Annals of Pure and Applied Mathematics
Vol. 16, No. 1, 2018, 81-90
ISSN: 2279-087X (P), 2279-0888(online)
Published on 1 January 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v16n1a10

Annals of
Pure and Applied Mathematics

Hamacher Operations of Intuitionistic Fuzzy Matrices

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Received 12 December 2017; accepted 26 December 2017

Abstract. In this paper, some operations on the intuitionistic fuzzy matrices such as Hamacher sum and Hamacher product are introduced and investigates the algebraic properties of intuitionistic fuzzy matrices under these operations as well as the properties of intuitionistic fuzzy matrices in the case where these new operations are combined with the well-known operations $\wedge, \vee$.

Keywords: Intuitionistic fuzzy matrix, Hamacher sum, Hamacher product

AMS Mathematics Subject Classification(2010): 03E73, 04A72

1. Introduction

After introduction of the concept of fuzzy matrices by Thomason [18] several researchers were conducted on the generalization of the notion of fuzzy matrices. Meenakshi [8] studied the theoretical developments of fuzzy matrices. The idea of IFM was first bublished by Im et al. and Pal [5,6,12,14] and many works by the same authors and his colleagues appeared in the literature. IFM is very useful in the discussion of intuitionistic fuzzy relation [7]. Xu [20] defined intuitionistic fuzzy similarity relation and utilize it in clustering analysis. Mondal [9] studied similarity relations, invertibility and eigenvalues of IFM. Monoids on IFMs are studied in [17]. [11] Muthuraji and sriram studied reduction operators of IFM. They discussed in [10] the properties of various (α,α') cuts on IFM and some representation and decomposition of an IFM using (α,α') cuts.

The paper is organized in four sections. We define Hamacher operations of fuzzy matrices and investigate the algebraic properties [16]. In this paper we extend Hamacher operations to intuitionistic fuzzy matrices. In section 2, we give the basic definitions and operations on intuitionistic fuzzy matrices which will be used in this paper. In section 3, we introduce the hamacher operations on intuitionistic fuzzy matrices and focusing on its properties. The De Morgan's law for the Hamacher operations are established in section 4. In section 5, some results on Necessity and Possibility operators of intuitionistic fuzzy matrices.

2. Preliminaries and definitions

In this section, we give some definitions and preliminaries which are applied in the paper.

Definition 2.1.[14] An IFM is a matrix of pairs $A=(<a_{ij},a_{ij}^{'}>)$ of non negative real numbers satisfying $a_{ij}+a_{ij}^{'}\leq 1$ for all i,j. The $m\times n$ zero intuitionistic fuzzy matrix O is an intuitionistic fuzzy matrix all of whose entries are $<0,1>$. The $m\times n$ universal intuitionistic fuzzy matrix J is an intuitionistic fuzzy matrix all of whose entries are $<1,0>$. Denote the set of all IFMs of order $m\times n$ by F_{mn} and square matrix of order n by F_n.

Definition 2.2. [14] Let $A,B\in F_{mn}$,

$(i)A\vee B=\left(\left\langle \max(a_{ij},b_{ij}),\min(a_{ij}^{'},b_{ij}^{'})\right\rangle\right)$,

$(ii)A\wedge B=\left(\left\langle \min(a_{ij},b_{ij}),\max(a_{ij}^{'},b_{ij}^{'})\right\rangle\right)$.

Definition 2.3. [14] The complement of intuitionistic fuzzy matrix A which is denoted by A^c and is defined by $A^c=(<a_{ij}^{'},a_{ij}>)$ for all i,j.

Definition 2.4. [14] If $A=(<a_{ij},a_{ij}^{'}>)$ is an $m\times n$ intuitionistic fuzzy matrix, then the $n\times m$ intuitionistic fuzzy matrix is $A^T=(<a_{ji},a_{ji}^{'}>)$,
where $a_{ij}=a_{ji},a_{ij}^{'}=a_{ji}^{'},1\leq i\leq m,1\leq j\leq m$ is called the transpose of A.

Definition 2.5. [14] Let $A,B\in F_{mn},A=(<a_{ij}>),B=(<b_{ij}>)$ then we write $A\leq B$ if and only if $a_{ij}\leq b_{ij}$ and $a_{ij}^{'}\geq b_{ij}^{'}$ for all i,j.

Definition 2.6. [14]For intuitionistic fuzzy matrix A, define $\Box A=(<a_{ij},1-a_{ij}>)$ and $\Diamond A=(<1-a_{ij}^{'},a_{ij}^{'}>)$.

3. Hamacher sum and hamacher product of intuitionistic fuzzy matrices

$t-norm(T)$ and $t-conorm(T^*)$ [4] operations are widely used for finding the various arithmetic operations in the fuzzy sets. For instance, Hamacher (1978) proposed t-norm and t-conorm by defining as

$$T(x,y)=\left(\left\langle\frac{xy}{(x+y-xy)}\right\rangle\right) \text{ and } T^*(x,y)=\left(\left\langle\frac{x+y-2xy}{1-xy}\right\rangle\right),$$

respectively, thus based on these operations, Hamacher sum and hamacher product operations are defined for two intuitionistic fuzzy matrices A and B as follows.

Definition 3.1. Let $A, B \in F_{mn}$,

(i) $A \oplus_H B = \left(\left\langle \dfrac{a_{ij} + b_{ij} - 2a_{ij}b_{ij}}{1 - a_{ij}b_{ij}}, \dfrac{a_{ij}^{'}b_{ij}^{'}}{a_{ij}^{'} + b_{ij}^{'} - a_{ij}^{'}b_{ij}^{'}} \right\rangle \right)$

(ii) $A \odot_H B = \left(\left\langle \dfrac{a_{ij}b_{ij}}{a_{ij} + b_{ij} - a_{ij}b_{ij}}, \dfrac{a_{ij}^{'} + b_{ij}^{'} - 2a_{ij}^{'}b_{ij}^{'}}{1 - a_{ij}^{'}b_{ij}^{'}} \right\rangle \right)$ for all i, j.

Lemma 3.2. For any two real numbers $a, b \in [0,1]$, the following inequality holds, $\dfrac{ab}{a+b-ab} \le \dfrac{a+b-2ab}{1-ab}$

Proof: We know that $(a+b)^2 \ge 4ab$ (1.1)

$a+b-ab \le 1 \Rightarrow 1+3(a+b-ab) \le 4$

$\Rightarrow (1+3(a+b-ab))ab \le 4ab$ (1.2)

From (1.1) and (1.2)

$(1+3(a+b-ab))ab \le 4ab \le (a+b)^2$

$0 \le (a+b)^2 - (1+3(a+b-ab))ab$

$= (a+b)^2 + 3ab(ab-a-b) - ab$

$ab \le (a+b)^2 + 3a^2b^2 - 3ab(a+b)$

$ab - a^2b^2 \le (a+b-2ab)(a+b-ab)$

$\dfrac{ab}{a+b-ab} \le \dfrac{a+b-2ab}{1-ab}$

Hence the result.

Lemma 3.3. For any three real numbers $a, b, c \in [0,1]$,the following inequalities hold, If $a \le b$ then ,

$(i) \dfrac{ac}{a+c-ac} \le \dfrac{bc}{b+c-bc} (ii) \dfrac{a+c-2ac}{1-ac} \le \dfrac{b+c-2bc}{1-bc}$

Proof: Let $a, b, c \in [0,1]$ and $a \le b$,

$(i) ac^2 \le bc^2$

$ac^2 + abc(1-c) \le bc^2 + abc(1-c)$

$ac^2 + abc - abc^2 \le bc^2 + abc - abc^2$

$ac(c+b-bc) \le bc(c+a-ac)$

$\dfrac{ac}{a+c-ac} \le \dfrac{bc}{b+c-bc}$

(ii)Since $a \le b$

$$a(1-c)^2 \le b(1-c)^2$$
$$a(1-2c+c^2) \le b(1-2c+c^2)$$
$$a-2ac+ac^2 \le b-2bc+bc^2$$
$$a-2ac+ac^2+(c-abc+2abc^2) \le b-2bcbc^2+(c-abc+2abc^2)$$
$$a+c-2ac-abc-bc^2+2abc^2 \le b+c-2bc-abc-ac^2+2abc^2$$
$$a+c-2ac-bc(a+c-2ac) \le b+c-2bc-ac(b+c-2bc)$$
$$(a+c-2ac)(1-bc) \le (b+c-2bc)(1-ac)$$
$$\frac{a+c-2ac}{1-ac} \le \frac{b+c-2bc}{1-bc}$$

Hence the result.

Property 3.4. For $A, B \in F_{mn}$, $A \odot_H B \le A \oplus_H B.$

Proof: By using Lemma 3.2,

$$\frac{a_{ij}b_{ij}}{a_{ij}+b_{ij}-a_{ij}b_{ij}} \le \frac{a_{ij}+b_{ij}-2a_{ij}b_{ij}}{1-a_{ij}b_{ij}} \text{ and}$$

$$\frac{a_{ij}^{'}+b_{ij}^{'}-2a_{ij}^{'}b_{ij}^{'}}{1-a_{ij}^{'}b_{ij}^{'}} \ge \frac{a_{ij}^{'}b_{ij}^{'}}{a_{ij}^{'}+b_{ij}^{'}-a_{ij}^{'}b_{ij}^{'}} \text{ for all } i,\ j$$

Hence, ij^{th} entry of $A \odot_H B \le ij^{th}$ entry of $A \oplus_H B$.

Therefore, $A \odot_H B \le A \oplus_H B.$

Property 3.5. For $A \in F_{mn}$,

$(i) A \oplus_H A \ge A,$

$(ii) A \odot_H A \le A.$

Proof: (i)

$$A \oplus_H A = \left(\left\langle \frac{2a_{ij}-2a_{ij}^2}{1-a_{ij}^2}, \frac{a_{ij}^{'\,2}}{2a_{ij}^{'}-a_{ij}^{'\,2}} \right\rangle \right)$$
$$= \left(\left\langle \frac{2a_{ij}}{1+a_{ij}}, \frac{a_{ij}^{'}}{2-a_{ij}^{'}} \right\rangle \right)$$
$$\ge (< a_{ij}, a_{ij}^{'} >) \text{ for all } i,\ j$$
$$\ge A.$$

Since $\frac{2a_{ij}}{1+a_{ij}} \ge a_{ij}$ and $a_{ij}^{'} \le \frac{a_{ij}^{'}}{2-a_{ij}^{'}}$.

Hence, ij^{th} entry of $A \oplus_H A \ge ij^{th}$ entry of $A.$

Therefore, $A \oplus_H A \geq A$.

(ii) It can be proved similarly.

The following properties are obvious. The operations $\oplus_H$ and $\odot_H$ are commutative as well as associative. Existence of the identity elements with respect to $\oplus_H$ and $\odot_H$ are determined in the following Theorems.

Property 3.6. Let $A, B \in F_{mn}$ and $C \in F_{mn}$

$$\begin{aligned} &(i) A \oplus_H B = B \oplus_H A, \\ &(ii) (A \oplus_H B) \oplus_H C = A \oplus_H (B \oplus_H C), \\ &(iii) A \odot_H B = B \odot_H A, \\ &(iv) (A \odot_H B) \odot_H C = A \odot_H (B \odot_H C). \end{aligned}$$

Property 3.7. For $A \in F_{mn}$,

$$\begin{aligned} &(i) A \oplus_H O = O \oplus_H A = A, \\ &(ii) A \odot_H J = J \odot_H A = A, \\ &(iii) A \oplus_H J = J, \\ &(iv) A \odot_H O = O. \end{aligned}$$

Thus $(F_{mn}, \oplus_H)$ and $(F_{mn}, \odot_H)$ form commutative monoids. The operations $\oplus_H$ and $\odot_H$ do not obey the De Morgan's laws over transpose.

Property 3.8. For $A, B \in F_{mn}$,

$$\begin{aligned} &(i) (A \oplus_H B)^T = A^T \oplus_H B^T, \\ &(ii) (A \odot_H B)^T = A^T \odot_H B^T. \end{aligned}$$

where A^T is the transpose of A.

Property 3.9. For $A, B \in F_{mn}$, if $A \leq B$, then $A \odot_H C \leq B \odot_H C$

Proof: Let $a_{ij} \leq b_{ij}$ for all i, j and $a'_{ij} \geq b'_{ij}$ for all i, j

By using Lemma 3.3 (i),

$$\frac{a_{ij}c_{ij}}{a_{ij} + c_{ij} - a_{ij}c_{ij}} \leq \frac{b_{ij}c_{ij}}{b_{ij} + c_{ij} - b_{ij}c_{ij}}$$

$$\Rightarrow \frac{b'_{ij}c'_{ij}}{b'_{ij} + c'_{ij} - b'_{ij}c'_{ij}} \geq \frac{a'_{ij}c'_{ij}}{a'_{ij} + c'_{ij} - a'_{ij}c'_{ij}} \quad \text{for all } i, j.$$

Therefore, the ij^{th} entry of $A \odot_H C$ $\leq$ ij^{th} entry of $B \odot_H C$.

Hencce the result.

Property 3.10. For $A,B\in F_{mn}$, If $A\le B$, then $A\oplus_H C\le B\oplus_H C$

Proof: Let $a_{ij}\le b_{ij}$ for all i,j and $a_{ij}^{'}\ge b_{ij}^{'}$ for all i,j

By using Lemma 3.3 (ii),

$$\frac{a_{ij}+c_{ij}-2a_{ij}c_{ij}}{1-a_{ij}c_{ij}}\le\frac{b_{ij}+c_{ij}-2b_{ij}c_{ij}}{1-b_{ij}c_{ij}} \text{ and}$$

$$\Rightarrow\frac{b_{ij}^{'}+c_{ij}^{'}-2b_{ij}^{'}c_{ij}^{'}}{1-b_{ij}^{'}c_{ij}^{'}}\ge\frac{a_{ij}^{'}+c_{ij}^{'}-2a_{ij}^{'}c_{ij}^{'}}{1-a_{ij}^{'}c_{ij}} \text{ for all } i,j.$$

Therefore, the ij^{th} entry of $A\oplus_H C$ $\le$ ij^{th} entry of $B\oplus_H C$.

Hence the result.

Property 3.11. For $A,B\in F_{mn}$,

$(i)(A\wedge B)\oplus_H(A\vee B)=A\oplus_H B(ii)(A\wedge B)\odot_H(A\vee B)=A\odot_H B$

Proof:

$$(i)(A\wedge B)\oplus_H(A\vee B)=\left(\left\langle\min(a_{ij},b_{ij}),\max(a_{ij}^{'},b_{ij}^{'}\right\rangle\right)\oplus_H\left(\left\langle\max(a_{ij},b_{ij}),\min(a_{ij}^{'},b_{ij}^{'})\right\rangle\right)$$

$$=\left(\left\langle\frac{(min(a_{ij},b_{ij})+max(a_{ij},b_{ij})-2min(a_{ij},b_{ij})max(a_{ij},b_{ij}))}{(1-min(a_{ij},b_{ij})max(a_{ij},b_{ij}))}\right\rangle\right),$$

$$\left(\left\langle\frac{min(a_{ij}^{'},b_{ij}^{'})max(a_{ij}^{'},b_{ij}^{'})}{(min(a_{ij}^{'},b_{ij}^{'})+max(a_{ij}^{'},b_{ij}^{'})-min(a_{ij}^{'},b_{ij}^{'})max(a_{ij}^{'},b_{ij}^{'}))}\right\rangle\right)$$

$$=\left(\left\langle\frac{a_{ij}+b_{ij}-2a_{ij}b_{ij}}{1-a_{ij}b_{ij}},\frac{a_{ij}^{'}b_{ij}^{'}}{a_{ij}^{'}+b_{ij}^{'}-a_{ij}^{'}b_{ij}^{'}}\right\rangle\right)=A\oplus_H B$$

(ii) It can be proved similarly.

4. Results on complement of intuitionistic fuzzy matrix

In this section, the complement of an intuitionistic fuzzy matrix is used to analyse the complementary nature of any system. Using the following results we can study the complement nature of a system with the help of original intuitionistic fuzzy matrix. The operator complement obey the De Morgan's law for the operations $\oplus_H$ and $\odot_H$. This is established in the following property.

Property 4.1. For $A,B\in F_{mn}$,

$(i)(A\oplus_H B)^c=A^c\odot_H B^c$,

$(ii)(A\odot_H B)^c=A^c\oplus_H B^c$,

$(iii)(A \oplus_H B)^c \le A^c \oplus_H B^c,$

$(iv)(A \odot_H B)^c \ge A^c \odot_H B^c.$

Proof:

$$(i) A^c \odot_H B^c = \left(\left\langle \frac{a_{ij}^{'} b_{ij}^{'}}{a_{ij}^{'} + b_{ij}^{'} - a_{ij}^{'} b_{ij}^{'}}, \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}} \right\rangle \right)$$

$$= (A \oplus_H B)^c$$

$$(ii) A^c \oplus_H B^c = \left(\left\langle \frac{a_{ij}^{'} + b_{ij}^{'} - 2a_{ij}^{'} b_{ij}^{'}}{1 - a_{ij}^{'} b_{ij}^{'}}, \frac{a_{ij} b_{ij}}{a_{ij} + b_{ij} - a_{ij} b_{ij}} \right\rangle \right)$$

$$= (A \odot B)^c$$

$$(iii)(A \oplus_H B)^c = \left(\left\langle \frac{a_{ij}^{'} b_{ij}^{'}}{a_{ij}^{'} + b_{ij}^{'} - a_{ij}^{'} b_{ij}^{'}}, \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}} \right\rangle \right)$$

$$A^c \oplus_H B^c = \left(\left\langle \frac{a_{ij}^{'} + b_{ij}^{'} - 2a_{ij}^{'} b_{ij}^{'}}{1 - a_{ij}^{'} b_{ij}^{'}}, \frac{a_{ij} b_{ij}}{a_{ij} + b_{ij} - a_{ij} b_{ij}} \right\rangle \right)$$

By Lemma 3.2, $\frac{a_{ij}^{'} b_{ij}^{'}}{a_{ij}^{'} + b_{ij}^{'} - a_{ij}^{'} b_{ij}^{'}} \le \frac{a_{ij}^{'} + b_{ij}^{'} - 2a_{ij}^{'} b_{ij}^{'}}{1 - a_{ij}^{'} b_{ij}^{'}}$ and

$\frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}} \ge \frac{a_{ij} b_{ij}}{a_{ij} + b_{ij} - a_{ij} b_{ij}}$ for all i, j

Hence $(A \oplus_H B)^c \le A^c \oplus_H B^c.$

(iv) It can be proved similarly.

5. Necessity and possibility operators

The necessity and possibility operators for an IFMs are defined [14] Pal. In this section, we studied the algebraic properties of Necessity and Possibility Operators of IFMs with respect to hamacher sum and hamacher product.

Theorem 5.1. For $A, B \in F_{mn}$,

$(i) \Box(A \oplus_H B) = \Box A \oplus_H \Box B,$

$(ii) \Diamond(A \oplus_H B) = \Diamond A \oplus_H \Diamond B.$

Proof:

$$(i) \Box(A \oplus_H B) = \left(\left\langle \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}}, 1 - \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}} \right\rangle \right)$$

$$=\left(\left\langle \frac{a_{ij}+b_{ij}-2a_{ij}b_{ij}}{1-a_{ij}b_{ij}}, \frac{(1-a_{ij})(1-b_{ij})}{1-a_{ij}b_{ij}} \right\rangle\right)$$

$$=\Box A \oplus_H \Box B.$$

(ii) It can be proved similarly.

Theorem 5.2. For $A, B \in F_{mn}$,

$(i) \Box(A \odot_H B) = \Box A \odot_H \Box B,$

$(ii) \Diamond(A \odot_H B) = \Diamond A \odot_H \Diamond B.$

Proof:

$$(i) \Box(A \odot_H B) = \left(\left\langle \frac{a_{ij}b_{ij}}{a_{ij}+b_{ij}-a_{ij}b_{ij}}, 1-\frac{a_{ij}b_{ij}}{a_{ij}+b_{ij}-a_{ij}b_{ij}} \right\rangle\right)$$

$$=\left(\left\langle \frac{a_{ij}b_{ij}}{a_{ij}+b_{ij}-a_{ij}b_{ij}}, \frac{a_{ij}+b_{ij}-2a_{ij}b_{ij}}{a_{ij}+b_{ij}-a_{ij}b_{ij}} \right\rangle\right)$$

$$=\Box A \odot_H \Box B.$$

(ii) It can be proved similarly.

Theorem 5.3. For $A, B \in F_{mn}$,

$(i) \left(\Box(A^c \oplus_H B^c)\right)^c = \Diamond A \odot_H \Diamond B,$

$(ii) \left(\Box(A^c \odot_H B^c)\right)^c = \Diamond A \oplus_H \Diamond B.$

Proof:

$$(i) \Box(A^c \oplus_H B^c) = \left(\left\langle \frac{a'_{ij}+b'_{ij}-2a'_{ij}b'_{ij}}{1-a'_{ij}b'_{ij}}, 1-\frac{a'_{ij}+b'_{ij}-2a'_{ij}b'_{ij}}{1-a'_{ij}b'_{ij}} \right\rangle\right)$$

$$=\left(\left\langle \frac{a'_{ij}+b'_{ij}-2a'_{ij}b'_{ij}}{1-a'_{ij}b'_{ij}}, \frac{(1-a'_{ij})(1-b'_{ij})}{1-a'_{ij}b'_{ij}} \right\rangle\right)$$

$$\left(\Box(A^c \oplus_H B^c)\right)^c = \left(\left\langle \frac{(1-a'_{ij})(1-b'_{ij})}{1-a'_{ij}b'_{ij}}, \frac{a'_{ij}+b'_{ij}-2a'_{ij}b'_{ij}}{1-a'_{ij}b'_{ij}} \right\rangle\right)$$

$$= \Diamond A \odot_H \Diamond B$$

$$(ii) \Box(A^c \odot_H B^c) = \left(\left\langle \frac{a'_{ij}b'_{ij}}{a'_{ij}+b'_{ij}-a'_{ij}b'_{ij}}, 1-\frac{a'_{ij}b'_{ij}}{a'_{ij}+b'_{ij}-a'_{ij}b'_{ij}} \right\rangle\right)$$

$$= \left(\left\langle \frac{a'_{ij} b'_{ij}}{a'_{ij} + b'_{ij} - a'_{ij} b'_{ij}}, \frac{a'_{ij} + b'_{ij} - 2a'_{ij} b'_{ij}}{a'_{ij} + b'_{ij} - a'_{ij} b'_{ij}} \right\rangle \right)$$

$$\left(\Box(A^c \odot_H B^c) \right)^c = \left(\left\langle \frac{a'_{ij} + b'_{ij} - 2a'_{ij} b'_{ij}}{a'_{ij} + b'_{ij} - a'_{ij} b'_{ij}}, \frac{a'_{ij} b'_{ij}}{a'_{ij} + b'_{ij} - a'_{ij} b'_{ij}} \right\rangle \right)$$

$$= \Diamond A \oplus_H \Diamond B.$$

Theorem 5.4. For $A, B \in F_{mn}$,

$(i) \left(\Diamond(A^c \oplus_H B^c) \right)^c = \Box A \odot_H \Box B,$

$(ii) \left(\Diamond(A^c \odot_H B^c) \right)^c = \Box A \oplus_H \Box B.$

Proof:

$$(i) \Diamond(A^c \oplus_H B^c) = \left(\left\langle 1 - \frac{a_{ij} b_{ij}}{a_{ij} + b_{ij} - a_{ij} b_{ij}}, \frac{a_{ij} b_{ij}}{a_{ij} + b_{ij} - a_{ij} b_{ij}} \right\rangle \right)$$

$$\left(\left\langle \Diamond(A^c \oplus_H B^c) \right\rangle \right)^c = \left(\left\langle \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{a_{ij} + b_{ij} - a_{ij} b_{ij}}, \frac{a_{ij} b_{ij}}{a_{ij} + b_{ij} - a_{ij} b_{ij}} \right\rangle \right)$$

$$= \Box A \odot_H \Box B$$

$$(ii) \Diamond(A^c \odot_H B^c) = \left(\left\langle 1 - \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}}, \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}} \right\rangle \right)$$

$$= \left(\left\langle \frac{(1 - a_{ij})(1 - b_{ij})}{1 - a_{ij} b_{ij}}, \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}} \right\rangle \right)$$

$$\left(\left\langle \Diamond(A^c \odot_H B^c) \right\rangle \right)^c = \left(\left\langle \frac{a_{ij} + b_{ij} - 2a_{ij} b_{ij}}{1 - a_{ij} b_{ij}}, \frac{(1 - a_{ij})(1 - b_{ij})}{1 - a_{ij} b_{ij}} \right\rangle \right)$$

$$= \Box A \oplus_H \Box B.$$

6. Conclusion

In this article, Hamacher operations of intuitionistic fuzzy matrices are defined and some properties are proved. The set of all Hamacher sum and Hamacher product of intuitionistic fuzzy matrices form a commutative monoids with respect to these operations and also proved necessity and possibility operators of intuitionistic fuzzy matrices.

Acknowledgements. The author is very grateful and would like to express his thanks to the referees for their valuable comments and suggestions provided in revising and improve the presentation of the paper

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