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A Comparison of Two New Ranking Methods on Solving Fuzzy Assignment Problem

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Abstract. In this paper two different new ranking methods namely "Ranking of Fuzzy Numbers with New Area Method" and "revised SD of Point of Intersection of Legs of Trapezium (PILOT) ranking procedure" are proposed to defuzzify the generalized trapezoidal fuzzy numbers. A comparative study based on the solution of fuzzy assignment problems with generalized trapezoidal fuzzy numbers has been made using the proposed ranking methods and the results are discussed along with appropriate numerical examples

Keywords: Defuzzification; Fuzzy Number; Generalized Trapezoidal Fuzzy Numbers; Ranking of Fuzzy Numbers; Fuzzy Assignment Problem

AMS Mathematics Subject Classification (2010): 03E72, 90C05

1. Introduction

The process of production management involves increase of profit by reducing the cost with the values vague atmosphere, the fuzzy set theory could be applied appropriately. Linear programming problems play an important task as a revolutionary tool. Fuzzy sets were introduced by Zadeh and Dieter Klaua in 1965 to represent, control data and information possessing nonstatistical uncertainties [15]. Since the parameters engaged in this process uncertainly, we can use fuzzy linear programming problems. Bellman and Zadeh proposed the concepts of decision making in fuzzy environments [1]. The idea of fuzzy linear programming was first initiated by H.O'heigeartaigh [6].

Assignment problem is a special type of linear programming problem. It is basically allocation of different resources to different activities on one-to-one basis. It helps to use the resources such a way that cost or time may be minimized so as to increase the profit. The process of production management involves maximizing of profit by minimizing the cost with the values imprecise; the fuzzy set theory could be used appropriately. A fuzzy assignment problem is a transportation problem formulated with fuzzy quantities of transportation costs, supply and demand. Many of the existing techniques provide solutions for the fuzzy assignment problem. Chen [3] presented a fuzzy assignment model that considers all persons to have same skills. Long Sheng

Huang and Li-pu Zhang [5] provided a mathematical model for the fuzzy assignment problem and transformed the model as assignment problem with restriction of qualification. Chen Liang-Hsuan and Lu Hai-Wen [8] developed a procedure to resolve fuzzy assignment problems with multiple inadequate inputs and outputs in crisp form using linear programming model to determine the assignments with the maximum efficiency. Xionghui ye and JuipingXu [14] developed a priority based genetic algorithm to a fuzzy vehicle routing assignment model with connection network.

Liu and Goa [4] proposed an equilibrium optimization problem and extended the assignment problem to the equilibrium multi job assignment problem, equilibrium multi job quadratic assignment problem and used genetic algorithm to solve the proposed models. Bai et al [2] proposed a method for solving fuzzy generalized assignment problem. NagoorKani and Mohamed solved a fuzzy assignment problem using a new ranking method [7].

Since fuzzy numbers are denoted by possibility distribution, it is tough to order clearly the ascending or descending order. A right method for ordering the fuzzy numbers is by the use of a ranking function. The ranking function maps each fuzzy number into the real line. A ranking function is a function R:(R) $\rightarrow R$ which maps each fuzzy number into the real line, where a natural order exists. There are so many ranking methods available, nowadays. Among them, the notable procedures are Lexicographic screening procedure discovered by Wang et al. [12], Area between centroid and its original point method [13] by Wang and Lee (2008); SD of PILOT procedure [9]; and Area method [10] and A Revised approach of PILOT ranking procedure [11] by Dinagar and Kamalanathan.

We solved the problem by using two new ranking methods New Area Method and revised (Shortest Distance of Point of Intersection of Legs of Trapezium) SD of PILOT ranking method by converting the fuzzy number into crisp form. The rest of the article is organized as follows: the next section provides the definitions of fuzzy numbers; Section 3 explains the ranking procedures; the algorithm is explained in Section 4; Section 5 is provided with the numerical examples; and Section 6 is for results and discussion for solving fuzzy transportation problems.

2. Preliminaries

Definition 1. Fuzzy set: A fuzzy set A in a nonempty set X is categorized by its membership function $\mu_A(x) \rightarrow [0,1]$ and $\mu_A(x)$ is meant as the degree of membership of element *x* in fuzzy set A for each *x* belongs to *X*.

Definition 2. A fuzzy number A is a fuzzy set of the real line with a normal, convex and continuous membership function of bounded support. The family of fuzzy numbers will be denoted by F.

Definition 3. A Generalized Trapezoidal Fuzzy Number (GTrFN) $\widetilde{A}$ as $\widetilde{A} = (a,b,c,d;\, w)$, $0 < w \leq 1$ and *a*, *b*, *c* and *d* are real numbers. The generalized fuzzy number $\widetilde{A}$ is a fuzzy subset of real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions:

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x) = 0$, where $-\infty \leq x \leq a$.

(iii) $\mu_{\tilde{A}}(x)$ isstrictlyincreasing with constant rate on $a \le x \le b$.

(iv) $\mu_{\tilde{A}}(x)$ = w, where $b \le x \le c$.

(v) $\mu_{\tilde{A}}(x)$ isstrictlydecreasing with constant rate on $c \le x \le d$ $\mu_{\tilde{A}}(x) = 0$, where $d \le x \le \infty$.

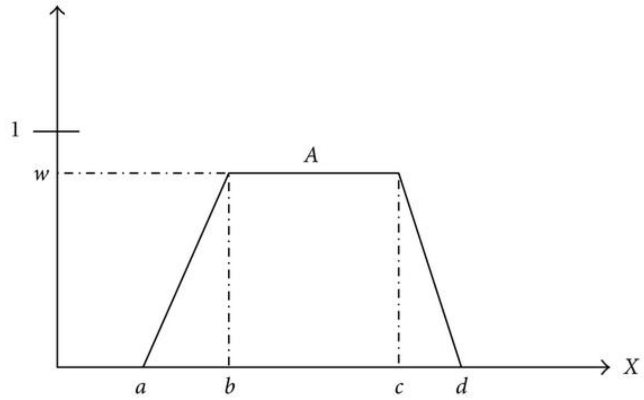

Figure 1: Generalized trapezoidal fuzzy number

A Generalized Trapezoidal Fuzzy Number $\widetilde{A} = (a,b,c,d;w)$ is a fuzzy set of the real line R whose membership function $\mu_{\tilde{A}}(x) : R \to [0, w]$ is defined as

$$\mu^{w}_{\tilde{A}}(x) = \begin{cases} \mu^{w}_{L\tilde{A}}(x) = w\left(\dfrac{x-a}{b-a}\right), \text{for } a \le x \le b \\ w, \qquad \text{for } b \le x \le c \\ \mu_{R\tilde{A}}(x) = w\left(\dfrac{d-x}{d-c}\right), \text{for } c \le x \le d \\ 0, \text{ Otherwise} \end{cases}$$

where $a < b < c < d$ and $w \in (0,1]$

3. Ranking procedure

New Area Method: The area of the trapezium is the ranking function [9] and it is given by

$$A = \frac{w}{2}[(d-a)+(c-b)] \qquad \text{...... (1)}$$

Revised SD of PILOT Ranking:

The area covered by the PILOT [10] is derived as follows. The line OP makes 90° with y-axis when it meets at Q and with y-axis when it meets at R. It is shown in Figure 2.

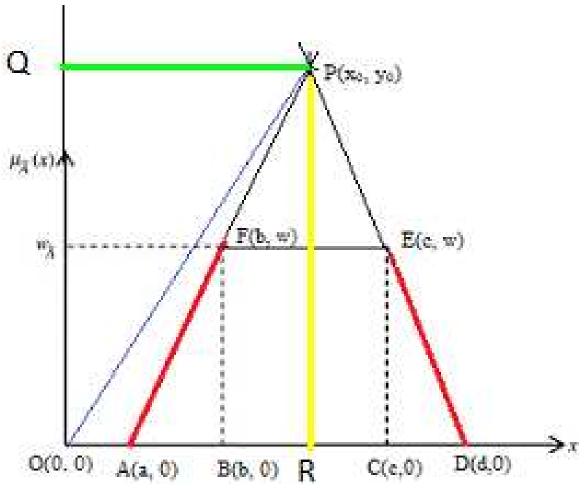

Figure 2: Area Covered by PILOT of a Generalized Trapezoidal Fuzzy Number $\tilde{A} = (a, b, c, d; w)$

Now the area of the rectangle ORPQ = OQ . OR = $x_0 . y_0$

$$= \frac{(bd - ac)}{[(b+d)-(a+c)]} \cdot \frac{w(d-a)}{[(b+d)-(a+c)]}$$

$$\Re(A) = \frac{(bd - ac)w(d-a)}{[(b+d)-(a+c)]^2} \qquad \text{...... (2)}$$

is the ranking function that area covered by PILOT $P(x_0, y_0)$.

If the ranking are equal or of the negative fuzzy number, the ranking will be decided by calculating mode, left spread, right spread and the weights of the fuzzy number in order.

Step 1. Find $\Re(A)$ and $\Re(B)$

Case (i) If $\Re(A) < \Re(B)$ then $A \prec B$;

Case (ii) If $\Re(A) > \Re(B)$ then $A \succ B$

Case (iii) If $\Re(A) = \Re(B)$, comparison is not possible, then go to step 2.

Step 2. Find Mode (A) and Mode (B)

Case (i) If Mode (A) < Mode (B), then $A \prec B$;

Case (ii) If Mode (A) > Mode (B) , then $A \succ B$;

Case (iii) If Mode (A) = Mode (B) then go to Step 3.

Step 3. Find spread (A) and spread (B)

Case (i) If Spread (A) < Spread (B), then $A \succ B$;

Case (ii) If Spread (A) > Spread (B) then $A \prec B$

Case (iii) If Spread (A) = Spread (B) then go to Step 4.

Step 4. Find Left Spread (A) and Left Spread (B)

Case (i) If left spread (A) < left spread (B) then $A \prec B$

Case (ii) If left spread (A) > left spread (B) then $A \succ B$

Case (iii) If left spread (A) = left spread (B) then go to Step 5.

Step 5. Examine w_1 and w_2

Case (i) If left w_1<w_2, then $A \prec B$; ***Case (ii)*** If w_1< w_2, then $A \succ B$

Case (iii) If $w_1 = w_2$, then $A \approx B$.

4. Method of solving fuzzy assignment problem

Suppose it is to allocate n different jobs to five different resources, there will be n assignments. Assume that each person can do one job at a time and each job can be assigned to one person only. Each machine or man can perform each job one at a time. There should be a definite procedure used to maximize the profit by minimizing the time or cost. The classical assignment problem is formulated as follows:

	Job or Work					
Worker	J_1	J_2	$\cdots$	J_j	$\cdots$	J_n
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\cdots$	$\tilde{c}_{1j}$	$\cdots$	$\tilde{c}_{1n}$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\cdots$	$\tilde{c}_{2j}$	$\cdots$	$\tilde{c}_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
i	$\tilde{c}_{i1}$	$\tilde{c}_{i2}$	$\cdots$	$\tilde{c}_{ij}$	$\cdots$	$\tilde{c}_{in}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$.	$\vdots$	$\vdots$.	$\vdots$
n	$\tilde{c}_{n1}$	$\tilde{c}_{n1}$	$\cdots$	$\tilde{c}_{nj}$	$\cdots$	$\tilde{c}_{nn}$

Minimize $Z = \sum_{i=1}^{n} \tilde{c}_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = 1; \quad i = 1,2,....,n$

$$\sum_{i=1}^{n} x_{ij} = 1; \quad j = 1,2,....,n$$

$$x_{ij} = \{0, 1\}; i = 1,2,...., n \qquad j = 1,2,...., n$$

where x_{ij} is the decision variable, ie, ith worker in jth job and c_{ij} is the cost of ith worker who are doing the jth job.

Algorithm to find the basic feasible solution and checking optimality

Step 1: Check if the number of rows is equal to number of columns (i.e,, balanced assignment problem). If not balanced add a row (or column), with zero cost, whichever is not there.

Step 2: Subtract the smallest cost element of each row from all the elements in the row of the given matrix. See that each row contains at least on zero.

Step 3: Subtract the smallest cost element of each column from all the elements in the column of the resulting cost obtained by step 2.

Step 4:(Assigning the zeros)

(i) Examine the rows successively until a row with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column of this encircled zero, as these will not be considered for any future assignment. Continue in this way until all the rows have been examined.

(ii) Examine the columns successively until a column with exactly with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it and cross any other zero in the row. Continue until all the columns have been examined.

Step 5:(Apply optimal test)

(i) If the row and each column contain exactly one encircled zero, then the current assignment is optimal.

(ii) If at least one row/column is without an assignment, then the current assignment is not optimal. Then go to Step 6.

Step 6: Cover all the zeros by drawing a minimum number of straight lines as follows:

(i) Mark (√) the rows that do not have assignment.

(ii) Mark (√) the columns (not already marked) that have zeros in marked rows.

(iii) Mark (√) the rows (not already marked) that have assignments in marked columns.

(iv) Repeat 6(i) and 6(ii) until no more marking is required.

(v) Draw lines through all unmarked rows and marked columns. If the number of these lines is equal to the order of the matrix then it is an optimum solution, otherwise not.

Step 7: Determine the smallest cost element not covered by the straight lines. Subtract this smallest cost element from all the uncovered elements and add this to all those elements which are lying in the intersection of these straight lines and do not change the remaining elements which lie on the straight lines.

Step 8: Repeat Steps (4) to (6) until an optimum assignment is attained.

5. Numerical example

We take a fuzzy assignment problem as example with generalized trapezoidal fuzzy number. First, the fuzzy numbers were converted in to the crisp value problem using the proposed ranking methods and then the assignment problem was solved.

Example 1.

	M_1	M_2	M_3	M_4
J_1	(15,35,70,80;0.1)	(60,75,90,115;0.2)	(25,35,40,75;0.4)	(15,30,45,60;0.2)
J_2	(6,21,45,62;0.2)	(15,35,60,90;0.1)	(50,60,75,95;0.3)	(11,18,38,51;0.2)
J_3	(20,30,40,50;0.2)	(10,25,45,60;0.2)	(20,30,40,50;0.5)	(2,6,12,31;0.4)
J_4	(10,40,65,85;0.2)	(20,40,60,80;0.1)	(40,60,80,100;0.2)	(25,30,40,45;0.2)

When the above fuzzy assignment problem is defuzzified using the new area ranking function in the relation (1), we get the following assignment problem:

		Machines			
		M_1	M_2	M_3	M_4
	J_1	5	7	11	6
Jobs	J_2	8	5	9	6
	J_3	4	7	10	7
	J_4	10	4	8	3

Solution: The cost matrix with allotment of jobs is $\begin{pmatrix} \cancel{0} & \cancel{0} & \cancel{0} & (0) \\ 5 & (0) & \cancel{0} & 2 \\ (0) & 1 & \cancel{0} & 3 \\ 8 & \cancel{0} & (0) & \cancel{0} \end{pmatrix}$

Since each row and column has allotment, the current assignment is optimal.

The optimum assignment schedule is given by $J_1 \to M_4$, $J_2 \to M_2$, $J_3 \to M_1$, $J_4 \to M_3$ and the optimum (minimum) assignment cost = (6 + 5 + 4 + 8) = Rs. 23/-

When the above fuzzy assignment problem is defuzzified using the revised SD of PILOT ranking method in the relation (2), we get the following assignment problem:

		Machines			
		M_1	M_2	M_3	M_4
	J_1	12.64	22.17	16.05	11.25
Jobs	J_2	11.29	6.75	29.25	10
	J_3	10.5	11.67	26.25	3.55
	J_4	16.5	7.5	21	14

The cost matrix with allotment of jobs is

$$\begin{pmatrix} \cancel{0} & 14.07 & (0) & \cancel{0} \\ (0) & \cancel{0} & 14.55 & 0.1 \\ 5.56 & 11.27 & 17.9 & (0) \\ 4.46 & (0) & 5.55 & 3.35 \end{pmatrix}$$

Since each row and column has allotment, the current assignment is optimal.

The optimum assignment schedule is given by $J_1 \rightarrow M_3$, $J_2 \rightarrow M_1$, $J_3 \rightarrow M_4$, $J_4 \rightarrow M_2$ and the optimum (minimum) assignment cost = (16.05 + 11.29 + 3.55 + 7.5) = Rs. 38.39/-

Example 2.

	1	2	3	4
A	(23,32,70,75;0.4)	(5,36,42,95;0.5)	(30,90,150,250;0.2)	(18,40,60,158;0.4)
B	(15,36,51,80;0.2)	(12,17,63,96;0.2)	(5,25,60,140;0.2)	(2,27,80,139;0.2)
C	(0,20,40,80;0.2)	(5,45,60,90;0.3)	(6,31,84,143;0.2)	(56,80,96,150;0.4)

When the above fuzzy assignment problem is defuzzified using the new area ranking method in the relation (1), we get the following assignment problem:

		Machines			
		1	2	3	4
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

Solution:

The given problem is unbalanced one. To balance this, add a dummy row with zero cost.

$$\begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The cost matrix with allotment of jobs is $\begin{pmatrix} (0) & 1 & 1 & 5 \\ \cancel{0} & (0) & \cancel{0} & 2 \\ \cancel{0} & \cancel{0} & (0) & 3 \\ 9 & 4 & \cancel{0} & (0) \end{pmatrix}$

Since each row and column has allotment, the current assignment is optimal.
The optimum assignment schedule is given by $J_1 \to M_1$, $J_2 \to M_2$, $J_3 \to M_3$, $J_4 \to M_4$ and the optimum (minimum) assignment cost = (18 + 13 + 19 + 0) = Rs. 50/-

When the above fuzzy assignment problem is defuzzified using the revised SD of PILOT ranking method in the relation (2), we get the following assignment problem:

		Machines			
		1	2	3	4
Jobs	A	26.74	12.86	7.7	14.93
	B	2.08	5.75	4.59	6.2
	C	2.67	5.46	6.2	10.61

The given problem is unbalanced one. To balance this, add a dummy row with zero cost.

$$\begin{pmatrix} 26.74 & 12.86 & 7.7 & 14.93 \\ 2.08 & 5.75 & 4.59 & 6.2 \\ 2.67 & 5.46 & 6.2 & 10.61 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The cost matrix with allotment of jobs is

$$\begin{pmatrix} 21.55 & 4.88 & (0) & 6.95 \\ (0) & 0.88 & \cancel{0} & 1.33 \\ \cancel{0} & (0) & 1.30 & 5.15 \\ 2.79 & \cancel{0} & \cancel{0} & (0) \end{pmatrix}$$

Since each row and column has allotment, the current assignment is optimal.
The optimum assignment schedule is given by $J_1 \to M_3$, $J_2 \to M_1$, $J_3 \to M_2$, $J_4 \to M_4$ and the optimum (minimum) assignment cost = (7.7 + 2.08 + 5.46 + 0) = Rs.15.24/-

6. Results and discussion

We solved two fuzzy assignment problems as examples which were defuzzified using two different proposed ranking methods before solving them. In example 1, it took an iteration ahead to optimize when using revised SD of PILOT method compared to new area method, but in example 2, it took two iterations when using both the methods. The results are given in the following Table.

Example	Assignment Cost	
	New Area Method	Revised SD of PILOT Method
1	23	38.39
2	50	15.24

7. Conclusion

We solved the fuzzy assignment problem by converting in to a crisp valued assignment problem using the new ranking methods and the results were listed. The ranking methods could be applied to some other field involving generalized trapezoidal fuzzy number. It is easy for computation.

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