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Economic Lot Size Inventory Problem–Discounted Cost Dynamic System

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Abstract. In this article we treat a simple lot-size inventory system without shortage by a finite horizon dynamic programming technique.A constant discount factor is assumed throughout the time horizon and the problem is solved to get optimal lot size.A numerical example is provided to illustrate the model proposed.

Keywords: inventory system, discounted cost, optimal lot-size, dynamic programming.

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

Stochastic inventory control system is a vastly studied model in which system performance measures are obtained in steady-state case. But steady state conditions are not satisfied by most of the problems in the real world. The essential characteristic of all economic management system is that they are continuously changing with time. However the changes occur slowly enough so that one can treat the system in steady state and model them. In other instances, however the changes occur more rapidly and cannot be accounted explicitly.In inventory systems, the process that generate demands are more important so in this article we study a realistic dynamic inventory model treating demand as a stochastic variable whose mean is time dependent.

2. Model formulation and notations

Inventory is maintained for a single item at a single location.The time period for maintaining the inventory is assumed to be finite withn periods and $\zeta_1, \zeta_2, \zeta_3, \ldots\ldots, \zeta_n$are time epochs of beginning of the periods.

The demand process is assumed to be deterministic,within a periodλ(t),t $\in$ [t_j, t_{j+1}].No back orders or lost sales are to be allowed.The procurement lead time τ_j associated with any order placed at time epoch ζ_j is constant in the period j.The lead time τ_j are described such that orders cannot cross.The decision maker decides whether or not to order additional stock at time epoch$\zeta_1, \zeta_2, \zeta_3, \ldots\ldots, \zeta_n$.but$\zeta$ denote the end of the last period where $\zeta > \zeta_n + \tau_n$.

Notations

Q_j – quantity to be ordered at the time epoch ζ_j

$T_j = t_{j+1} - t_j$ where $t_j = \zeta_j + \tau_j$

I_jC – cost of carrying one unit in inventory for period j and $\lambda(t)$ – demand rate.

3. Problem formulation

Let $t_j = \zeta_j + \tau_j$, the possible times at which orders can arrive. A_j denote the cost of placing an order at time ζ_j. Q_j ($Q_j \geq 0$) the order quantity at time ζ_j will be determined by minimizing the sum of the ordering and carrying costs over the planning horizon, which will be assumed to end at time ζ ($\zeta > \zeta_n + \tau_n$). Let y_{n+1} denote the onhand inventory at time ζ. By period j we mean the time from t_j to t_{j+1} ($t_{n+1} = \zeta$) and $T_j = t_{j+1} - t_j$.

3.1. Analysis

The objective of the problem is to find the optimal order quantity Q_j for each j = 1,2,…..,n. Let y_1 be the onhand inventory at time t_1. $\lambda(t)$ the demand rate as a function of t. Then the demand d_j in period j is $d_j = \int_{t_j}^{t_{j+1}} \lambda(t)dt$. Let I_jC be the cost of carrying one unit in inventory for period j. Then the inventory carrying charges for the period j are

$$Ꝁ_{I_j} = \frac{I_jC}{T_j}\int_{t_j}^{t_{j+1}}\left(y_j + Q_j - \int_{t_j}^{t}\lambda(u)du\right)dt$$

$$= I_jC[y_j+Q_j] - \frac{I_jC}{T_j}\int_{t_j}^{t_{j+1}}\int_{t_j}^{t}\lambda(u)\,dudt \tag{1}$$

By material balance at the beginning of period j+1 is given by

$$y_{j+1} = y_j + Q_j - d_j,\ j=1,2,….,n. \tag{2}$$

The carrying cost for period j becomes

$$Ꝁ_{I_j} = I_jCy_{j+1} + I_jC\left(d_j - \frac{1}{T_j}\int_{t_j}^{t_{j+1}}\int_{t_j}^{t}\lambda(u)\,dudt\right) \tag{3}$$

In this expression the first term I_jCy_{j+1} is the inventory carrying cost in period j for those units carried into period j+1. The next term is the cost in period j of carrying the d_j units which are demanded in period j. This latter cost is independent of Q_j and is unavoidable and hence need not be included in variable cost expression. Let i be the interest rate and $a = (1+i)^{-1}$. Then the variable cost of ordering and holding inventory which are incurred over the planning horizon are:

$$Ꝁ = \sum_{j=1}^{n} a^j\left[A_j\delta_j + I_jCy_{j+1}\right] \tag{4}$$

where $\delta_j = \begin{cases} 0 & if \quad Q_j = 0 \\ 1 & if \quad Q_j > 0 \end{cases}$ and y_i are described as in equation **2**.

3.2. Computational procedure to determine Q_j^*

Let us define the sequence of functions

$$Z_k(\xi) = \underset{Q_k}{Min}\left\{\sum_{j=1}^{k} a^j\left(A_j\delta_j + I_jCy_{j+1}\right)\right\}. \quad k = 1,2,……,n.$$

where $y_{k+1} = \xi$ and equation 2 holds for other y_i .

[$Z_k(\xi)$ can be interpreted physically as the minimum costs for periods 1 through k if the inventory position at the end of period k is ξ.]

The recurrence relations are:

$Z_k(\xi) = a^k[I_kC\xi + \underset{Q_k}{Min}\{A_k\delta_k + Z_{k-1}(\xi + d_k - Q_k)\}]$

Also $Z_1(\xi) = aI_1C\xi + \begin{cases} A_1 & if \quad Q_1 > 0 \\ 0 & if \quad Q_1 = 0 \end{cases}$

where $Q_1 = \begin{cases} \xi + d_1 - y_1 & if \quad \xi > y_1 - d_1 \\ 0 & otherwise \end{cases}$

4. Computational simplifications

When ξ=0, an optimal policy for k periods, when nothing is on hand at the end of the period k, must have the form that an order arrives at the beginning of period ω which satisfies the demands in periods ω through k, and optimal policy is followed in periods 1 through ω-1given that nothing is on hand at the end of period ω-1

Thus $Z_k(0) = \underset{\omega}{Min} Y_k(\omega)$, $\omega = 1,2,\ldots\ldots,k$ and $Z_0(0) = 0$,

where$Y_k(\omega) = a^\omega A_\omega + C \sum_{j=\omega}^{k-1}\left(a^j I_j \sum_{i=j+1}^{k} d_i\right) + Z_{\omega-1}(0)$ (5)

If it is desired to have ξ units on hand at the end, then at the last step we compute instead $Z_n(\xi) = \underset{\omega}{Min}[a^\omega A_\omega + C \sum_{j=\omega}^{n-1}\left(a^j I_j \sum_{i=j+1}^{n} d_i + \xi\right) + Z_{\omega-1}(0)]$.

4.1. Computational format for the dynamic lotsize model

	1	2	3	4	….	n-1	N
$Y_k(\omega)$	$Y_1(1)^*$	$Y_2(1)^*$	$Y_3(1)$		---		
		$Y_2(2)$	$Y_3(2)$		….		
			$Y_3(3)^*$	$Y_4(3)^*$	….		
				$Y_4(4)$	…..		
					…..	$Y_{n-1}(v)$	
					…..	…..	
					…..		
					….	$Y_{n-1}(n-1)^*$	$Y_n(n-1)^*$
					…..		$Y_n(n)$
$Z_k(0)$	$Z_1(0)= Y_1(1)$	$Z_2(0)= Y_2(1)$	$Z_3(0)= Y_3(3)$	$Z_4(0)= Y_4(3)$	…..	$Z_{n-1}(0)= Y_{n-1}(n-1)$	$Z_n(0)= Y_n(n-1)$
Q_k^*	(1)	(1,2)	(1,2)(3)	(1,2) (3,4)	….	(1,2)(3,4,.) (n-1)	(1,2)(3,4,.) (n-1,n)

In the last row of the table the optimal Q_j values are indicated by enclosing parentheses all periods whose demands are met by the order arriving at the beginning of the period whose number appear first in the parentheses.

5. Numerical example

Consider an inventory system in whicha particular item is ordered in lots.Demand varies monthly and is given in the table.The ordering cost for a consignment ofQ items is \$300.The variable purchase costs on the item amount to \$120 per unit andinventory carrying charge is \$0.20.Assume that the rate of interest for money involved in the system is i=0.10.

Here A_j=A=\$300 independently when an order is made.

$$I_jC=\frac{IC}{12}=\frac{0.20\ x\ 120}{12}=\$2.00 \text{ per unit per month.}$$

	1	2	3	4	5	6	7	8	9	10	11	12
demand	80	100	125	100	50	50	100	125	125	100	50	100
$Y_k(\omega)$	$Y_1(1)=$ 273^*	$Y_2(1)=$ 455^*	$Y_3(1)=$ 889.53									
		$Y_2(2)=$ 521.43	$Y_3(2)=$ 728.46									
			$Y_3(3)=$ 681.07^*	$Y_4(3)=$ 831.79^*	$Y_5(3)=$ 975.72							
				$Y_4(4)=$ 886.79	$Y_5(4)=$ 955.37 *	$Y_6(4)=$ 1086.35						
					$Y_5(5)=$ 1019	$Y_6(5)=$ 1081.40^*	$Y_7(5)=$ 1319.78					
						$Y_6(6)=$ 1125.73	$Y_7(6)=$ 1239.30					
							$Y_7(7)=$ 1236.43^*	$Y_8(7)=$ 1365.62^*	$Y_9(7)=$ 1619.37			
								$Y_8(8)=$ 1377.51	$Y_9(8)=$ 1495.07			
									$Y_9(9)=$ 1494^*	$Y_{10}(9)=$ 1579.58^*	$Y_{11}(9)=$ 1661.32	
										$Y_{10}(10)=$ 1610.82	$Y_{11}(10)$ $=1649.77^*$	$Y_{12}(10)=$ 1798.52
											$Y_{11}(11)$ =1685.89	$Y_{11}(12)=$ 1756.76
												$Y_{12}(12)=$ 1746.51^*
$Z_k(0)$	$Z_1(0)=$ 273	$Z_2(0)=$ 455	$Z_3(0)=$ 681.07	$Z_4(0)=$ 831.79	$Z_5(0)=$ 955.37	$Z_6(0)=$ 1081.40	$Z_7(0)=$ 1236.43	$Z_8(0)=$ 1365.62	$Z_9(0)=$ 1494	$Z_{10}(0)=$ 1579.58	$Z_{11}(0)=$ 1649.77	$Z_{12}(0)=$ 1746.51
Q^*_k	$Q^*_1=180$	$Q^*_2=0$	$Q^*_3=$ 225	$Q^*_4=0$	$Q^*_5=10$ 0	$Q^*_6=0$	$Q^*_7=225$	$Q^*_8=0$	$Q^*_9=125$	$Q^*_{10}=150$	$Q^*_{11}=0$	$Q^*_{12}=100$
Q_k^*	(1,2)		(1,2) (3,4)		(1,2) (3,4) (5,6)		(1,2) (3,4) (5,6) (7,8)		(1,2) (3,4) (5,6) (7,8)(9)	(1,2) (3,4) (5,6) (7,8) (9) (10,11)		(1,2)(3,4) (5,6)(7,8) (9)(10,11) (12)

6. Conclusion

In this article an optimal reorder quantity (Economic lot size) is got by considering a discount for the money involved in the system in the process of optimization. We obtained a down to earth model which is more realistic. We suggest a more general case in which the demand process may be stochastic with specific distribution.

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