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(δ_i,δ_j) F-γ-Semiopen and (δ_i,δ_j) F-γ-Semiclosed Sets in Fuzzy Bitopological Spaces

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Abstract. The aim of this paper is to introduce the concepts of (δ_i,δ_j) F-γ-semiopen (respectively (δ_i, δ_j) F -γ-semiclosed) sets in fuzzy bitopological spaces, which is weaker than the concept of (δ_i, δ_j) F-strongly semiopen (respectively (δ_i, δ_j) F-strongly semiclosed) sets and stronger than the concept of (δ_i, δ_j) F-γ open (respectively (δ_i, δ_j) F-γ closed) sets and (δ_i, δ_j) F-semi-pre-open (respectively (δ_i, δ_j) F-semi-pre-closed)sets. Their properties and relationship between other sets and relevant concepts are studied in fuzzy bitopological spaces. Also, the notion of (δ_i, δ_j) F-γ-semi interior and (δ_i, δ_j) F-γ-semi closure are introduced and their properties are discussed.

Keywords: Fuzzy bitopological spaces, (δ_i, δ_j)-F-γ-open, (δ_i, δ_j)-F-γ-closed, (δ_i, δ_j)-F-γ-semiopen, (δ_i, δ_j)-F-γ-semiclosed, (δ_i, δ_j)-F-γ-semi neighbourhood, (δ_i, δ_j)-F-γ-semi q neighbourhood, (δ_i,δ_j)-F-γ-semi interior, (δ_i,δ_j)-F-γ-semi closure.

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1. Introduction

Fuzzy topology, as an important research field in fuzzy set theory, has been established by Chang [3] in 1968, who introduced the concept of fuzzy topological space which is a natural generalization of topological spaces based on Zadeh's [15] concept of fuzzy sets. Let X be a non-empty set and I be the unit interval [0, 1]. A fuzzy set in X is a mapping from X into I. Since then much attention [1,6,2,8] has been paid to generalize the basic concepts of general topology in fuzzy settings.

Azad [1] introduced the notions of fuzzy semi open and fuzzy semi closed sets with specific attention to weaker forms of fuzzy continuity in fuzzy topological spaces. Hanafy [4] introduced the notion of Fuzzy γ-open sets and Fuzzy γ-continuity in fuzzy topological spaces and discussed the fundamental properties of these sets. In 1989, Kandil and El-Shafee [5] introduced the concept of fuzzy bitopological spaces (Fbts, in short) as an extension of fuzzy topological spaces and as a generalization of bitopological spaces which was introduced by Kelly.

Throughout this paper (X, δ_i, δ_j) (or simply X), denote fuzzy bitopological spaces (Fbts). For a fuzzy set A in a fuzzy bitopological space X, δ_i-cl(A), δ_i- int(A) denote the closure, interior with respect to the topology δ_i respectively. Using the union of the concepts of (δ_i, δ_j) F semiopen[12], (δ_i, δ_j) F preopen [11], F.S. Mahmoud, M.A. Fath Alla and M.M. Khalaf [7] introduced and studied Fuzzy-γ-open sets and fuzzy-γ-continuity in fuzzy bitopological spaces which is weaker than each of them. The objective of this paper is to introduce and study the notion of (δ_i, δ_j) F-γ-semiopen sets in fuzzy bitopological spaces which is weaker than the concept of (δ_i, δ_j) F-strongly-semi open and stronger than the concept of (δ_i, δ_j) F-γ-open and (δ_i, δ_j) F-semi-pre-open. We discuss the concepts and properties that are needed in this paper in the third section. We introduce and study the concepts of (δ_i,δ_j)F-γ-semiopen (respectively (δ_i, δ_j)F-γ-semiclosed) sets in fuzzy bitopological spaces along with their properties in the fourth section. Using this concept, in section 4.1 and 4.2 we define and deal with the concepts of (δ_i, δ_j)-F-γ-semi interior and (δ_i, δ_j)-F-γ-semi closure and investigate some of the fundamental properties of these concepts.

2. Preliminaries

In this section, we give some elementary concepts and results which will be used in the sequel. Let X be a nonempty set and I= [0, 1]. A fuzzy set (briefly F-set) A in X is a mapping from X to I. A fuzzy set A of X is contained in a fuzzy set B of X denoted by A $\leq$ B if and only if A(x) $\leq$ B(x) for each x$\in$ X. A fuzzy point [14] with singleton support x$\in$ X and the value $\alpha\in[0,1]$ is denoted by x_α. The complement A' of a fuzzy set X is 1−A defined by (1−A) (x)= 1−A(x) for each x $\in$ X. A fuzzy point $x_\beta\in$ A if and only if $\beta\leq$A(x).A fuzzy set A is the union of all fuzzy points which belong to A. A fuzzy point x_β is said to be quasicoincident with the fuzzy set A denoted by x_βqA if and only if β + A(x) > 1 [10]. A fuzzy set A is said to be quasicoincident with B denoted by AqB if and only if there exists x $\in$ X such that A(x) + B(x) > 1. A $\leq$ B if and only if ˥(AqB′) [10].

Definition 2.1. Let λ be a fuzzy set of a fuzzy topological space (X, δ). Then A is called
(a) a F semiopen (briefly FSO) set of X if $\lambda \leq$ cl (int (λ)) [1] ;
(b) a F semiclosed (briefly FSC) set of X if $\lambda \geq$ int (cl (λ)) [1];
(c) a F preopen (briefly FPO) set of X if $\lambda \leq$ int (cl (λ)) [6];
(d) a F preclosed (briefly FPC) set of X if $\lambda \geq$ cl (int (λ)) [6];
(e) a F strongly semiopen (briefly FSSO) set of X if $\lambda \leq$ int (cl (int (λ))) [2];
(f) a F strongly semiclosed (briefly FSSC) set of X if $\lambda \geq$ cl (int (cl (λ))) [2];
(g) a F semi-preopen (briefly FSPO) set of X if $\lambda \leq$ cl (int (cl (λ))) [8];
(h) a F semi-preclosed (briefly FSPC) set of X if $\lambda \geq$ int (cl (int (λ))) [8];
The set of all F-so(resp. F-sc), F-po(resp. F-pc), F-sso(resp. F-ssc), F-spo(resp. F-spc) of a fuzzy topological space will be denoted by FSO(X) (resp. FSC(X)), FPO(X) (resp. FPC(X)), F-SSO(X) (resp. F-SSC(X)), F-SPO(X) (resp. F-SPC(X)).

Definition 2.2. [4] Let (X, δ) be a fuzzy topological space.Then v is called a F-γ open(F-γ closed) set of X if $v \leq$ int(cl(v)) $\vee$ cl(int(v)) ($v \geq$ cl(int(v)) $\wedge$ int(cl(v))).
The family of all F-γ open (respectively F-γ closed) sets of X is denoted by F-γO(X) (respectively F-γC(X)).

Lemma 2.3. [1] For a family $\{\lambda_\alpha\}$of fuzzy sets of a Fts X,$\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$ and $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$.

Lemma 2.4. [1] For a fuzzy set λ of a F-ts X, (i)$(int(\lambda))' = cl(\lambda')$ and (ii) $(cl(\lambda))' = int(\lambda')$

Lemma 2.5. [1] For a fuzzy set λ of a F-ts X, (a)$1-int\ \lambda=cl(1-\lambda)$ and (b)$1-cl\ \lambda=int(1-\lambda)$.

Definition 2.6. [5] A set X on which are defined two (arbitrary) F-topologies δ_1 and δ_2 is called F- bitopological space (briefly F-bts) and denoted by (X, δ_1, δ_2).As to the notions, we shall write δ_i-int(λ) and δ_i-cl(λ) to mean respectively the interior and closure of a F-set λ with respect to the F-topology δ_i in F-bts (X, δ_i, δ_j), with δ_i-F-o set and δ_i-F-c set, we mean respectively δ_i-F-open and δ_i-F-closed set. The indices i and j take values {1, 2} throughout this paper and $i \neq j$, $i = j$ gives the known results in F-ts.

Definition 2.7. [12] Let λ be a fuzzy set of a F-bts (X, δ_i,δ_j). Then λ is called
(a) a (δ_i, δ_j) F semiopen (briefly (δ_i, δ_j) F-so) set of X if $\lambda \leq \delta_j\text{-cl}(\delta_i\text{-int}(\lambda))$;
(b) a (δ_i, δ_j) F semiclosed (briefly (δ_i, δ_j) F-sc) set of X if $\lambda \geq \delta_j\text{-int}(\delta_i\text{-cl}(\lambda))$;
The set of all (δ_i, δ_j) F-so, (resp. (δ_i, δ_j)F-sc) sets of a F-bts X will be denoted by (δ_i, δ_j) FSO(X), (resp. (δ_i, δ_j) FSC(X)).

Definition 2.8. [11] Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called
(a) a (δ_i, δ_j) F strongly semiopen (briefly (δ_i, δ_j) F-sso) set of X if $\lambda \leq (\delta_i\text{-int}(\lambda)))$;
(b) a (δ_i,δ_j)F strongly semiclosed (briefly (δ_i, δ_j)F-ssc) set of X if$\lambda \geq \delta_i\text{-cl}(\delta_j\text{-int}(\delta_i\text{-cl}(\lambda)))$;
(c) a (δ_i, δ_j) F preopen (briefly (δ_i, δ_j) F-po) set of X if $\lambda \leq \delta_i\text{-int}(\delta_j\text{-cl}(\lambda))$;
(d) a (δ_i, δ_j) F preclosed (briefly (δ_i, δ_j) F-pc) set of X if $\lambda \geq \delta_i\text{-cl}(\delta_j\text{-int}(\lambda))$;
The set of all (δ_i, δ_j) F-sso, (δ_i, δ_j) F-ssc, (δ_i, δ_j) F-po, (δ_i, δ_j) F-pc sets of a F-bts X will be denoted by (δ_i, δ_j) FSSO(X), (δ_i,δ_j) FSSC(X), (δ_i,δ_j) FPO(X) and (δ_i,δ_j) FPC(X) respectively.

Definition 2.9. [9] Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j). Then λ is called
(a) a (δ_i, δ_j) F semi-preopen (briefly (δ_i, δ_j) F-spo) set of X if $\lambda \leq \delta_j\text{-cl}(\delta_i\text{-int}(\delta_j\text{-cl}(\lambda)))$;
(b) a (δ_i, δ_j) F semi-preclosed (briefly (δ_i, δ_j) F-spc) set of X if $\lambda \geq \delta_j\text{-int}(\delta_i\text{-cl}(\delta_j\text{-int}(\lambda)))$;
The set of all (δ_i, δ_j) F-spo, (resp. (δ_i, δ_j) F-spc) sets of a F-bts X will be denoted by (δ_i, δ_j) FSPO(X), (resp. (δ_i, δ_j) FSPC(X)).

Definition 2.10. [7] Let λ be a fuzzy set of a F-bts (X, δ_i, δ_j).Then λ is called a (δ_i, δ_j) Fγ open (resp. (δ_i, δ_j) Fγ closed), briefly (δ_i, δ_j) F-γo (resp. (δ_i, δ_j) F-γc) if
$\lambda \leq \delta_i\text{-int}(\delta_j\text{-cl}(\lambda)) \vee \delta_j\text{-cl}(\delta_i\text{-int}(\lambda))$, respectively $\lambda \geq \delta_i\text{-cl}(\delta_j\text{-int}(\lambda)) \wedge \delta_j\text{-int}(\delta_i\text{-cl}(\lambda))$.
The family of all (δ_i, δ_j) F-γo (resp. (δ_i, δ_j) F-γc) sets of X is denoted by (δ_i, δ_j) F-γO(X) and (resp. (δ_i, δ_j) F-γC(X)).

Remark 2.11. [7] (i) The union of (δ_i, δ_j) F-γo sets is a (δ_i, δ_j) F-γo set.
(ii)The intersection of (δ_i, δ_j) F-γc sets is a (δ_i, δ_j) F-γc set.

Definition 2.12. [7] Let λ be a fuzzy set of a F-bts (X,δ_i, δ_j). Then the (δ_i, δ_j) γ-closure ((δ_i, δ_j) γ-cl for short) and (δ_i, δ_j) γ-interior ((δ_i, δ_j) γ-int for short) of λ are defined as
(δ_i, δ_j) γ-cl$(\lambda) = \wedge\{v : v$ is (δ_i, δ_j) F-γ closed and $\lambda \leq v\}$
(δ_i, δ_j) γ-int$(\lambda) = \vee\{v : v$ is (δ_i, δ_j) F-γ open and $v \leq \lambda\}$

Remark 2.13. (i) (δ_i,δ_j) γ-cl(λ) is the intersection of all (δ_i,δ_j)F-γ c sets of X containing λ.
(ii) (δ_i, δ_j) γ-int(λ) is the union of all (δ_i, δ_j) F-γo sets of X contained in λ.
(iii) δ_i-γ cl(λ) is the intersection of all (δ_i,δ_j)F-γc sets of X containing λ with respect to δ_i.
(iv) δ_i-γ int(λ) is the union of all (δ_i, δ_j)F-γo sets of X contained in $\lambda$with respect to the δ_i.

3. Properties of (δ_i, δ_j) F-γ closure and (δ_i, δ_j) F-γ interior

Proposition 3.1. Let (X, δ_i, δ_j) be a F-bts. Then every δ_iF o set is (δ_i, δ_j) F-γo.
Proof: Let A be δ_i-F o in X, then δ_i-int(δ_j-cl(A))$\vee\delta_j$-cl(δ_i-int(A)) = δ_i-int(δ_j-cl(A)) $\vee$ δ_j-cl(A) $\geq$ δ_i-int(δ_j-cl(A))$\vee$A= A. Then A$\leq$ δ_i-int(δ_j-cl(A)) $\vee$ δ_j-cl(δ_i-int(A)). Thus,A is (δ_i, δ_j) F-γo.

Remark 3.2. The converse of the above is not true.Every (δ_i,δ_j)F-γo(c) need not be δ_i-Fo(c).
Let (X,δ_1,δ_2) be a F-bts with X={a,b,c}, $\delta_1=\{\tilde{0},\tilde{1},A\}$,$\delta_2=\{\tilde{0},\tilde{1},B\}$ and fuzzy sets A=$\{a_{0.5},b_{0.2}, c_{0.4}\}$, B =$\{a_{0.5},b_{0.6},c_{0.3}\}$.Here B (resp. B') is (δ_1,δ_2)F-γo (resp. (δ_1,δ_2)F-γc) but not δ_1-Fo (resp. δ_1-F c).

Proposition 3.3. Let A be a F-set of a Fbts(X,δ_i,δ_j).Then A is δ_iFo if and only if A is δ_iF γo.

Corollary 3.4. (a) δ_i-int(A) = δ_i-γint(A) (b) δ_i-cl(A) = δ_i-γcl(A).
Proof: Follows from Definition 2.12

Theorem 3.5. Let (X, δ_i, δ_j) be a F-bts. Then for fuzzy sets A and B of X,
(i) (δ_i, δ_j) γ-int$(\tilde{0}) = \tilde{0}$, (δ_i, δ_j) γ-int$(\tilde{1}) = \tilde{1}$ and (δ_i, δ_j) γ-int(A) $\leq$ A
(ii) A is (δ_i, δ_j) F-γ o if and only if A = (δ_i, δ_j) γ-int(A)
(iii) (δ_i, δ_j) γ-int(A) is (δ_i, δ_j) F-γ-o set and $(\delta_i, \delta_j)\gamma$-int($(\delta_i, \delta_j)\gamma$-int(A))=$(\delta_i, \delta_j)\gamma$-int(A)
(iv) If A$\leq$B, then (δ_i, δ_j) γ-int(A) $\leq$ (δ_i, δ_j) γ-int(B)
(v) (δ_i, δ_j) γ-cl$(\tilde{0}) = \tilde{0}$,(δ_i, δ_j) γ-cl$(\tilde{1}) = \tilde{1}$ and A$\leq(\delta_i, \delta_j)$ γ-cl(A)(vi) A is (δ_i, δ_j) F-γ closed if and only if A = (δ_i, δ_j) γ-cl(A).
(vii) (δ_i, δ_j) γ-cl(A) is (δ_i, δ_j) F-γ-c set and $(\delta_i, \delta_j)\gamma$-cl($(\delta_i, \delta_j)\gamma$-cl(A)) = $(\delta_i,\delta_j)\gamma$-cl(A).
(viii) If A $\leq$ B, then (δ_i, δ_j) γ-cl(A) $\leq$ (δ_i, δ_j) γ-cl(B).
Proof: Follows from Remark 2.11, Remark 2.13 and Definition 2.12.

Remark 3.6. Theorem 3.5 also holds when single topology δ_i is considered.

Proposition 3.7. Let A and B be any two fuzzy sets of a F-bts (X,δ_i,δ_j).Then
(i) (δ_i, δ_j) γ-int(A $\wedge$ B) = (δ_i, δ_j) γ-int(A) $\wedge$ (δ_i, δ_j) γ-int(B)
(ii) (δ_i, δ_j) γ-int(A$\vee$ B) $\geq$ (δ_i, δ_j) γ-int(A) $\vee$ (δ_i, δ_j) γ-int(B)
Proof: Follows from Definition 2.12

Remark 3.8. Equality need not hold in Proposition 3.9 (ii). Let (X, δ_1, δ_2) be a F-bts with X={a,b,c}, δ_1={$\tilde{0}$,$\tilde{1}$,Y}, δ_2 ={$\tilde{0}$,$\tilde{1}$,Z} and fuzzy sets Y={$a_{0.5}$,$b_{0.6}$,$c_{0.7}$}, Z={$a_{0.5}$,$b_{0.3}$,$c_{0.2}$}, A={$a_{0.5}$,$b_{0.4}$,$c_{0.8}$}, B={$a_{0.5}$,$b_{0.7}$,$c_{0.7}$} and A∨B={$a_{0.5}$,$b_{0.7}$,$c_{0.8}$}= Z'. Then (δ_1, δ_2)γ-int(A∨B) =A∨B, (δ_1,δ_2)γ-int(A)=Z, (δ_1,δ_2)γint(B)=B. So, (δ_1,δ_2)γ-int(A) ∨(δ_1,δ_2)γ-int(B)=B and A∨B ≥ B.

Proposition 3.9. Let A and B be any two fuzzy sets of a F-bts (X, δ_i, δ_j). Then
(i) (δ_i, δ_j) γ-cl(A ∨ B) = (δ_i, δ_j) γ-cl(A) ∨ (δ_i, δ_j) γ-cl(B)
(ii) (δ_i, δ_j) γ-cl(A ∧ B) ≤ (δ_i, δ_j) γ-cl(A) ∧ (δ_i, δ_j) γ-cl(B)
Proof: Follows from Definition 2.12

Remark 3.10. Equality need not hold in Proposition 3.13 (ii). Let (Z, δ_1, δ_2) be a F-bts with Z={a,b,c},δ_1={$\tilde{0}$,$\tilde{1}$,X} and δ_2={$\tilde{0}$,$\tilde{1}$,Y} where X={$a_{0.5}$,$b_{0.6}$,$c_{0.7}$}, Y={$a_{0.5}$,$b_{0.3}$,$c_{0.2}$}, A={$a_{0.5}$,$b_{0.6}$, $c_{0.8}$}, B ={$a_{0.5}$,$b_{0.7}$,$c_{0.7}$}, A∧B ={$a_{0.5}$,$b_{0.6}$,$c_{0.7}$}=X. Then (δ_1,δ_2)γ-cl(A∧B)=A∧B, (δ_1, δ_2)γ-cl(A)=Y', (δ_1, δ_2)γ-cl(B)=Y' and so (δ_1,δ_2)γ-cl(A)∧(δ_1,δ_2)γ-cl(B)=Y' ≥(δ_1, δ_2) γ-cl(A∧B).

4. (δ_i, δ_j) fuzzy-γ-semiopen and (δ_i, δ_j) fuzzy-γ-semiclosed sets

Definition 4.1. Let A be a fuzzy set of a F-bts (X, δ_i, δ_j). Then A is called a
(i) (δ_i, δ_j) Fuzzy-γ-semiopen (briefly (δ_i, δ_j) F-γ-so) set if A ≤ δ_j-cl (δ_i-γ int(A))
(ii) (δ_i, δ_j) Fuzzy-γ-semiclosed (briefly (δ_i, δ_j) F-γ-sc) set if A ≥ δ_j- int (δ_i-γ cl(A))
The family of all (δ_i, δ_j) F-γ so (respectively (δ_i, δ_j) F-γ sc)sets of X is denoted by (δ_i, δ_j)F-γ SO(X) and (respectively (δ_i, δ_j) F-γ SC(X)).

Example 4.2. Let (X,δ_1,δ_2) be a F-bts where X={a, b}, δ_1= {$\tilde{0}$,$\tilde{1}$,A} and δ_2={$\tilde{0}$,$\tilde{1}$,B}. A and B are fuzzy sets defined in X as, A={$a_{0.2}$,$b_{0.5}$)}and B={$a_{0.4}$,$b_{0.5}$}. Here (δ_1, δ_2) F-γ-o sets ={$\tilde{0}$, $\tilde{1}$,A,B} The sets A and B (resp. A' and B') are (δ_1, δ_2)F-γ-so (resp. (δ_1, δ_2) F-γ-sc).

Theorem 4.3. A fuzzy set A of a F-bts (X, δ_i, δ_j) is (δ_i,δ_j)F-γ-sc if and only if A' is (δ_i,δ_j)F-γ-so.
Proof: Follows from Definition 4.1.

Remark 4.4. The concepts of (δ_i, δ_j) F-γ-so (resp. (δ_i, δ_j)F-γ-sc) and (δ_j,δ_i) F-γ-so (resp. (δ_j,δ_i) F-γ-sc) sets are independent. The following example illustrates this.

Example 4.5. Let (X, δ_1, δ_2) be a F-bts with X={a, b}, δ_1 ={$\tilde{0}$, $\tilde{1}$, A} and δ_2 ={$\tilde{0}$, $\tilde{1}$, B}and A, B are fuzzy sets defined in X as, A={$a_{0.5}$, $b_{0.5}$)}, B ={$a_{0.6}$, $b_{0.3}$)}.Here A (resp.A') is (δ_1, δ_2)F-γ-so (resp. (δ_1,δ_2)F-γ-sc). But A (resp. A') is not (δ_2, δ_1)F-γ-so ((δ_2, δ_1)F-γ-sc). Also, B (resp. B') is (δ_2, δ_1) F-γ-so(resp. (δ_2, δ_1)F-γ-sc) and not (δ_1, δ_2)F-γ-so ((resp. (δ_1, δ_2) F-γ-sc))

Theorem 4.6. Let (X, δ_i, δ_j) be a F-bts. Then a fuzzy subset A of X is (δ_i, δ_j) F-γ-so if and only if there exists a δ_i F-γ o set U, such that U ≤ A ≤ δ_j-γ cl(U).
Proof: Let A be (δ_i, δ_j)F-γ-so in X. Then δ_i- γ int(A) ≤ A ≤ δ_j-cl(δ_i- γ int(A)). Let δ_i γ int(A) = U and U is δ_i F-γ o. Then U ≤ A ≤ δ_j-cl(U), that is U ≤ A ≤ δ_j- γ cl(U).

Conversely, suppose there exists a δ_i F-γo set U such that U $\leq$ A $\leq$ δ_j-γ cl(U).Then U $\leq$ δ_i-γint(A). Then by Theorem 3.5, δ_j-γ cl(U) $\leq$ δ_j-γ cl(δ_i-γ int(A))= δ_j-cl(δ_i-γ int(A)). Thus, A $\leq$ δ_j-γ cl(U) $\leq$ δ_j-cl(δ_i-γ int(A)). Hence A is (δ_i, δ_j)F-γ-so.

Theorem 4.7. Let (X, δ_i, δ_j) be a F-bts. Then a fuzzy subset B of X is (δ_i, δ_j) F-γ-sc if and only if there exists a δ_i F-γ c set F, such that δ_j- γ int(F) $\leq$ B $\leq$ F.

Proof: Let B be (δ_i, δ_j)F-γ-sc in X. Then δ_i-γ cl(B) $\geq$ B $\geq$$\delta_j$- int($\delta_i$- γ cl(B)). Let F = δ_i-γ cl(B), then F is δ_i F-γ c and F $\geq$ B $\geq$$\delta_j$- int(F)) which implies δ_j- γ int(F) $\leq$ B $\leq$ F. Conversely, suppose there exists a δ_iF-γc set F such that δ_j-γ int(F) $\leq$ B $\leq$ F. Then δ_i-γcl(B)$\leq$ F. That isδ_j-int(δ_i-γ cl(B))$\leq$$\delta_j$ int(F))=δ_j-γ int(F)$\leq$B.Then B$\geq$$\delta_j$-int($\delta_i$-$\gamma$ cl(B)). Thus, B is (δ_i,δ_j)F-γ-sc.

Proposition 4.8. The union of two (δ_i,δ_j)F-γ-so sets is a (δ_i,δ_j)F-γ-so set in a F-bts (X,δ_i,δ_j).

Remark 4.9. The intersection of two (δ_i,δ_j)F-γ-so sets need not be (δ_i,δ_j)F-γ-so in a F-bts (X,δ_i,δ_j)as given below.

Example 4.10. Let (X, δ_1,δ_2) be a F-bts with X={a, b, c},δ_1={$\tilde{0}$, $\tilde{1}$, A, B, A$\vee$B} and δ_2 ={$\tilde{0}$,$\tilde{1}$,C}. A,B,C,E and F are fuzzy sets defined in X as, A={a_0,$b_{0.2}$,$c_{0.1}$}, B={$a_{0.3}$,b_0,c_0)}, C={$a_{0.1}$,$b_{0.2}$,$c_{0.1}$}, E={a_0,$b_{0.3}$,$c_{0.1}$}, F={$a_{0.3}$,$b_{0.3}$,c_0}, E$\wedge$F={a_0, $b_{0.3}$, c_0)}. Here E and F are (δ_1, δ_2) F-γ-so but E$\wedge$F is not (δ_1,δ_2) F-γ-so.

Theorem 4.11. Arbitrary union of (δ_i,δ_j)F-γ-so sets is a (δ_i,δ_j)F-γso in a F-bts (X,δ_i,δ_j).

Proof: Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of (δ_i, δ_j)F-γ-so sets in (X, δ_i, δ_j). For each $\alpha\in\Delta$, A_α is (δ_i, δ_j) F-γ-so. Then for each $\alpha\in\Delta$, $A_\alpha \leq \delta_j$- cl (δ_i- γ int(A_α)). That is $\vee_{\alpha\in\Delta}A_\alpha\leq\vee_{\alpha\in\Delta}\delta_j$-cl($\delta_i$-$\gamma$ int(A_α)). Then $\vee_{\alpha\in\Delta}A_\alpha\leq\delta_j$-cl($\vee_{\alpha\in\Delta}\delta_i$-$\gamma$ int(A_α)) which implies $\vee_{\alpha\in\Delta}A_\alpha \leq \delta_j$-cl($\delta_i$-$\gamma$ int($\vee_{\alpha\in\Delta}A_\alpha$)), by Remark 2.11. Thus, $\vee_{\alpha\in\Delta}A_\alpha$ is (δ_i,δ_j)F-γ-so.

Proposition 4.12. The intersection of two (δ_i,δ_j)F-γ-sc sets is a (δ_i,δ_j)F-γ-sc in a F-bts

Remark 4.13. The union of two (δ_i, δ_j)F-γ-sc sets need not be (δ_i,δ_j)F-γ-sc as shown below.

Example 4.14. Let (X, δ_1,δ_2) be a F-bts with X={a,b,c}, δ_1={$\tilde{0}$, $\tilde{1}$,A,B, A$\vee$B}, δ_2={$\tilde{0}$,$\tilde{1}$,C,D} and A,B,C,D,E,F are fuzzy sets defined in X as, A={a_0,$b_{0.2}$,$c_{0.1}$)}, B={$a_{0.3}$,b_0,c_0)}, A$\vee$B={$a_{0.3}$,$b_{0.2}$, $c_{0.1}$}, C={$a_{0.1}$, $b_{0.2}$, $c_{0.1}$}, D={$a_{0.8}$, $b_{0.9}$, $c_{0.9}$},E={a_0, $b_{0.3}$, $c_{0.1}$}, F={$a_{0.3}$,$b_{0.3}$,c_0}. Since E' $\geq$$\delta_2$-int($\delta_1$-$\gamma$ cl(E'))=C and F' $\geq$$\delta_2$-int($\delta_1$-$\gamma$ cl(F'))= C, E' and F' are (δ_1, δ_2)F-γ-sc, but E' $\vee$ F' is not (δ_1, δ_2) F-γ-sc as, δ_2-int(δ_1- γcl(E'$\vee$F')) = 1 $\nleq$ E' $\vee$ F'

Theorem 4.15. Let (X,δ_i,δ_j) be a F-bts. Arbitrary intersection of (δ_i,δ_j) Fγsc sets is (δ_i,δ_j) Fγsc.

Proof: Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a collection of (δ_i,δ_j)F-γ-sc sets in X. For each $\alpha\in\Delta$, A_α is (δ_i,δ_j)F-γ-sc.Then for each $\alpha\in\Delta$, A_α' is (δ_i,δ_j)F-γ-so which implies $\vee_{\alpha\in\Delta}A_\alpha$' is ($\delta_i$,$\delta_j$)F-$\gamma$-so. Let

$B=\vee_{\alpha\in\Delta}A_\alpha'$.Then $B'=(\vee_{\alpha\in\Delta}A_\alpha')'$ is (δ_i,δ_j)F-γ-sc. So, $B'=\wedge_{\alpha\in\Delta}(A_\alpha')'$ is (δ_i,δ_j)F-γ-sc. Thus, $\wedge_{\alpha\in\Delta}A_\alpha$ is (δ_i,δ_j)F-γ-sc.

Theorem 4.16. In a F-bts (X,δ_i,δ_j), every δ_i Fo is (δ_i,δ_j) Fγ-so and every δ_i-Fγo is (δ_i,δ_j)F-γso.
Proof: Follows from Definition 4.1.

Example 4.17. The converse of the above theorem is not true. Let (X,δ_1,δ_2) be a F-bts, X={a,b,c}, $\delta_1=\{\tilde{0}, \tilde{1}, A\}$, $\delta_2=\{\tilde{0},\tilde{1},B\}$ and A, B, C are fuzzy sets in X as, A={$a_{0.5}$, $b_{0.3}$, $c_{0.4}$}, B={$a_{0.5}$,$b_{0.6}$,$c_{0.3}$}, C={$a_{0.5}$, $b_{0.4}$, $c_{0.5}$)}. The set C (resp. C') is(δ_1, δ_2)F-γ-so (resp. (δ_1,δ_2) F-γ-sc) but C (resp. C') is not δ_1-F o (resp. δ_1-F c) as well as not δ_1-F- γ o (resp. δ_1-F- γ c).

Theorem 4.18. Let (X, δ_i, δ_j) be a F-bts. Then every (δ_i, δ_j) F-γ-so set is (δ_i, δ_j) Fγo.
Proof: Follows from Definition 4.1.

Example 4.19. The converse of the above result need not be true.Let (X, δ_1, δ_2) be a F-bts with X = {a, b, c}, $\delta_1=\{\tilde{0}, \tilde{1}, A\}$, $\delta_2=\{\tilde{0}, \tilde{1}, B\}$and A, B, C are fuzzy sets defined in X as, A={$a_{0.5}$,$b_{0.2}$,$c_{0.6}$}, B={$a_{0.5}$,$b_{0.4}$,$c_{0.3}$}, C={$a_{0.3}$,$b_{0.1}$,$c_{0.5}$}. Here C (resp. C') is (δ_1, δ_2)F-γo (resp. (δ_1, δ_2) F-γc) and C (resp. C') is not (δ_1, δ_2) F-γ-so (resp. (δ_1, δ_2) F-γ-sc).

Theorem 4.20. Let (X, δ_i, δ_j) be a F-bts. Then every (δ_i,δ_j)F-s-so set is (δ_i,δ_j)F-γ-so.
Proof: Let A be (δ_i,δ_j) F-s-so in X. Then $A\leq\delta_i$-int(δ_j-cl(δ_i-int(A))). Define $B=\delta_j$cl($\delta_i$int(A)). Then $A\leq\delta_i$-int(B) and $A\leq B$. That is $A\leq\delta_j$-cl(δ_i-int(A)). Thus, $A\leq\delta_j$-cl(δ_i-γ int(A)).

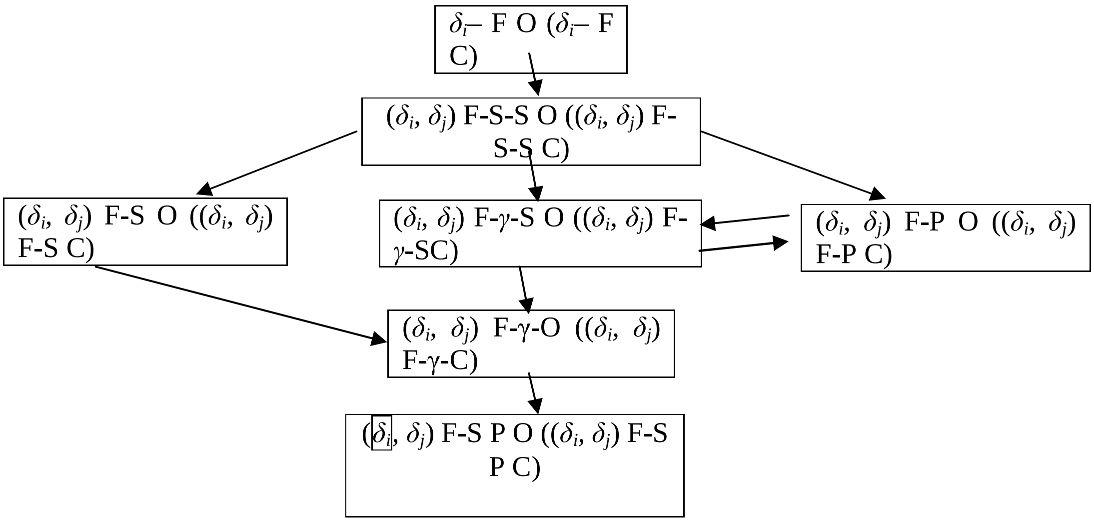
Example 4.21. The converse need not be true always which is shown by the example. Let (X,δ_1,δ_2) be a F-bts with X={a,b,c},$\delta_1=\{\tilde{0},\tilde{1},A\}$, $\delta_2=\{\tilde{0},\tilde{1},B\}$ and A,B,D are fuzzy setsdefined in X as, A={$a_{0.5}$,$b_{0.2}$,$c_{0.6}$},B={$a_{0.5}$,$b_{0.4}$,$c_{0.3}$},D={$a_{0.5}$,$b_{0.3}$,$c_{0.6}$}. The set D (resp.D') is (δ_1,δ_2)F-γ-so (resp.(δ_1,δ_2) F-γ-sc). But D (resp.D') is not (δ_1,δ_2) Fsso (resp. (δ_1,δ_2) Fssc).

Theorem 4.22. Let (X, δ_i, δ_j) be a F-bts. Let A be a fuzzy set in X. If A is (δ_i, δ_j) F-γ-so then A is (δ_i, δ_j) F-so and the converse also holds.
Proof: Suppose A is (δ_i,δ_j)F-γ-so then $A\leq\delta_j$-cl(δ_i-γ int(A)). By Corollary 3.4, A is (δ_i,δ_j) F-so. Now suppose A is (δ_i, δ_j) F-so then $A\leq\delta_j$-cl(δ_i- int(A)). Then A is (δ_i, δ_j) F- γ so.

Theorem 4.23. Let (X,δ_i,δ_j) be a F-bts.Then every (δ_i, δ_j) F-γ-so is(δ_i, δ_j) F-spo.
Proof: Let A be a (δ_i,δ_j) F-γso set in X then $A\leq\delta_j$-cl($\delta_i\gamma$ int(A)). Now δ_j-cl($\delta_i$int($\delta_j$cl(A))) $\geq$ δ_j-cl($\delta_i$int(A) $\geq$ A. Since δ_j-cl(A)$\geq$A), $A\leq\delta_j$-cl(δ_i-int(δ_j-cl(A))). Thus, A is (δ_i,δ_j)F-s-po.

Example 4.24. The converse of the above result need not be true. Let (X, δ_1,δ_2) be a F-bts with X ={a,b,c}, $\delta_1=\{\tilde{0}, \tilde{1},A\}$,$\delta_2=\{\tilde{0}, \tilde{1},B\}$and A,B,C are fuzzy sets defined in X as, A={$a_{0.3}$, $b_{0.4}$,$c_{0.5}$}, B={$a_{0.3}$,$b_{0.5}$,$c_{0.4}$},C={$a_{0.6}$,$b_{0.5}$,$c_{0.2}$}.The set C (resp.C') is (δ_1,δ_2) F-s-po (resp.(δ_1,δ_2)F-s-pc). But C(resp. C') is not (δ_1,δ_2)F-γ-so((δ_1,δ_2)F-γsc).

Remark 4.25. It is now clear that a (δ_i, δ_j) F-γ-s open set is weaker than the concept of (δ_i,δ_j) F-s-s open and stronger than the concept of (δ_i, δ_j) F-γ-open and (δ_i, δ_j) F-s-p open.



The following results can be easily verified.

Proposition 4.26. If A is (δ_i, δ_j) F-γ o set and A is not (δ_i, δ_j) F-p o then A is (δ_i,δ_j)F-γ-s o.

Corollary 4.27. If A is (δ_i, δ_j)F-γ o set and A is not (δ_i, δ_j) F-γ-s o then A is (δ_i, δ_j) F-p o.

Corollary 4.28. If A is (δ_i, δ_j) F-γ o set and δ_i-int(δ_j-cl(A)) = $\tilde{0}$, then A is (δ_i, δ_j) F-γ-s o.

Corollary 4.29.(a) If A is (δ_i, δ_j)F-γo set and A is not (δ_i, δ_j)F-so then A is (δ_i, δ_j) F-po. (b) If A is (δ_i, δ_j) F-γ o set and δ_i-int(δ_j-cl(A)) = $\tilde{0}$, then A is (δ_i, δ_j) F-s o.

Proposition 4.30. Each (δ_i, δ_j) F-γ o set which is δ_j F-c is (δ_i, δ_j) F-γ-so.

Proposition 4.31. Each (δ_i, δ_j) F-s-p o set which is δ_j F-c is (δ_i, δ_j) F-γ-so.

Theorem 4.32. Let (X,δ_i,δ_j) be a F-bts. Let A be a fuzzy set in X. Then A is (δ_i,δ_j)F-γ-so if and only if for each fuzzy point $x_\beta \in$ A there exists a (δ_i,δ_j)F-γ-so set U such that $x_\beta \in U \leq$ A.

Proof: Necessity: Assume A is (δ_i, δ_j)F-γ-so. Let $x_\beta \in$ A. By Theorem 4.6, there exists a δ_i Fγo set U such that U ≤ A. By Theorem 4.16, U is (δ_i, δ_j)F-γ-so. Suppose $x_\beta \notin$ U, then $\beta \nleq$ U≤A. That is$\beta \nleq$A. Then$x_\beta \notin$ A, a contradiction.

Sufficiency: Suppose for every $x_\beta \in$ A there exists a (δ_i, δ_j)F-γ-so set U such that $x_\beta \in U \leq$ A. Then $\{U_\beta\}$ is a collection of (δ_i,δ_j)F-γ-so set such that for every $x_\beta \in$ A, $x_\beta \in U_{\beta i} \leq A$, $\beta_i \in \Delta$. Further $\bigcup_{\beta i \in \Delta} U_{\beta i} = A$ and $U_{\beta i}$ is (δ_i, δ_j) F-γ-so. Then by Theorem 4.11, A is (δ_i, δ_j)F-γ-so.

4.1. (δ_i, δ_j) fuzzy- γ-semi interior and (δ_i, δ_j) fuzzy- γ-semi closure

Definition 4.1.1. Let A be a fuzzy set of a F-bts (X, δ_i, δ_j). Then the (δ_i, δ_j) γ-semi closure ($(\delta_i, \delta_j)\gamma$-scl for short) and $(\delta_i, \delta_j)\gamma$-semi interior ($(\delta_i,\delta_j)\gamma$-sint for short) of A are defined as
(δ_i, δ_j) γ-scl(A) = $\wedge$ { B : B is (δ_i, δ_j) F- γ-s closed and A $\leq$ B }
(δ_i, δ_j) γ-sint(A) = $\vee$ { B : B is (δ_i, δ_j) F- γ-s open and B $\leq$ A }

Example 4.1.2. Let (X,δ_1,δ_2) be a F-bts with X ={a,b,c}, $\delta_1=\{\tilde{0},\tilde{1},A\}$, $\delta_2=\{\tilde{0},\tilde{1}, B\}$. A,B,C and D are fuzzy sets defined in X as, A={$a_{0.5}$,$b_{0.3}$,$c_{0.4}$},B={$a_{0.5}$,$b_{0.6}$,$c_{0.3}$)},C={$a_{0.2}$,$b_{0.1}$, $c_{0.3}$}, D={$a_{0.5}$,$b_{0.4}$,$c_{0.5}$}. $(\delta_1,\delta_2)\gamma$-sint(A)=A; $(\delta_1,\delta_2)\gamma$-sint(B)=0;$(\delta_1,\delta_2)\gamma$-sint(C)=0;$(\delta_1,\delta_2)\gamma$-sint(D)=D, $(\delta_1,\delta_2)\gamma$-scl(A′)=A′, $(\delta_1,\delta_2)\gamma$-scl(B′)=1, $(\delta_1,\delta_2)\gamma$-scl(C′)=1; $(\delta_1,\delta_2)\gamma$-scl(D′) = D′

Proposition 4.1.3. Let A be a fuzzy set of a F-bts (X, δ_i, δ_j). Then
(i) (δ_i, δ_j) γ-scl(A′) = ((δ_i, δ_j) γ-sint(A))′(ii) (δ_i, δ_j) γ-sint(A′) = ((δ_i, δ_j) γ-scl(A))′.
Proof: Follows from Definition 4.1.1

Definition 4.1.4. Let (X, δ_i, δ_j) be a fuzzy bitopological space and x_β is a fuzzy point of X. A fuzzy set A of X is called
(a) (δ_i, δ_j) F-γ semi neighbourhood (briefly (δ_i, δ_j) F-γ-semi nbhd) of $x_\beta$if there exists a (δ_i,δ_j)F-γ-so set O such that $x_\beta \in O \leq A$
(b) (δ_i, δ_j)F-γ semi q neighbourhood (briefly (δ_i, δ_j) F-γ-semi q nbhd)of $x_\beta$if there exists a (δ_i, δ_j)F-γ-so set O such that x_βq O $\leq$ A

Example 4.1.5. Let (X,δ_1,δ_2) be a F-bts, X={a,b,c},$\delta_1=\{\tilde{0},\tilde{1},A\}$and $\delta_2=\{\tilde{0},\tilde{1},B\}$. A,B,C and D are fuzzy sets defined in X as, A = {$a_{0.5}$, $b_{0.3}$, $c_{0.4}$}, B = {$a_{0.5}$, $b_{0.6}$, $c_{0.3}$}, C = {$a_{0.5}$, $b_{0.4}$, $c_{0.5}$} and D = {$a_{0.8}$,$b_{0.7}$,$c_{0.6}$}. Let β = 0.4. Then $x_\beta = x_{0.4} \in$ C $\leq$ D. Thus, D is a (δ_1, δ_2) F-γ-semi nbhd of $x_{0.4}$. Now, let ρ=0.7, then $x_\rho=x_{0.7}$. Thus, $x_{0.7}$ q C since 0.7+0.4=1.1>1.

Theorem 4.1.6. In a F-bts (X, δ_i, δ_j) a fuzzy set A is (δ_i, δ_j) F-γ-so if and only if for each fuzzy point $x_\beta \in$ A, A is a (δ_i, δ_j) F-γ-semi neighbourhood of x_β.
Proof: Follows from Theorem 4.32and Definition 4.1.4.

Theorem 4.1.7. In a F-bts (X, δ_i, δ_j) a fuzzy set A is (δ_i, δ_j) F-γ-so if and only if for every fuzzy point x_βq A, A is a (δ_i, δ_j) F- γ-semi q nbhd of x_β.
Proof: Let A be (δ_i,δ_j)F-γso. Suppose x_βqA. By Definition4.1.4, A is a (δ_i,δ_j)F-γ-semi q nbhd of x_β. Conversely, suppose for every fuzzy point x_βq A, A is a (δ_i, δ_j) F-γ-semi q nbhdof x_β.Then for each fuzzy point x_βqA, there exists a (δ_i, δ_j)F-γ-so set B such that x_βqB and B $\leq$ A. Now if $x_{\beta 1}$qA, then there exists a (δ_i,δ_j)F-γ-so set B_1 such that $x_{\beta 1}$qB_1 and $B_1 \leq$ A. Similarly, if $x_{\beta n}$qA, then there exists a (δ_i,δ_j)F-γ-so set B_n such that $x_{\beta n}$qB_n and $B_n \leq$A. Then A = $\bigcup_{\alpha\in\Delta} B_\alpha$. Thus, A is (δ_i, δ_j) F-γ-so.

Theorem 4.1.8. Let (X, δ_i, δ_j) be a F-bts. Let A be a fuzzy set in X, then $x_\beta \in$ $(\delta_i, \delta_j)\gamma$-scl(A) if and only if every (δ_i, δ_j)F-γ-semi q nbhd of x_β is quasicoincident with A. **Proof: Necessity:** Suppose $x_\beta \in$ (δ_i, δ_j) γ-scl(A). If possible let there exist a (δ_i,δ_j) F-γ-semi q nbhd B of x_β such that $\rceil$(BqA). Then B$\leq$ A′. By Definition4.1.4,there exists a (δ_i,δ_j)F-γ-so set B_1 such that x_βqB_1 and $B_1 \leq$B. As β+B_1(x)>1, β>B_1′(x). Now, $B_1 \leq$ A′implies $\rceil$(B_1qA) Then A$\leq$ B_1′ and B_1′ is (δ_i,δ_j)F-γ-sc. So $(\delta_i,\delta_j)\gamma$-scl(A)$\leq$ B_1′. By assumption, $\beta \leq (\delta_i, \delta_j)\gamma$-scl(A)$\leq B_1'$.Thus $\beta = B_1'(x)$. Hence $x_\beta \notin (\delta_i, \delta_j)\gamma$-scl(A), a contradiction.

Sufficiency: Suppose every (δ_i, δ_j)F-γ-s q nbhd of x_β is quasicoincident with A. Assume $x_\beta \notin (\delta_i, \delta_j)$ γ-scl(A), then $\beta > (\delta_i, \delta_j)\gamma$-scl(A). That is, there exists at least one (δ_i, δ_j)F-γ-sc set $B \geq A$ and $\beta > B$. Then $x_\beta \notin B$ and so $\beta + B'(x) > 1$.Thus, B' is (δ_i,δ_j)F-γ-so and $x_\beta q B'$. As $B' \leq A'$, $\rceil(B'qA)$. Then $B'(x)+A(x) \leq 1$, $B'(x)+A(x) < \beta + B'(x)$. That is $\beta > A(x)$ .Then $x_\beta \notin A$,so $x_\beta \in A'$, thus,$\beta \leq A'$. That is $\rceil(x_\beta qA)$ which is a contradiction.

Theorem 4.1.9. Let $x_\rho$be a fuzzy point of X and A be a fuzzy set in a F-bts (X,δ_i, δ_j). Then $x_\rho q$ $(\delta_i,\delta_j)\gamma$-scl(A) if and only if for every (δ_i, δ_j) F- γ-s q nbhd B of x_ρ, BqA.
Proof: Necessity: Suppose $x_\rho q$ $(\delta_i, \delta_j)\gamma$-scl(A). Then $\rho + (\delta_i, \delta_j)\gamma$-scl(A(x))>1. If possible there exists a (δ_i,δ_j) F-γ-semi q nbhd B of x_ρ, $\rceil$(BqA) which implies $B \leq A'$. Since B is (δ_i, δ_j)F-γ-semi q nbhd of x_ρ, there exists a (δ_i,δ_j)F-γso set B_1 such that $x_\rho q B_1$, $B_1 \leq B$. By Theorem4.1.8, $x_\rho \notin (\delta_i,\delta_j)\gamma$-scl(A). Then $x_\rho \in [(\delta_i,\delta_j)\gamma$scl(A)$]'$.That is$\rho \leq [(\delta_i,\delta_j)\gamma$-scl(A(x))$]'$. Thus,$\rceil(x_\rho q(\delta_i,\delta_j)\gamma$-scl(A)). A contradiction, which proves the theorem.

Sufficiency: Suppose every (δ_i, δ_j)F-γ-s q nbhd of x_ρ is quasicoincident with A. If $(x_\rho q(\delta_i, \delta_j)$ γ-scl(A)). Then $\rho \leq [(\delta_i,\delta_j)\gamma$-scl(A(x))$]'$. That is $x_\rho \in [(\delta_i,\delta_j)\gamma$-cl(A)$]'$, which implies $x_\rho \notin (\delta_i,\delta_j)\gamma$-scl(A). By Theorem 4.1.8, this leads to $\rceil$ (x_ρ q A) which is a contradiction.

Theorem 4.1.10. Let (X, δ_i, δ_j) be a F-bts. Let A be a fuzzy set in X and B $\in$ (δ_i, δ_j)F-γ-so(x), such that $\rceil$ (A q B) then $\rceil$ ((δ_i, δ_j) γ-scl(A) q B).
Proof: Suppose B $\in$ (δ_i,δ_j)F-γ-so(x), then B is (δ_i,δ_j)F-γ-so. Now $\rceil$(AqB) implies $A \leq B'$. Since B' is (δ_i,δ_j)F-γ-sc, $(\delta_i,\delta_j)\gamma$-scl(A) $\leq B'$. Thus,$\rceil((\delta_i,\delta_j)\gamma$-scl(A) q B).

4.2. Properties of (δ_i, δ_j) γ-semi interior and (δ_i, δ_j) γ-semi closure operators

Theorem 4.2.1. Let (X, δ_i,δ_j) be a F-bts.Then for any fuzzy sets A and B of X,
(i) (δ_i, δ_j) γ-s int$(\tilde{0}) = \tilde{0}$ and (δ_i, δ_j) γ-s int$(\tilde{1}) = \tilde{1}$ (ii) δ_i int(A) $\leq (\delta_i, \delta_j)$ γ-s int(A) $\leq$ A
(iii) A is (δ_i, δ_j) F-γ so if and only if A = (δ_i, δ_j) γ-s int(A)
(iv) $(\delta_i, \delta_j)\gamma$-s int(A) is (δ_i,δ_j)F-γ-so set and $(\delta_i,\delta_j)\gamma$- s int($(\delta_i,\delta_j)\gamma$-s int(A)) = $(\delta_i,\delta_j)\gamma$-s int(A)
(v) If $A \leq B$, then (δ_i, δ_j) γ-s int(A) $\leq (\delta_i, \delta_j)$ γ-s int(B)
Proof: (i) and (ii). Follows from Definition 4.1.1.
(iii) Let A be (δ_i, δ_j)F-γ so. Then (δ_i, δ_j) γ-s int(A) = A. Conversely, if A = $(\delta_i,\delta_j)\gamma$-s int(A), then by Definition 4.1.1, A is (δ_i, δ_j)F-γ so.
(iv) From Definition 4.1.1 $(\delta_i,\delta_j)\gamma$-sint(A) is (δ_i,δ_j)F-γso. From (iii) other result holds.
(v) Let $A \leq B$. From (ii), $(\delta_i,\delta_j)\gamma$-s int(A)$\leq A \leq B$. By (iv), $(\delta_i,\delta_j)\gamma$-s int(A) $\leq (\delta_i,\delta_j)\gamma$-s int(B).

Proposition 4.2.2. Let (X,δ_i,δ_j) be a F-bts and A and B be any two fuzzy sets of X. Then
(i) $(\delta_i, \delta_j)\gamma$-s int($A \wedge B$) = (δ_i, δ_j) γ-s int(A) $\wedge$ (δ_i, δ_j) γ-sint(B)
(ii) (δ_i, δ_j) γ-s int($A \vee B$) $\geq (\delta_i, \delta_j)$ γ-s int(A) $\vee$ (δ_i, δ_j) γ-sint(B)
Proof: (i) By Theorem 4.2.1, $(\delta_i,\delta_j)\gamma$ -sint($A \wedge B$) $\leq (\delta_i,\delta_j)\gamma$-sint(A),$(\delta_i,\delta_j)\gamma$-sint($A \wedge B$) $\leq (\delta_i, \delta_j)\gamma$ sint(B). Thus, (δ_i, δ_j) γ-s int($A \wedge B$) $\leq (\delta_i, \delta_j)$ γ-s int(A) $\wedge$ (δ_i, δ_j) γ-s int(B).
Let C $\in$ [(δ_i, δ_j) γ-s int(A) $\wedge$ (δ_i, δ_j) γ-s int(B)]. Then C is a (δ_i,δ_j)F-γ so set and $C \leq A \wedge B$. Then $C \leq (\delta_i,\delta_j)\gamma$-s int($A \wedge B$). Thus, [$(\delta_i, \delta_j)\gamma$-s int(A) $\wedge$ $(\delta_i,\delta_j)\gamma$-s int(B)] $\leq (\delta_i, \delta_j)$ γ-s int($A \wedge B$). Hence (δ_i, δ_j) γ-s int($A \wedge B$) = (δ_i, δ_j) γ-s int(A) $\wedge$ (δ_i, δ_j) γ-s int(B).
(ii) By Theorem 4.2.1(v), (δ_i, δ_j) γ -s int($A \vee B$) $\geq (\delta_i, \delta_j)$ γ -s int(A) $\vee$ (δ_i, δ_j) γ -s int(B).

Remark 4.2.3. Equality need not hold in Proposition 4.2.2(ii) which is given by the example below. Let (Z,δ_1,δ_2) be a F-bts with Z={a,b,c}, $\delta_1=\{\tilde{0},\tilde{1},X\}$, $\delta_2=\{\tilde{0},\tilde{1},Y\}$ and fuzzy sets X={$a_{0.5}$,$b_{0.6}$,$c_{0.7}$)}, Y={$a_{0.5}$,$b_{0.3}$,$c_{0.2}$}, A={$a_{0.5}$,$b_{0.4}$,$c_{0.8}$}, B={$a_{0.5}$,$b_{0.7}$,$c_{0.7}$}, A∨B={$a_{0.5}$,$b_{0.7}$, $c_{0.8}$}=Y'. Here (δ_1,δ_2)F-γ-so sets={$\tilde{0},\tilde{1}$, X,B,A∨B} and $(\delta_1, \delta_2)\gamma$-s int(A∨B)=A∨B, $(\delta_1, \delta_2)\gamma$-s int(A)=0, (δ_1, δ_2) γ-s int(B) = B. Then, $(\delta_1,\delta_2)\gamma$-sint(A)∨$(\delta_1,\delta_2)\gamma$-sint(B)=B. Thus, $(\delta_1,\delta_2)\gamma$-sint(A∨B) ≥ $(\delta_1,\delta_2)\gamma$-sint(A)∨$(\delta_1,\delta_2)\gamma$-sint(B).

Theorem 4.2.4. Let (X,δ_i,δ_j) be a F-bts. For fuzzy sets A and B of X, the following holds:
(i) (δ_i, δ_j) γ-s cl($\tilde{0}$) = $\tilde{0}$ and (δ_i, δ_j) γ-s cl($\tilde{1}$) = $\tilde{1}$ (ii) A ≤ (δ_i, δ_j) γ-s cl(A) ≤ δ_i cl(A)
(iii) A is (δ_i, δ_j) F-γ sc if and only if A = (δ_i, δ_j) γ-s cl(A)
(iv) $(\delta_i, \delta_j)\gamma$-s cl(A) is (δ_i, δ_j)F-γ-sc set and (δ_i, δ_j) γ-s cl((δ_i, δ_j) γ-s cl(A)) = (δ_i, δ_j) γ-s cl(A)
(v) If A ≤ B, then (δ_i, δ_j) γ -s cl(A) ≤ (δ_i, δ_j) γ -s cl(B)
Proof: Follows from Definition 4.1.1.

Proposition 4.2.5. Let (X, δ_i, δ_j) be a F-bts and A and B be any two fuzzy sets of X. Then
(i) (δ_i, δ_j) γ-s cl(A ∨ B) = (δ_i, δ_j) γ -s cl(A) ∨ (δ_i, δ_j) γ -s cl(B)
(ii) (δ_i, δ_j) γ-s cl(A ∧ B) ≤ (δ_i, δ_j) γ -s cl(A) ∧ (δ_i, δ_j) γ -s cl(B)
Proof: (i) Consider (δ_i, δ_j) γ-s cl(A ∨ B) = [(δ_i, δ_j) γ-s int(A∨ B)']'=[(δ_i, δ_j) γ-s int(A'∧ B')]'
= [(δ_i, δ_j) γ-s int(A') ∧ (δ_i, δ_j) γ-s int(B')]' = [(δ_i, δ_j) γ-s int(A')]' ∨ [(δ_i, δ_j) γ-s int(B')]'
= (δ_i, δ_j) γ-s cl(A′)′ ∨ (δ_i, δ_j) γ-s cl(B′)′ = (δ_i, δ_j) γ-s cl(A) ∨ (δ_i, δ_j) γ-s cl(B).
Thus, (δ_i, δ_j) γ-s cl(A∨B) = (δ_i, δ_j) γ-s cl(A) ∨ $(\delta_i, \delta_j)\gamma$-s cl(B)
(ii) Consider (δ_i, δ_j) γ-s cl(A ∧ B) = [((δ_i, δ_j) γ-s cl(A ∧ B))'
= [$(\delta_i, \delta_j)\gamma$-s int(A ∧ B)']' = [$(\delta_i, \delta_j)\gamma$-s int(A' ∨ B') ≥ [$(\delta_i, \delta_j)\gamma$-s int(A') ∨ $(\delta_i, \delta_j)\gamma$-sint(B')]≤ [$(\delta_i, \delta_j)$ γ-s int(A')]' ∧ [(δ_i, δ_j) γ-s int(B')]' = [(δ_i, δ_j) γ-s cl(A)']'∧[$(\delta_i, \delta_j)\gamma$-s cl(B)']'
= $(\delta_i, \delta_j)\gamma$-s cl(A)∧$(\delta_i, \delta_j)\gamma$-s cl(B). Thus, (δ_i,δ_j) γ-s cl(A∧B) ≤$(\delta_i,\delta_j)\gamma$-s cl(A)∧$(\delta_i,\delta_j)\gamma$-s cl(B)

Remark 4.2.6. Equality need not hold in Proposition 4.2.4 (ii). Let (Z,δ_1,δ_2) be a F-bts with Z={a,b,c},$\delta_1=\{\tilde{0},\tilde{1},X\}$, $\delta_2=\{\tilde{0},\tilde{1},Y\}$ with fuzzy sets X={$a_{0.6}$,$b_{0.5}$,$c_{0.6}$}, Y={$a_{0.3}$,$b_{0.5}$,$c_{0.1}$}, A={$a_{0.6}$,$b_{0.3}$,$c_{0.8}$}, B={$a_{0.7}$,$b_{0.5}$,$c_{0.7}$} and A∨B={$a_{0.7}$,$b_{0.5}$,$c_{0.8}$}=Y', A∧B ={$a_{0.6}$,$b_{0.3}$, $c_{0.7}$}.Then (δ_1,δ_2)γ-s cl(A′∧B′) = A′∧B′, (δ_1,δ_2)γ-s cl(A′)=1 and (δ_1,δ_2)γ-scl(B′) = B′. Then (δ_1, δ_2) γ-cl(A′)∧(δ_1,δ_2)γ-s cl(B′)=B′ and A′∧B′≤B'. Thus, (δ_i,δ_j)γ-cl(A'∧B') ≤ (δ_i,δ_j)γ-cl(A')∧(δ_i,δ_j)γcl(B').

7. Conclusion

In this paper, the notion of (δ_i, δ_j) F-γ-semiopen (δ_i, δ_j) F-γ-semi closed sets in fuzzy bitopological spacesare introduced and their properties are discussed their relationship with other sets are studied.

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REFERENCES

1. A.K.Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, *Journal of Mathematical Analysis And Applications*, 82 (1981) 14–32.
2. B.S.Zong, Fuzzy strongly semiopen sets and fuzzy strongly semicontinuity, *Fuzzy Sets Syst.*, 52 (1992) 345–351.

3. C.L.Chang, Fuzzy topological spaces, *J. Math. Anal. Appl*., 24 (1968) 182–190.
4. I.M.Hanafy, Fuzzy γ-open sets and fuzzy γ-continuity, *J. Fuzzy Math.*, 7 (2) (1999)
5. A.Kandil and M.E.El-Shafee, Biproximities and fuzzy bitopological spaces, *Simon Stevin*, 63 (1) (1989) 45–66.
6. A.S.Mashour, M.H.Ghanim and M.A.Fath Alla, On fuzzy non-continuous mappings, *Bull Calcutta Math. Soc.*, 78 (1986) 57–69.
7. F.S.Mahmoud, M.A.Fath Alla, M.M.Khalaf, Fuzzy-γ-open sets and fuzzy-γ-continuity in fuzzy bitopological spaces, *Applied Mathematics and Computation*, 153 (2004) 117–126.
8. J.H.Park and B.Y.Lee, Fuzzy semi-preopen sets and fuzzy semi-precontinuous mappings, *Fuzzy Sets Syst.*, 67 (1994) 359–364.
9. J.H.Park, On fuzzy pairwise semi-precontinuity, *Fuzzy Sets Syst.*, 93 (1998) 375–379.
10. P.M.Pu and Y.M.Liu, Fuzzy topology I. Neighbourhood structure of a fuzzy point and Moore Smith convergence, *J. Math. Anal. Appl.*, 76 (1980) 571–599.
11. S.Sampath Kumar, On fuzzy pairwise α-continuity and fuzzy pre-continuity, *Fuzzy Sets Syst.*, 62 (1994) 231–238.
12. S.Sampath Kumar, Semi-open sets, semi-continuity and semi-open mapping in fuzzy bitopological spaces, *Fuzzy Sets Syst.*, 64 (1994) 421–426.
13. S.S.Thakur and R.Malviya, Semi-open sets and semi-continuity in fuzzy bitopological spaces, *Fuzzy Sets Syst.*, 75 (1996) 451–456.
14. C.K.Wong, Fuzzy point and local properties of fuzzy topology, *J. Math. Anal. Appl.*, 46 (1974) 328–361.
15. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338–353.