^*-T_3-space, there exists subsets A of X such that $x_1 \notin A^{*x_2}$ and $C_1 \nsubseteq A^{*x_1}$, so that $f(x_1) \notin f(A^{*x_2}) \subseteq B^{*f(x_2)}$ and $f(C_1) \nsubseteq f(A^{*x_1}) \subseteq B^{*f(x_1)}$. This implies $y_1 \notin B^{*y_2}$ and $C_2 \nsubseteq B^{*y_1}$. Thus Y is a G^*-T_3-space.

Theorem 3.5.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G^*-T_3-space Y, then X is a G^*-T_3-space.
Proof: Let x_1 and C_1 be a disjoint pair of X. Since f is one-one and onto, there exists a disjoint pair y_1 and C_2 of Y such that $f(x_1) = y_1$ and $f(C_1) = C_2$. Since (Y, σ) is G^*-T_3-space, there exists subset B of Y such that $y_1 \notin B^{*y_2} \subseteq f(A^{*f^{-1}(y_2)})$ and $C_2 \nsubseteq B^{*y_1} \subseteq f(A^{*f^{-1}(y_1)})$, so that $f^{-1}(y_1) \notin f^{-1}(f(A^{*f^{-1}(y_2)}))$ and $f^{-1}(C_2) \nsubseteq f^{-1}(f(A^{*f^{-1}(y_1)}))$. This implies $x_1 \notin A^{*x_2}$ and $C_1 \nsubseteq A^{*x_1}$. Thus X is a G^*-T_3-space.

3.6. G^*-T_4-space

In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G^*-T_4-space.

Theorem 3.6.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a G^*-T_4-space X onto a space Y, then Y is a G^*-T_4-space.
Proof: Let C_2 and D_2 be two disjoint sets in Y. Since f is one-one and onto, there exists disjoint sets C_1 and D_1 of X such that $f(C_1) = C_2$ and f(D_1) = D_2. Since (X, τ) is G^*-T_4-space, there exists a subset A of X such that$C_1 \nsubseteq A^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$, so that $f(C_1) \nsubseteq f(A^{*x_2}) = (f(A))^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A^{*x_1}) = (f(A))^{*f(x_1)}$. Thus

$C_2 \nsubseteq (f(A))^{*f(x_2)=y_2}$and $D_2 \nsubseteq (f(A))^{*f(x_1)=y_1}$. Thus Y is a G^*-T_4-space.

Theorem 3.6.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G^*-T_4-space Y, then X is a G^*-T_4-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G^*-T_4-space, there exists a subset A of Y such that $C_2 \nsubseteq A^{*y_2}$ and $D_2 \nsubseteq A^{*y_1}$, so that $f^{-1}(C_2) \nsubseteq f^{-1}(A^{*y_2}) = (f^{-1}(A))^{*f^{-1}((y_2))}$ and $f^{-1}(D_2) \nsubseteq f^{-1}(A^{*y_1}) = (f^{-1}(A))^{*f^{-1}(y_1)}$. This implies $C_1 \nsubseteq (f^{-1}(A))^{*x_2}$ and $D_1 \nsubseteq (f^{-1}(A))^{*x_1}$. Thus X is a G^*-T_4-space.

Theorem 3.6.3. If f : (X, τ) → (Y, σ) is one-one A-map of a G^*-T_4-space X onto a space Y, then Y is a G^*-T_4-space.
Proof: Let C_2 and D_2 be two disjoint sets in Y. Since f is one-one and onto, there exists a disjoint sets C_1 and D_1 of X such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (X, τ) is G^*-T_4-space, there exists a subset A of X such that $C_1 \nsubseteq A^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$, so that $f(C_1) \nsubseteq f(A^{*x_2}) \subseteq B^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A^{*x_1}) \subseteq B^{*f(x_1)}$. This implies $C_2 \nsubseteq B^{*y_2}$ and $D_2 \nsubseteq B^{*y_1}$. Thus Y is a G^*-T_4-space.

Theorem 3.6.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G^*-T_4-space Y, then X is a G^*-T_4-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists a disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G^*-T_4-space, there exists subsets B of Y such that $C_2 \nsubseteq B^{*y_2} \subseteq f(A^{*f^{-1}(y_2)})$ and $D_2 \nsubseteq B^{*y_1} \subseteq f(A^{*f^{-1}(y_1)})$, so that $f^{-1}(C_2) \nsubseteq f^{-1}(f(A^{*f^{-1}(y_2)}))$ and $f^{-1}(D_2) \nsubseteq f^{-1}(f(A^{*f^{-1}(y_1)}))$. This implies $C_1 \nsubseteq A^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$. Thus X is a G^*-T_4-space.

3.7. G^*-T_5-space
In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G^*-T_5-space.

Theorem 3.7.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a G^*-T_5-space X onto a space Y, then Y is a G^*-T_5-space.
Proof: Let C_2 and D_2 be separated sets in Y. Since f is one-one and onto, there exists separated sets C_1 and D_1 of X such that $f(C_1) = C_2$ and f(D_1) = D_2. Since (X, τ) is G^*-T_5-space, there exists subset A of X such that $C_1 \nsubseteq A^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$, so that $f(C_1) \nsubseteq f(A^{*x_2}) = (f(A))^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A^{*x_1}) = (f(A))^{*f(x_1)}$. Thus $C_2 \nsubseteq (f(A))^{*f(x_2)=y_2}$ and$D_2 \nsubseteq (f(A))^{*f(x_1)=y_1}$. Thus Y is a G^*-T_5-space.

Theorem 3.7.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G^*-T_5-space Y, then X is a G^*-T_5-space.
Proof: Let C_1 and D_1 be separated sets in X. Since f is one-one and onto, there exists separated sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G^*-T_5-space, there exists subset A of Y such that $C_2 \nsubseteq A^{*y_2}$ and $D_2 \nsubseteq A^{*y_1}$, so that

$f^{-1}(C_2) \nsubseteq f^{-1}(A^{*y_2}) = (f^{-1}(A))^{*f^{-1}((y_2))}$ and $f^{-1}(D_2) \nsubseteq f^{-1}(A^{*y_1}) = (f^{-1}(A))^{*f^{-1}(y_1)}$. This implies $C_1 \nsubseteq (f^{-1}(A))^{*x_2}$ and $D_1 \nsubseteq (f^{-1}(A))^{*x_1}$. Thus X is a G^*-T_5-space.

Theorem 3.7.3. If f : (X, τ) → (Y, σ) is one-one A-map of an G^*-T_5-space X onto a space Y, then Y is a G^*-T_5-space.
Proof: Let C_2 and D_2 be separated sets in Y. Since f is one-one and onto, there exists separated sets C_1 and D_1 of X such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (X, τ) is G^*-T_5-space, there exists subset A of X such that $C_1 \nsubseteq A^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$, so that $f(C_1) \nsubseteq f(A^{*x_2}) \subseteq B^{*f(x_2)}$ and $f(D_1) \nsubseteq f(A^{*x_1}) \subseteq B^{*f(x_1)}$. This implies $C_2 \nsubseteq B^{*y_2}$ and $D_2 \nsubseteq B^{*y_1}$. Thus Y is a G^*-T_5-space.

Theorem 3.7.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G^*-T_5-space Y, then X is a G^*-T_5-space.
Proof: Let C_1 and D_1 be separated sets in X. Since f is one-one and onto, there exists separated sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G^*-T_5-space, there exists subset B of Y such that $C_2 \nsubseteq B^{*y_2} \subseteq f(A^{*f^{-1}(y_2)})$ and $D_2 \nsubseteq B^{*y_1} \subseteq f(A^{*f^{-1}(y_1)})$, so that $f^{-1}(C_2) \nsubseteq f^{-1}(f(A^{*f^{-1}(y_2)}))$ and
$f^{-1}(D_2) \nsubseteq f^{-1}(f(A^{*f^{-1}(y_1)}))$. This implies $C_1 \nsubseteq A^{*x_2}$ and $D_1 \nsubseteq A^{*x_1}$. Thus X is a G^*-T_5-space.

3.8. G^*-T_6-space

In this section we proved some theorems in connection with I^*-map, I^{**}-map, A-map and AO-map for G^*-T_6-space.

Theorem 3.8.1. If f : (X, τ) → (Y, σ) is one-one I^*-map of a space X onto G^*-T_6-space Y, then X is a G^*-T_6-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since f is an I^*-map, so that f($C_1^{*x_1}$) = $(f(C_1))^{*f(x_1)} = C_2^{*y_1}$ and f($D_1^{*x_2}$) = $(f(D_1))^{*f(x_2)} = D_2^{*y_2}$ Since (Y, σ) is G^*-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and $D_2^{*y_2} = g^{-1}(\{1\})$. This implies f($C_1^{*x_1}$) $\neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) $= g^{-1}(\{1\})$. Now g(f($C_1^{*x_1}$)) $\neq$ ({1}) and g(f($D_1^{*x_2}$)) = ({1}). This implies h($C_1^{*x_1}$) $\neq$ ({1}) and h($D_1^{*x_2}$) = ({1}). Thus $C_1^{*x_1} \neq h^{-1}(\{1\})$ and $D_1^{*x_2} = h^{-1}(\{1\})$ where h = g $\circ f$: X → [0, 1] is a continuous map. Hence by definition we have (X, τ) is a G^*-T_6-space.

Theorem 3.8.2. If f : (X, τ) → (Y, σ) is one-one I^{**}-map of a space X onto G^*-T_6-space Y, then X is a G^*-T_6-space.
Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G^*-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and $D_2^{*y_2}$ = $g^{-1}(\{1\})$. So that $f^{-1}(C_2^{*y_1}) \neq f^{-1}(g^{-1}(\{1\}))$ and $f^{-1} (D_2^{*y_2}) = f^{-1}(g^{-1}(\{1\}))$. This implies $f^{-1}(C_2^{*y_1}) \neq h^{-1}(\{1\})$ and $f^{-1}(D_2^{*y_2}) = h^{-1}(\{1\})$. Since f is an I^{**}-map, we have $(f^{-1}(C_2))^{*f^{-1}(y_1)} \neq h^{-1}(\{1\})$ and $(f^{-1}(D_2))^{*f^{-1}(y_2)} = h^{-1}(\{1\})$. This implies

$C_1^{*x_1} \neq h^{-1}(\{1\})$ and $D_1^{*x_2} = h^{-1}(\{1\})$ where h = g ∘ f : X → [0, 1] is a continuous map. Thus by definition we have (X, τ) is a G^*-T_6-space.

Theorem 3.8.3. If f : (X, τ) → (Y, σ) is one-one A-map of a space X onto G^*-T_6-space Y, then X is a G^*-T_6-space.

Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since f is an A-map, so that f($C_1^{*x_1}$) ⊆ $C_2^{*f(x_1)} = C_2^{*y_1}$ and f($D_1^{*x_2}$) ⊆ $D_2^{*f(x_2)} = D_2^{*y_2}$ Since (Y, σ) is G^*-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and $D_2^{*y_2} = g^{-1}(\{1\})$. This implies f($C_1^{*x_1}$) ⊆ $C_2^{*y_1} \neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) ⊆ $D_2^{*y_2} = g^{-1}(\{1\})$. This implies f($C_1^{*x_1}$) $\neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) $= g^{-1}(\{1\})$. Now g(f($C_1^{*x_1}$)) ≠ ({1}) and g(f($D_1^{*x_2}$)) = ({1}). This implies h($C_1^{*x_1}$) ≠ ({1}) and h($D_1^{*x_2}$) = ({1}). Thus $C_1^{*x_1} \nsubseteq h^{-1}(\{1\})$ and $D_1^{*x_2} \subseteq h^{-1}(\{1\})$ where h = g ∘ f : X → [0, 1] is a continuous map. Hence by definition we have (X, τ) is a G^*-T_6-space.

Theorem 3.8.4. If f : (X, τ) → (Y, σ) is one-one AO-map of a space X onto G^*-T_6-space Y, then X is a G^*-T_6-space.

Proof: Let C_1 and D_1 be two disjoint sets in X. Since f is one-one and onto, there exists disjoint sets C_2 and D_2 of Y such that $f(C_1) = C_2$ and $f(D_1) = D_2$. Since (Y, σ) is G^*-T_6-space, there exists a continuous map g : Y → [0, 1] such that $C_2^{*y_1} \neq g^{-1}(\{1\})$ and $D_2^{*y_2}$ = $g^{-1}(\{1\})$. Since f is an AO-map, so that $C_2^{*y_1} \subseteq f(C_1^{*f^{-1}(y_1)}) \neq g^{-1}(\{1\})$ and $D_2^{*y_2} \subseteq f(D_1^{*f^{-1}(y_2)}) = g^{-1}(\{1\})$. This implies f($C_1^{*x_1}$) $\neq g^{-1}(\{1\})$ and f($D_1^{*x_2}$) = $g^{-1}(\{1\})$. Now g(f($C_1^{*x_1}$)) ≠ ({1}) and g(f($D_1^{*x_2}$)) = ({1}). This implies h($C_1^{*x_1}$) ≠ ({1}) and h($D_1^{*x_2}$) = ({1}). Thus $C_1^{*x_1} \neq h^{-1}(\{1\})$ and $D_1^{*x_2} = h^{-1}(\{1\})$ where h = g∘ f : X → [0, 1] is a continuous map. Hence by definition we have (X, τ) is a G^*-T_6-space.

4. Conclusion

In this article, we studied some basic concepts and relations involving Gem-separation axioms. We also rename I^*-T_0-space, I^*-T_1-space, I^*-T_2-space by Gem-Kolmogorov space(G-T_0-space), Gem-accessible space or Gem-Frechlet space(G-T_1-space) and Gem-Hausdorff space(G-T_2-space) and I^{**}-T_i-spaces by G^*-T_i-spaces. In future the concepts used in nano-topology can be adopted to prove that Gem-set in nano topological space.

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