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The Fermat S-Prime Meet Matrices and Reciprocal Fermat S-Prime Meet Matrices on Posets

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Abstract. We consider fermat S-prime meet matrices and reciprocal fermat S–prime meet matrices on posets as an abstract generalization of fermat S-prime greatest common divisor (fermat S-prime GCD) matrices. Some of the most important properties of fermat S-prime GCD matrices are presented in terms of S-prime meet matrices.

Keywords: S-prime meet, reciprocal S-prime, fermat S-prime, reciprocal fermat S-prime, fermat numbers

AMS Mathematics Subject Classification (2010): 06A06, 06A11

1. Introduction

Let $S = \{x_1, x_2, ..., x_n\}$ be a set of distinct positive integer, and let f be an arithmetical function. Then $n \times n$ matrix (S) whose i,j-entry is the greatest common divisor (x_i, x_j) of x_i and x_j is called the GCD matrix on S[3,5,8].

In 1876, H. J. S. Smith [12] showed that the determinant of the GCD matrix defined on S = {1, 2, . . .n} (Smith's determinant) is equal to φ(1)φ(2) . . .φ(n),where φ is Euler's totient function.

The set S is said to be factor-closed if it contains every divisor of any element of S, and the set S is said to be GCD-closed if it contains the greatest common divisor of any two elements of S[8].

The GCD matrix with respect to f is

$$(f(x_i, x_i)) = \begin{bmatrix} f(x_1,x_1) & f(x_1,x_2) & ,...., & f(x_1,x_n) \\ f(x_2,x_1) & f(x_2,x_2) & ,...., & f(x_2,x_n) \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ f(x_n,x_1) & f(x_n,x_2) & ,...., & f(x_n,x_n) \end{bmatrix}$$

and $\det[f(x_i, x_j)] = \prod_{k=1}^{n} (f * \mu)(x_k)$

In 1960, Carlitz [9], gave a new form of gcd-matrices and determinant value,$[f(i,j)]_n = C\ (\mathrm{diag}(g(1),\ldots,g(n)))\ C^T$ where $C = (C_{ij})_{nxn}$;

$C_{ij} = \begin{cases} 1 \text{ if } j|i \\ 0 \text{ if } j\nmid i \end{cases}$ and $D = (d_{ij})$ diagonal matrix where $d_{ij} = \begin{cases} g(i) & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

$\therefore \det [f(i,j)]_{nxn} = g(1).g(2)\ldots g(n)$

In 1992, Beslin and Ligh [4] generalized in this results on GCD matrices by showing that the determinant of the GCD Matrix on a GCD closed set $S = \{x_1, x_2, x_3,\ldots,x_n\}$ is the product $\prod_{k=1}^{n} (\alpha_k)$ Where $\alpha_k = \sum_{\substack{d/x_k \\ d \nmid x_t \\ x_t < x_k}} \Phi(d)$

2. Structure of fermat S-prime meet and reciprocal fermat S-prime meet matrices

Definition 2.1. Let $(P, \prec) = (Z^+, |)$ be a finite poset. We call P be a meet - semi lattice if for any x, y $\in$ P there exist a unique z $\in$ p. such that (i) z$\leq$ x and z $\leq$y and (ii) If w $\leq$x and w $\leq$y for some w$\in$ P .then w $\leq$z. In such a case z is called the meet of x and y is denoted by $x \wedge y$.

Definition 2.2. Let S be a subset of subset of P .we call S be a lower- closed if for every x, y$\in$ P and x$\in$ S and y$\leq$S .we have y$\in$ S.

Definition 2.3. Let S be a subset of P then S is said to be meet-closed if for every x, y$\in$ S we have $x \wedge y \in$ S.

Definition 2.4. Let x and y be two elements of the poset P and μ is the Mobius function of the poset(S,$\prec$) then

$$\mu(x,y) = \begin{cases} 0 & \text{if} \quad x \neq y \\ 1 & \text{if} \quad x = y \\ -\sum_{z:z\leq y} \mu(x,z) & \text{otherwise} \end{cases}$$

Definition 2.5. Let $S = \{x_1, x_2, x_3,\ldots,x_n\}$ and $T = \{y_1,y_2,y_3,\ldots,y_n\}$ be any two subsets of P.Define the incidence matrix whose i,j entry is 1 if $y_j \leq x_i$ and 0 otherwise, namely

$$E(S,T) = (e_{ij})_{n\times m} = \begin{cases} 1 & ; \; y_j \leq x_i \\ 0 & ; \; otherwise \end{cases}$$

Example 2.6. We consider S={ 5,9,13},T={ 9,17,21} are the S-prime number subsets.

Then the incidence matrix of (S, T) is

$$E(S,T) = (e_{ij}) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Definition 2.7. Let $S = \{x_1, x_2, x_3,\ldots,x_n\}$ be a subset of P and the $n\times n$ matrix $(S)_f = (f_{ij})$ where

$f_{ij} = 2^{2^{4(x_i \wedge x_j)+1}} + 1$,is called the Fermat S-prime Meet matrix on S.

Definition 2.8. Let S = {x_1, x_2, x_3,…,x_n } be an ordered set of distinct positive integer. The Reciprocal Fermat S-prime Meet matrix on S is defined as $(S)_{1/f} = (f_{ij})$ where

$$f_{ij} = \frac{1}{2^{2^{4(x_i \wedge x_j)+1}} + 1}$$

Definition 2.9. Let S = {x_1, x_2, x_3,…,x_n } be a subset of P, and let f be a function on P with complex values. Then the function $g_{s,f}$ on S is defined inductively by

$$g_{s,f}(x_j) = f(x_j) - \sum_{x_i \le x_j} g_{s,f}(x_i)$$

where $x_i < x_j$ means that $x_i \ne x_j$ or $f(x_j) = \sum_{x_i \le x_j} g_{s,f}(x_i)$ $(p.2,[8])$

3. Determinant and inverse of the fermat S-prime meet matrices on posets

Theorem 3.1. Let S={x_1, x_2, x_3,…,x_n } be S-prime Meet-closed . Without loss of generality we may assume that i < j whenever $x_i < x_j$, then
$g_{s,f}(x_j) = \sum_{z \le x_j} \sum_{\substack{w \le z \\ z \le x_t \\ t<j}} f(w)\mu(w,z)$ where μ is the mobius function of P.

Proof: By using the definition (2.9),

$$f(x_j) = \sum_{x_i \le x_j} g_{s,f}(x_i) = \sum_{x_i \le x_j} \sum_{\substack{z \le x_j \\ z \nleq x_t \\ t<j}} \sum_{w \le z} f(w)\mu(w,z) \qquad (1)$$

We write, $f(x) = \sum_{z \le x} g_{s,f}(z)$ or $g_{s,f}(x) = \sum_{z \le x} f(z)\mu(z,x)$ for all x∈ P

It has to be prove that $\sum_{z \le x_j} g_{s,f}(z) = \sum_{x_i \le x_j} \sum_{\substack{z \le x_i \\ z \nleq x_t \\ t<i}} g_{s,f}(z)$

Now consider the sum of R.H.S of equation (1)

Let $x_i \le x_j$ and $z \le x_i \Rightarrow z \le x_j$. Thus every z occurring on the right side of equation (1) occurs on the left side of equation (1).

Conversely, consider the sum on the left side of equation (1).

Suppose that $z \le x_j$ we have $z \le x_i$ by minimality of i, we have r = i or $x_r = x_i$, therefore $x_r \le x_j$ means $x_r \le x_j$ thus every z occurring on the side of equation (1).

This completes the proof of the theorem.

Theorem 3.2. If S is lower closed subset of P. Then

$$g_{s,f(x_j)} = \sum_{x_i \le x_j} f(x_i)\mu(x_i, x_j)$$

Proof: Already we know that the result,

$$g_{s,f(x_j)} = \sum_{\substack{z \le x_j \\ z \nleq x_t \\ t<j}} \sum_{w \le z} f(w)\mu(w,z)$$

It reduces we get the proof of theorem [11]. Then S is lower closed.

Example 3.3. Let S={x_1,x_2,……..x_n} be a chain with $x_1<x_2<\ldots\ldots<x_n$. Then $g_{s,f}(x_1)=f(x_1)$, $g_{s,f}(x_2)=f(x_2)-f(x_1)$. In general $g_{s,f}(x_j)=f(x_j)-f(x_{j-1})$ where, j=2, 3, 4,…, n.

Example 3.4. Let S={x_1 ,x_2 ,…..x_n } be an incomparable set and let S={x_0, x_1 ,x_2 ,…..x_n}. Then, $g_{s,f}(x_0)=f(x_0)$, $g_{s,f}(x_1)=f(x_1)-f(x_0)$, and $g_{s,f}(x_2)=f(x_2)-f(x_0)$. In general $g_{s,f}(x_j)=f(x_j)-f(x_0)$ for j=1, 2, 3, …,n

Theorem 3.5. Let S = {x_1, x_2, x_3,…,x_n} be a meet – closed subset of P then $\det(S)_f= g_{s,f}(x_1)g_{s,f}(x_2)\ldots g_{s,f}(x_n)$ where $g_{s,f}(x_i)$ defined by

$$g_{s,f}(x_i)=\left(2^{2^{4x_i+1}}+1\right)-\sum_{x_j\in S,\,x_j\prec x_i} g_{s,f}(x_j).[11]$$

Theorem 3.6. Let S = {x_1, x_2, x_3,…,x_n} be a lower – closed subset of P then $\det(S)_f = g_{s,f}(x_1)g_{s,f}(x_2)\ldots g_{s,f}(x_n)$ where $g_{s,f}(x_i)=\sum_{x_j\wedge x_i}\left(2^{2^{4x_i+1}}+1\right)\mu(x_j,x_i)$

Proof: The theorem is proved and verified with a suitable example.
Consider the set S = {1,2}

By using the definition (2.7), we have

$$(S)_f=\begin{pmatrix} 2^{2^{4(1\wedge 1)+1}}+1 & 2^{2^{4(1\wedge 2)+1}}+1\\ 2^{2^{4(2\wedge 1)+1}}+1 & 2^{2^{4(2\wedge 2)+1}}+1\end{pmatrix}=\begin{pmatrix} 2^{32}+1 & 2^{32}+1\\ 2^{32}+1 & 2^{512}+1\end{pmatrix}$$

since $g_{s,f}(x_i)=\sum_{x_j\wedge x_i}\left(2^{2^{4x_i+1}}+1\right)\mu(x_j,x_i)$,

$$\therefore\quad g_{s,f}(x_1)=g_{s,f}(1)=\left(2^{2^{4(1)+1}}+1\right)\mu(1,1)=\left(2^{2^5}+1\right)(1)=\left(2^{32}+1\right)$$

$$g_{s,f}(x_2)=g_{s,f}(2)=-\left(2^{512}+1\right)+\left(2^{32}+1\right)$$

$$g_{s,f}(x_1).\,g_{s,f}(x_2)=\left(2^{32}+1\right)\left[\left(2^{512}+1\right)-\left(2^{32}+1\right)\right]$$

$$\therefore\ \det(S)_f=g_{s,f}(x_1).g_{s,f}(x_2)$$

Theorem 3.7. Let S = {x_1, x_2, x_3,…,x_n} and define m×m matrix

$\Lambda=diag(g(d_1),g(d_2),\ldots,g(d_m))$ where $g(n)=\sum_{d/n}\left(2^{2^{4d+1}}+1\right)\mu\left(\frac{n}{d}\right)$ and $n\times m$ matrix $E=(e_{ij})$ by $e_{ij}=\begin{cases}1 & if\ \ d/i\\ 0 & otherwise\end{cases}$ $then\quad (S)_f=E\Lambda E^T$

Proof: The ij- entry in $E\Lambda E^T$ is

$$(E\Lambda E^T)_{ij}=\sum_{k=1}^{n} e_{ik}\Lambda_k e_{kj}=\sum_{\substack{d_k/x_i\\ d_k/x_j}} g(d_k)=\sum_{d_k/x_i\wedge x_j} g(d_k)=2^{2^{4d+1}}+1=f_{ij}$$

Theorem 3.8. Let S = {x_1, x_2, x_3,…,x_n } be a subset of P with $\overline{S}$ ={x_1,x_2,…,x_n,x_{n+1},…,x_{n+r}}. Let g be a function on $\overline{S}$ defined as in theorem(3.5).Then $(S)_f$ = $E \Lambda E^T$ and E^T is the transpose of E.

Proof: The theorem is proved and verified with a suitable example.

Consider the set S = {1,2}, $\overline{S}$ = {1,2,3}

By using the definition (2.6), we have

$$(S)_f = f\left(2^{2^{4(x_i \wedge x_j)+1}} + 1\right) = \begin{pmatrix} f\left(2^{2^5}+1\right) & f\left(2^{2^5}+1\right) \\ f\left(2^{2^5}+1\right) & f\left(2^{2^9}+1\right) \end{pmatrix} = \begin{pmatrix} f\left(2^{32}+1\right) & f\left(2^{32}+1\right) \\ f\left(2^{32}+1\right) & f\left(2^{512}+1\right) \end{pmatrix}$$

since $g_{s,f}(x_j) = f(x_j) - f(x_{j-1})$ where j = 1, 2,….,n,

$$\therefore g_{s,f}(x_1) = f(x_1) = f(1) = f(2^{2^5}+1) = f\left(2^{32}+1\right)$$

$$g_{s,f}(x_2) = f(x_2) - f(x_1) = \left(2^{512}+1\right) - f\left(2^{32}+1\right),\ g_{s,f}(x_3) = f\left(2^{8192}+1\right) - f\left(2^{512}+1\right)$$

$$\Lambda = diag\left(g_{s,f}(x_1), g_{s,f}(x_2), g_{s,f}(x_3)\right) \begin{pmatrix} f\left(2^{32}+1\right) & 0 & 0 \\ 0 & f\left(2^{512}+1\right) - f\left(2^{32}+1\right) & 0 \\ 0 & 0 & f\left(2^{8192}+1\right) - f\left(2^{512}+1\right) \end{pmatrix}$$

$$E \Lambda E^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{pmatrix} f\left(2^{32}+1\right) & 0 & 0 \\ 0 & f\left(2^{512}+1\right) - f\left(2^{32}+1\right) & 0 \\ 0 & 0 & f\left(2^{8192}+1\right) - f\left(2^{512}+1\right) \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = (S)_f$$

Theorem 3.9. Let T ={ y_1, y_2, y_3,…,y_m } be a S-prime Meet –closed subset of P containing S={ x_1, x_2, x_3,…,x_n }. Then,

$$\det (S)_f = \sum_{1 \le k_1 \le \ldots \le k_n \le m} \det \left[E(k_1, k_2, \ldots, k_n)^2\right] g_{T,f}(y)_{k_1}, , g_{T,f}(y)_{k_2}, ,\ldots g_{T,f}(y)_{k_n}$$

where, E=E(S,T) is the submatrix of E = E(S, $\overline{S}$) consisting of the k_1th ,k_2th, …,k_nth columns of E.

Proof: Since $(S)_f = E \Lambda E^T$ and also $\det(E) = \det\left(E^T\right)$, by using Cauchy Binet Formula[10] to get the proof of the theorem.

Example 3.10. Let S = {1,3}

By using the definition(2.7), we have

$$(S)_f = \begin{pmatrix} 2^{2^{4(1\wedge 1)+1}}+1 & 2^{2^{4(1\wedge 3)+1}}+1 \\ 2^{2^{4(3\wedge 1)+1}}+1 & 2^{2^{4(3\wedge 3)+1}}+1 \end{pmatrix} = \begin{pmatrix} 2^{32}+1 & 2^{32}+1 \\ 2^{32}+1 & 2^{8192}+1 \end{pmatrix}$$

$$\therefore \det (S)_f = \left(2^{32}+1\right)\left(2^{8192}+1\right) - \left(2^{32}+1\right)^2$$

since $g_{T,f}(x_i) = \sum_{x_j \wedge x_i} \left(2^{2^{4x_i+1}}+1\right)\mu\left(x_j, x_i\right)$,

$\therefore \quad g_{T,f}(x_1) = g_{T,f}(1) = (2^{2^{4(1)+1}}+1)\mu(1,1) = (2^{2^5}+1)(1) = (2^{32}+1)$

$g_{T,f}(x_2) = g_{T,f}(3) = (2^{2^5}+1)(-1) + (2^{2^{13}}+1)(1) = -(2^{32}+1) + (2^{8192}+1)$

$$\sum_{1\le k_1\le\ldots\le k_n\le m} \det[E(k_1,k_2,\ldots,k_n)^2] g_{T,f}(y)_{k1},, g_{T,f}(y)_{k2},,\ldots g_{T,f}(y)_{kn}$$

$$= \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}^2 g_{s,f}(x_1) g_{s,f}(x_2) + \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}^2 g_{s,f}(x_1) g_{s,f}(x_3) + \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}^2 g_{s,f}(x_2) g_{s,f}(x_3)$$

$$= g_{s,f}(x_1) g_{s,f}(x_2) = (2^{32}+1)\left[-(2^{32}+1)+(2^{8192}+1)\right]$$

Hence $\det (S)_f = \sum_{1\le k_1\le\ldots\le k_n\le m} \det[E(k_1,k_2,\ldots,k_n)^2] g_{T,f}(y)_{k1},, g_{T,f}(y)_{k2},,\ldots g_{T,f}(y)_{kn}$

Theorem 3.11. Let $S = \{x_1, x_2, x_3, \ldots, x_n\}$ be a lower closed subset of P and let

$$g_{s,f}(x_i) = \sum_{x_j \wedge x_i} (2^{2^{4x_i+1}}+1)\mu(x_j, x_i) \neq 0 \quad \text{for all } x_j \in S$$

Then $(S)_f$ is invertible and $(S)_f^{-1} = (c_{ij})$,

where $c_{ij} = \sum_{\substack{x_i \le x_k \\ x_j \le x_k}} \frac{1}{g_{s,f}(x_k)} \mu(x_i, x_k)\mu(x_j, x_k)$

Proof:

The $n \times n$ *matrix* $[Y] = (y_{ij})$ *defined by* $y_{ij} = \begin{cases} \mu(x_i, x_j); & x_i x_j \\ 0 & ; \textit{otherwise} \end{cases}$

Calculating the ij-entry of product EY gives,

$$(EY)_{ij} = \sum_{k=1}^{n} e_{ik} y_{kj} = \sum_{\substack{x_k / x_j \\ x_j / x_k}} \mu(x_k, x_j) = \sum_{x_k / \frac{x_i}{x_j}} \mu(x_k) = \begin{cases} 1 & \textit{if } i = j \\ 0 & \textit{otherwise} \end{cases}$$

Thus $E^{-1} = Y$.

$(S)_f = E \Lambda E^T$ and $E^{-1} = Y$ then $(S)_f^{-1} = (E \Lambda E^T)^{-1} = Y^T \Lambda^{1/2} Y = c_{ij}$

Thus, the proof is complete.

Example 3.12.

Let $S = \{1,3\}$ be a lower closed subset of P then, $(S)_f^{-1} = (c_{ij})$ where

$$c_{11} = \frac{1}{g_{s,f}(1)} \mu(1,1)\mu(1,1) = \frac{1}{(2^{32}+1)} \mu(1,1)^2 = \frac{1}{2^{32}+1}$$

$$c_{12} = \sum_{\substack{1\le x_k \\ 2\le x_k}} \frac{1}{g_{s,f}(x_k)}\mu(1,x_k)\mu(3,x_k) = \frac{\mu(1,3)\mu(3,3)}{g_{s,f}(3)} = \frac{(-1)(1)}{-(2^{32}+1)+(2^{8192}+1)}$$

$$similarly \quad ,c_{21} = \frac{(-1)(1)}{-(2^{32}+1)+(2^{8192}+1)}, c_{22} = \frac{1}{-(2^{32}+1)+(2^{8192}+1)}$$

$$\therefore \quad (S)_f^{-1} = (c_{ij}) = \begin{pmatrix} \frac{1}{2^{32}+1} & \frac{-1}{-(2^{32}+1)+(2^{8192}+1)} \\ \frac{-1}{-(2^{32}+1)+(2^{8192}+1)} & \frac{1}{-(2^{32}+1)+(2^{8192}+1)} \end{pmatrix}$$

4. Determinant and inverse of the reciprocal fermat S-prime meet matrices on posets

Theorem 4.1. Let S = $\{x_1, x_2, x_3,\ldots,x_n\}$ be a meet – closed subset of P then $\det(S)_{1/f}$= $g_{s,1/f}(x_1)g_{s,1/f}(x_2)\ldots g_{s,1/f}(x_n)$ where $g_{s,1/f}(x_i)$ defined by $g_{s,\frac{1}{f}}(x_i) = \left(\frac{1}{2^{2^{4x_i+1}}+1}\right)\mu(x_i,x_j)$

Proof: The theorem is proved and verified with a suitable example.

Consider the set S = {1,2}

By using the definition(2.8),we have

$$(S)_{1/f} = \begin{pmatrix} \frac{1}{2^{2^{4(1\wedge 1)+1}}+1} & \frac{1}{2^{2^{4(1\wedge 2)+1}}+1} \\ \frac{1}{2^{2^{4(2\wedge 1)+1}}+1} & \frac{1}{2^{2^{4(2\wedge 2)+1}}+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2^{32}+1} & \frac{1}{2^{32}+1} \\ \frac{1}{2^{32}+1} & \frac{1}{2^{512}+1} \end{pmatrix}$$

$$\therefore \quad g_{s,\frac{1}{f}}(x_1) = g_{s,\frac{1}{f}}(1) = \left(\frac{1}{2^{2^{4(1)+1}}+1}\right)\mu(1,1) = \left(\frac{1}{2^{2^5}+1}\right)(1) = \frac{1}{2^{32}+1}$$

$$g_{s,\frac{1}{f}}(x_2) = g_{s,\frac{1}{f}}(2) = \left(\frac{1}{2^{2^5}+1}\right)(-1) + \left(\frac{1}{2^{2^9}+1}\right)(1) = \left(\frac{-1}{2^{32}+1}\right) + \left(\frac{1}{2^{512}+1}\right)$$

$$Now \quad g_{s,\frac{1}{f}}(x_1)g_{s,\frac{1}{f}}(x_2) = g_{s,\frac{1}{f}}(1).\,g_{s,\frac{1}{f}}(2) = \left(\frac{1}{2^{32}+1}\right)\left[\left(\frac{-1}{2^{32}+1}\right)+\left(\frac{1}{2^{512}+1}\right)\right]$$

Hence $\det(S)_{1/f} = g_{s,1/f}(x_1)g_{s,1/f}(x_2)$.

Theorem 4.2. Let S $=\{x_1, x_2, x_3,\ldots,x_n\}$ be a subset of P with $\overline{S}$ $=\{x_1,x_2,\ldots,x_n,x_{n+1},\ldots,x_{n+r}\}$. Let g be a function on $\overline{S}$ defined as in theorem(4.1).Then $(S)_{1/f} = E \wedge E^T$ and E^T is the transpose of E.

Proof: The theorem is proved and verified with a suitable example.

Consider the set S = {1,3}, $\overline{S}$ = {1,3,5}

By using the definition(2.8), we have

$$(S)_{1/f} = f\left(\frac{1}{2^{2^{4(x_i \wedge x_j)+1}}+1}\right) = \begin{pmatrix} f\left(\frac{1}{2^{32}+1}\right) & f\left(\frac{1}{2^{32}+1}\right) \\ f\left(\frac{1}{2^{32}+1}\right) & f\left(\frac{1}{2^{8192}+1}\right) \end{pmatrix}$$

since $g_{s,1/f}(x_j) = f(x_j) - f(x_{j-1})$ where $j = 1,2,\ldots,n$

$$\therefore g_{s,1/f}(x_1) = f(x_1) = f(1) = f\left(\frac{1}{2^{2^5}+1}\right) = f\left(\frac{1}{2^{32}+1}\right)$$

$$g_{s,1/f}(x_2) = f\left(\frac{1}{2^{8192}+1}\right) - f\left(\frac{1}{2^{32}+1}\right), \; g_{s,1/f}(x_3) = f\left(\frac{1}{2^{2^{21}}+1}\right) - f\left(\frac{1}{2^{2^{13}}+1}\right)$$

$$\therefore \quad \Lambda = \begin{pmatrix} f\left(\frac{1}{2^{32}+1}\right) & 0 & 0 \\ 0 & f\left(\frac{1}{2^{8192}+1}\right) - f\left(\frac{1}{2^{32}+1}\right) & 0 \\ 0 & 0 & f\left(\frac{1}{2^{2^{21}}+1}\right) - f\left(\frac{1}{2^{2^{13}}+1}\right) \end{pmatrix}$$

$$E\Lambda E^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{pmatrix} f\left(\frac{1}{2^{32}+1}\right) & 0 & 0 \\ 0 & f\left(\frac{1}{2^{8192}+1}\right) - f\left(\frac{1}{2^{32}+1}\right) & 0 \\ 0 & 0 & f\left(\frac{1}{2^{2^{21}}+1}\right) - f\left(\frac{1}{2^{2^{13}}+1}\right) \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = (S)_f$$

Theorem 4.3.
Let T ={ $y_1, y_2, y_3, \ldots, y_m$ } be a S-prime Meet –closed subset of P containing S={ $x_1, x_2, x_3, \ldots, x_n$ }. Then,

$$\det (S)_{1/f} = \sum_{1 \le k_1 \le \ldots \le k_n \le m} \det\left[E(k_1, k_2, \ldots, k_n)^2\right] g_{T,1/f}(y)_{k_1}, , g_{T,1/f}(y)_{k_2}, , \ldots g_{T,1/f}(y)_{k_n} \text{ where,}$$

E=E(S,T) is the submatrix of E = E(S, $\overline{S}$) consisting of the k_1th ,k_2th, …,k_nth columns of E.

Proof: The theorem is proved and verified with a suitable example. Let S = {1,5}
By using the definition(2.8), we have

$$(S)_{1/f} = \begin{pmatrix} \frac{1}{2^{2^{4(1\wedge 1)+1}}+1} & \frac{1}{2^{2^{4(1\wedge 5)+1}}+1} \\ \frac{1}{2^{2^{4(5\wedge 1)+1}}+1} & \frac{1}{2^{2^{4(5\wedge 5)+1}}+1} \end{pmatrix} = \begin{pmatrix} \frac{1}{2^{32}+1} & \frac{1}{2^{32}+1} \\ \frac{1}{2^{32}+1} & \frac{1}{2^{2^{21}}+1} \end{pmatrix}$$

$$\det (S)_{1/f} = \left(\frac{1}{2^{32}+1}\right)\left(\frac{1`}{2^{2^{21}}+1}\right) - \left(\frac{1}{2^{32}+1}\right)^2$$

since

$$g_{s,1/f}(x_i) = \sum_{x_j \wedge x_i} \left(\frac{1}{2^{2^{4x_i+1}}+1}\right) \mu(x_j, x_i)$$

$$\therefore \quad g_{s,1/f}(x_1) = g_{s,1/f}(1) = \left(\frac{1}{2^{2^{4(1)+1}}+1}\right)\mu(1,1) = \left(\frac{1}{2^{2^5}+1}\right)(1) = \left(\frac{1}{2^{32}+1}\right)$$

$$g_{s,1/f}(x_2) = g_{s,1/f}(5) = \left(\frac{1}{2^{2^5}+1}\right)(-1) + \left(\frac{1}{2^{2^{21}}+1}\right)(1) = \left(\frac{-1}{2^{32}+1}\right) + \left(\frac{1}{2^{2^{21}}+1}\right)$$

$$\sum_{1\le k_1 \le \ldots \le k_n \le m} \det\left[E(k_1, k_2, \ldots, k_n)^2\right] g_{T,1/f}(y)_{k_1}, , g_{T,1/f}(y)_{k_2}, , \ldots g_{T,1/f}(y)_{k_n} = \left(\frac{1}{2^{32}+1}\right)\left[\left(\frac{-1}{2^{32}+1}\right) + \left(\frac{1}{2^{2^{21}}+1}\right)\right]$$

Hence $\det (S)_{1/f} = \sum_{1\le k_1 \le \ldots \le k_n \le m} \det\left[E(k_1, k_2, \ldots, k_n)^2\right] g_{T,1/f}(y)_{k_1}, , g_{T,1/f}(y)_{k_2}, , \ldots g_{T,1/f}(y)_{k_n}$

Theorem 4.4. Let $S = \{x_1, x_2, x_3, \ldots, x_n\}$ be a lower closed subset of P and let

$$g_{s,1/f}(x_i) = \sum_{x_j \wedge x_i} \left(\frac{1}{2^{2^{4x_i+1}}+1}\right) \mu(x_j, x_i) \neq 0 \quad \text{for all} \quad x_j \in S.$$

Then $(S)_{1/f}$ is invertible and $(S)_{1/f}^{-1} = (d_{ij})$, where

$$d_{ij} = \sum_{\substack{x_i \le x_k \\ x_j \le x_k}} \frac{1}{g_{s,1/f}(x_k)} \mu(x_i, x_k)\mu(x_j, x_k).[6]$$

Example 4.5. Let S = {1,5} be a lower closed subset of P then $(S)_{1/f}^{-1} = (d_{ij})$ where

$$d_{11} = 2^{32} + 1, \; d_{12} = \frac{-1}{\left(\frac{-1}{2^{32}+1}\right) + \left(\frac{1}{2^{2^{21}}+1}\right)}$$

$$d_{21} = \frac{-1}{\left(\frac{-1}{2^{32}+1}\right)+\left(\frac{1}{2^{2^{21}}+1}\right)}, \quad d_{22} = \frac{1}{\left(\frac{-1}{2^{32}+1}\right)+\left(\frac{1}{2^{2^{21}}+1}\right)}$$

$$\therefore \quad (S)_{1/f}^{-1} = (d_{ij}) = \begin{pmatrix} 2^{32}+1 & \frac{-1}{\left(\frac{-1}{2^{32}+1}\right)+\left(\frac{1}{2^{2^{21}}+1}\right)} \\ \frac{-1}{\left(\frac{-1}{2^{32}+1}\right)+\left(\frac{1}{2^{2^{21}}+1}\right)} & \frac{1}{\left(\frac{-1}{2^{32}+1}\right)+\left(\frac{1}{2^{2^{21}}+1}\right)} \end{pmatrix}$$

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