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Results on Generalised *p*-closed Sets

Vinitha.T **[1,2] and *T.P.Johnson*** [3]

[1]Department of Mathematics
Cochin University of Science and Technology
Cochin-22, Kerala, India
[2]Department of Mathematics,
Al-Ameen College, Edathala, Kerala, India
[3]Applied Sciences and Humanities Division, School of Engineering, Kerala, India
email: vinitha01@gmail.com

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Abstract. In this paper, we analyse some equivalent conditions for a set to be generalised p-closed, generalised p-open and a space to be p-$T_{1/2}$. Also the notion of generalised p-continuous function is initiated. We proved that corresponding to any topology on any arbitrary set X, there always exist a finer p-$T_{1/2}$ topology on*X*.

Keywords: g-p.closed, g-p.open, gp-continuous, p-$T_{1/2}$

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1. Introduction

The notion of irreducible open set was first introduced by Gierz et al. in [3]. A lot of research work has been executed using the concept of irreducible open sets. Irreducible open set becomes useful tools in various domains like frames and locales, lattice theory etc [5,6,8]. The concept of generalised closed sets was initiated by Levine in [7]. A set A is called generalised closed if whenever A is contained in an open set its closure is also contained in that open set. In [15] we studied prime open set in a topological space inspired by the definition of G.Gierz and we introduce *generalised p-closed* sets shortly called *g-p.closed* sets. Also we studied some separation axioms using p-open sets and g-p.open sets and proved that all this separation axioms are equivalent in a prime symmetric space [15]. Again in [14] we use p-open sets to study some basic topological notions like continuity, compactness etc and thus introduce p-continuity, p-compactness etc.

In section 2 of present work we recall some of the basic definitions and results we obtained using the concept of p-open sets. In section 3 our main aim is to study the behaviour of generalised p-closed sets and p-$T_{1/2}$ spaces. We obtained equivalent condition for a set to be g-p.closed and also we introduce generalised p-continuous functions. We studied the behaviour of g-p.closed sets under p-continuous, p-closed functions. Then in section 4 we proved equivalent condition for a space to be p-$T_{1/2}$ and we obtain that all p-$T_{1/2}$ spaces are $T_{1/2}$. Moreover we proved that corresponding to any topological space there exist a finer p-$T_{1/2}$ topological space.

2. Preliminaries

Definition 2.1.[15] Let (X,T) be any arbitrary topological space. The open sets in T forms a complete lattice with smallest element 0 and largest element 1 ; where $0=\phi$ and $1=X$. We define an open set G $\neq$ 1 in T to be *prime open set* if $H \cap K \subseteq G \Rightarrow H \subseteq G$ or $K \subseteq G$; where H, K are open sets in T such that $H \cap K \neq \phi$. Clearly 0 and 1 are prime in T. Prime open sets are denoted by *p-open*sets. Complements of p-open sets are called *p-closed sets.*

Definition 2.2. [15] Let (X,T) be a topological space and let $A \subseteq X$, then the *p-closure* of A with respect to T is defined as the minimal p-closed super set of A in X and is denoted as p-$cl(A)$.

Proposition 2.3.[15] Let (X,T) be a topological space, then for every p-open set $A \subseteq X$ there always exists a unique p-closed set containing A.

Definition 2.4. [15] Let (X,T) be a topological space and let $A \subseteq X$, then the *p-interior* of A with respect to T is defined as the maximal p-open subset of A in X and is denoted as p-int(A).

Proposition 2.5.[15] Let (X,T) be a topological space, then for every p-closed set $A \subseteq X$ there always exists a unique p-open set contained in A.

Theorem 2.6.[15] Let (X,T) be a topological space and $Y \subseteq X$. U p-open in X implies $U \cap Y$ p-open in Y.

Proposition 2.7.[15] Let (X,T) be a topological space, $A \subseteq X$ and $x \in X$. Then $x \in$ p-$cl(A)$ if and only if every p-open set containing 'x' intersects A.

Definition 2.8.[15] Let (X,T) be a topological space and $A \subseteq X$ then A is said to be *generalised p-closed* shortly *g-p.closed* if p- $cl(A) \subseteq O$ whenever $A \subseteq O$; O p-open in X .

Theorem 2.9. [15] Let (X,T) be a topological space and $A \subseteq X$ then A is generalised p-closed if and only if p-$cl(A)-A$ contains no non-empty p-closed set.

Theorem 2.10.[15] Let (X,T) be a topological space and $A \subseteq X$ be such that A is generalised p-closed then A is g-closed.

Theorem 2.11. [15] Let (X,T) be a topological space. If A is g-p.closed and $A \subseteq B \subseteq$ p-$cl(A)$ then B is g-p.closed

Definition 2.12. [15] Let (X,T) be any topological space then X is *p*-$T_{1/2}$ if every g-p.closed set is p-closed.

Theorem 2.13. [15] Let (X,T) be a p-T_1 topological space then it is always p-$T_{1/2}$.

Theorem 2.14. [15] Any topological space is T_1 if and only if it is p-T_1.

Definition 2.14. [14] Let $(X,T),(Y,T^{'})$ be two topological spaces and let $f:(X,T) \to (Y,T^{'})$ be a mapping between this two topological spaces. f is called *p-continuous* if the inverse image of p-open sets in $T^{'}$ are p-open in T.

Definition 2.16. [14] Let $(X,T),(Y,T^{'})$ be two topological spaces and $f:(X,T) \to (Y,T^{'})$ be a mapping. f is said to be a *p-homeomorphism* if f is one-one, onto and both f, f^{-1} are p-continuous.

3.Characterisations of generalised p-closed sets, genaralised p-open sets and generalised p-continuous functions

Theorem 3.1. Let (X,T) be a topological space, then the following conditions are equivalent for any subset $A \subseteq X$

1. A is g-p.closed.
2. For each x belongs to p-$cl(A)$, p-$cl(\{x\}) \cap A \neq \phi$.
3. $B \subseteq$ p-$cl(A) - A$, $B \subseteq X$ implies $B = \phi$.

Proof: We proceed through the following steps :

Step 1: Proof of $(1) \Rightarrow (2)$

Suppose $x \in p\text{-}cl(A)$ and A is g-p.closed. To prove that p-$cl(\{x\}) \cap A \neq \phi$. On contradiction we assume that p-$cl(\{x\}) \cap A = \phi$ which implies $A \subseteq (p\text{-}cl(\{x\}))^c$. Since A is g-p.closed and (p-$cl(\{x\}))^c$ is p-open we obtain p-$cl(A) \subseteq (p\text{-}cl(\{x\}))^c$ implies $x \notin p\text{-}cl(A)$ which is not possible. Hence p-$cl(\{x\}) \cap A \neq \phi$ always for $x \in p\text{-}cl(A)$.

Step 2: To prove $(2) \Rightarrow (3)$

Assume that p-$cl(A) - A$ contains a non-empty p-closed set C and let $x \in C$; that is

$$x \in C \subseteq p\text{–}cl(A) - A \,(1)$$

(1) implies $x \in p\text{-}cl(A) \Rightarrow p\text{-}cl(\{x\}) \cap A \neq \phi$.

Thus $\phi \neq p\text{-}cl(\{x\}) \cap A$
$\subseteq C \cap A$
$\subseteq (p\text{-}cl(A) - A) \cap A$ by (1)
$= \phi$. Hence we obtain a contradiction and therefore the only possibility is $C = \phi$

Thus p-$cl(A) - A$ contains no non-empty p-closed set.

Step 3: $(3) \Rightarrow (1)$ holds by Theorem 2.9.

Hence by steps 1, 2 and 3 ; equivalent conditions for a set to be g-p.closed are verified.

Theorem 3.2. Let (X,T) be a topological space and let $A \subseteq X$, then A is g-p.closed if and only if $A = P - G$ where P is a p-closed subset of X and G is such that G contains no non-empty p-closed subset of X .

Proof: Assume that A is g-p.closed to prove that $A = P - G$ where P is p-closed and G is such that G contains no non-empty p-closed subset of X . Now take $P = p$-$cl(A)$ and $G = p\text{-}cl(A) - A$ then P is a p-closed set and since A is g-p.closed ; G contains no non-empty p-closed set by last theorem . Thus P and G are the required sets. Now consider $P - G = p\text{-}cl(A) - (p\text{-}cl(A) - A)$ =A ; that is A is of the required form and hence the necessary part is proved.

For sufficiency part let $A \subseteq X$ and A be of the form $A = P - G$ where P is p-closed and G contains no non empty p-closed set. We have to prove that A is g-p.closed. Let $A \subseteq O$ where 'O' is a p-open subset of X to prove that p-$cl(A) \subseteq O$. P and O^c are p-closed subsets of X hence $P \cap O^c$ is a closed subset of X and moreover $P \cap O^c$ is a closed subset of G , then two cases arise either $P \cap O^c$ is p-closed or it is not p-closed but closed. If the second case occurs, since the existence of atoms which are p-closed is inevitable for a topological space ; $P \cap O^c$ contains at least atoms in T . Thus in both cases $P \cap O^c$ contains a non-empty p-closed subset but as $P \cap O^c \subseteq G$; G contains a non-empty p-closed set if $P \cap O^c$ contains. Hence the only possibility is that $P \cap O^c = \phi$ which implies $P \subseteq O$. But $A \subseteq P \Rightarrow p$ - $cl(A) \subseteq P \Rightarrow p - cl(A) \subseteq O$ and hence A is g-p.closed.

Definition 3.3. A set $A \subseteq X$ in a topological space (X,T) is said to be *generalised p-open* shortly *g-p.open* if and only if A^c is g-p.closed.

Theorem 3.4. Let (X,T) be a topological space and $A \subseteq X$ be a g-p.open set then it is g-open.

Proof: Given that A is g-p.open $\Rightarrow A^c$ is g-p.closed , but then A^c is g-closed by Theorem 2.10. Thus A is g-open.

Remark 3.5. Converse of above theorem is not true. Consider $X = \{x, y, z\}$ with discrete topology. $A = X - \{x, y\}$ is g-open but not g-p.open.

Theorem 3.6. Let (X,T) be a topological space and let $A\subseteq X$. Then A is g-p.open if and only if $F\subseteq p\text{-}int(A)$ whenever F is p-closed and $F\subseteq A$.

Proof: Assume that A is g-p.open which implies A^c is g-p.closed

$\Rightarrow p\text{-}cl(A^c)\subseteq O$ whenever $A^c\subseteq O$ and O is p-open.

$\Rightarrow O^c\subseteq[p\text{-}cl(A^c)]^c$ whenever $A^c\subseteq O$ and O is p-open.

$\Rightarrow O^c\subseteq p\text{-}int(A)$ whenever $A^c\subseteq O$ and O is p-open. By taking $F=O^c$ as the p-closed set , the necessary part is proved.

Conversely we assume that F is p-closed and

$F\subseteq p\text{-}int(A)$ whenever $F\subseteq A$

$\Rightarrow[p\text{-}int(A)]^c\subseteq F^c$ whenever $A^c\subseteq F^c$

$\Rightarrow A^c$ is g-p.closed. Thus A is g-p.open.

Theorem 3.7. Let (X,T) be a topological space and let $A\subseteq X$, then A is g-p.open if and only if $O=X$ whenever O is p-open and p-$int(A)\cup A^c\subseteq O$.

Proof: Suppose A is g-p.open and p-$int(A)\cup A^c\subseteq O$ whenever O is p-open

$\Rightarrow O^c\subseteq[p\text{-}int(A)\cup A^c]^c$

$=(p\text{-}int(A))^c\cap A=p\text{-}cl(A^c)-A^c.$

Hence p-$cl(A^c)-A^c$ contains a non-empty p-closed set but A^c is g-p.closed and thus $O^c=\phi\Rightarrow O=X$.

For sufficiency part assume that F is a p-closed set and $F\subseteq A$. It is enough to prove that $F\subseteq p\text{-}int(A)$ for showing A is g-p.open. Consider p-$int(A)\cup A^c\subseteq p$-$int(A)\cup F^c$. Clearly p-$int(A)\cup F^c$ is open , then there arise two cases :

1. If p-$int(A)\cup F^c$ is prime then by assumption p-$int(A)\cup F^c=X$ and hence $F\subseteq p\text{-}int(A)$ which implies A is g-p.open.
2. If p-$int(A)\cup F^c$ is not prime then there exists two open sets G_1 and G_2 containing p-$int(A)\cup F^c$. Now if at least one of G_1 or G_2 is prime then by assumption the corresponding set becomes equal to X which is not possible by definition of prime. If both G_1 and G_2 are not prime then again there exists G_3 and G_4 containing the corresponding non-prime open set and again by the same reasoning as above that is not possible. Continuing this argument we reach the conclusion that whenever there exists open set containing p- $int(A)\cup F^c$ which is not prime, that will lead to a contradiction. Hence the only possibility is that p-$int(A)\cup F^c$ is prime and hence the result follows from case 1.

Theorem 3.8. Let (X,T) be a topological space and $A\subseteq X$. If A is g-p.closed then p-$cl(A)-A$ is g-p.open.

Proof: Assume that A is g-p.closed to prove that p-$cl(A)-A$ is g-p.open. That is to prove that $F \subseteq p\text{-}cl(A)-A \Rightarrow F \subseteq p\text{-}int(p\text{-}cl(A)-A)$ whenever F is p-closed. But $F \subseteq p\text{-}cl(A)-A$ implies $F=\phi$, since A is g-p.closed and F is p-closed . Hence result is trivial.

Remark 3.9. Converse of above theorem not true. For example $X=\{x_1,x_2,x_3\}$ and let the topology on it be the discrete topology. Let $A=\{x_1,x_2\}$, clearly A is not g-p.closed but p-$cl(A)-A$ is g-p.open.

Proposition 3.10. Let (X,T) be a topological space and $A,B \subseteq X$. If p-$int(A) \subseteq B \subseteq A$ and A is g-p.open then B is g-p.open.

Proof: Given that p- $int(A) \subseteq B \subseteq A \Rightarrow A^c \subseteq B^c \subseteq (p\text{ - }int(A))^c \Rightarrow A^c \subseteq B^c \subseteq p$ - $cl(A^c)$. A^c is g-p.closed $\Rightarrow B^c$ is g-p.closed by Theorem 2.11. Hence B is g-p.open.

Definition 3.11. Let (X,T), (Y,T') be a mapping between two topological spaces X and Y. Then f is said to be p-closed if every p-closed set in X is mapped on to p-closed set in Y.

Theorem 3.12. Let $(X,T),(Y,T')$ be two topological spaces. If A is a g-p.closed subset of X and $f: X \rightarrow Y$ be a p-continuous and p-closed function, then $f(A)$ is g-p.closed in Y.

Proof: Assume that $f(A) \subseteq O'$ where O' p-open in Y which implies $A \subseteq f^{-1}(O')$. Since f is p-continuous and O' is p-open in Y, $f^{-1}(O')$ is p-open in X and again since A is g-p.closed, $p\text{–}cl(A) \subseteq f^{-1}(O')$ implies

$$f(p\text{–}cl(A)) \subseteq O' \quad (2)$$

but $f(p\text{-}cl(A))$ is a p-closed set and for any set $A \subseteq X$, $A \subseteq p\text{-}cl(A)$ which implies p-$cl(f(A)) \subseteq p\text{-}cl(f(p\text{-}cl(A))) = f(p\text{-}cl(A)) \subseteq O'$ by (2)

$\Rightarrow f(A)$ is g-p.closed.

Theorem 3.13. Let (X,T), (Y,T') be any two topological spaces and $f:(X,T) \rightarrow (Y,T')$ be a p-continuous, p-closed mapping. If B is a g-p.closed subset of Y then $f^{-1}(B)$ is a g-p.closed subset of X.

Proof: Given B is a g-p.closed subset of Y we have to prove that $f^{-1}(B)$ is g-p.closed in X, that is whenever $f^{-1}(B) \subseteq O$ where O is a p-open set in X we have to prove that p-$cl(f^{-1}(B)) \subseteq O$. For that it is enough to prove that p-$cl(f^{-1}(B)) \cap O^c = \phi$.

But $f(p\text{-}cl(f^{-1}(B))\cap O^c) \subseteq p\text{-}cl(B)-B$. Since B is g-p.closed the only possibility is that $f(p\text{-}cl(f^{-1}(B))\cap O^c)=\phi$ which implies p- $cl(f^{-1}(B))\subseteq O$ whenever $f^{-1}(B)\subseteq O$. Hence $f^{-1}(B)$ is g-p.closed.

Definition 3.14. A map $f: X \to Y$ from a topological space X to another topological space Y is called *generalised p-continuous* shortly *gp-continuous* if inverse image of every p-closed set in Y is g-p.closed in X.

Remark 3.15. Let $f: X \to Y$ from a topological space X to another topological space Y be a p-continuous function then it is also gp-continuous.

Example 3.16. Let R be the real line and let I be the identity mapping from the topological space R with co finite topology to the topological space R with usual topology. Then I is gp-continuous but not g-continuous.

Example 3.17. Let $X=\{a,b,c,d\}$ and let I be the identity mapping from (X,D) to (X,T) where D is the discrete topology on X and
$T=\{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$.
Then I is g-continuous but not gp-continuous.

Remark 3.18. The concepts of g-continuity and gp-continuity are independent of each other as the above two examples illustrates.

Remark 3.19. When the domain space is p-$T_{1/2}$, the concepts of p-continuity and gp-continuity coincides.

Theorem 3.20. Let (X,T), (Y,T') be any two topological spaces and $f:(X,T)\to(Y,T')$ be a mapping between the two topological spaces. Then the following conditions are equivalent :

1. f is gp-continuous.
2. Inverse image of every p-open set in Y is g-p.open in X.

Proof: Assume that $f: X \to Y$ is gp-continuous and let G be a p-open set in *Y*, then $Y-G$ is p-closed set in Y. Since f is gp-continuous, $f^{-1}(Y-G)$ is g-p.closed in X. Trivially $f^{-1}(Y-G)=X-f^{-1}(G)$. $Y-G$ is p-closed in Y which implies $f^{-1}(Y-G)$ is g-p.closed in X. Hence $X-f^{-1}(G)$ is g-p.closed in X and thus $f^{-1}(G)$ is g-p.open in X. Conversely we assume that inverse image of every p-open set in Y is g-p.open in X. To prove that f is gp-continuous. Let H be a p-closed set in Y, then $Y-H$ is p-open in Y which implies $f^{-1}(Y-H)$ is g-p.open in X. But $f^{-1}(Y-H)=X-f^{-1}(H)$; which implies $f^{-1}(H)$ is g-p.closed in X. Thus f is g-p.continuous.

Definition 3.21. Let (X,T) be a topological space and let $A \subseteq X$ then *generalised p-closure* of A is defined as the intersection of all g-p.closed supersets of A and is denoted as *g-* $p.cl(A)$.

Remark 3.22. Since all p-closed sets are g-p.closed ; g- $p.cl(A) \subseteq p\text{-}cl(A)$ for any subset $A \subseteq X$.
Example 3.23. Let $X = \{a,b,c,d\}$ and $T = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$ be the topology on X . Consider $A = \{c,d\}$ then p- $cl(A) = \{d,b,c\}$ and $\{a,d,c\}$and g- $p.cl(A) = A$ itself.

Theorem 3.24. If $f:(X,T) \to (Y,T')$ is a gp-continuous function between the topological spaces (X,T) and (Y,T') then $f(g\text{ - }p.cl(A)) \subseteq p\text{-}cl(f(A))$ for every subset $A \subseteq X$.
Proof: Given f is gp-continuous. Let $A \subseteq X$ to prove that $f(g\text{ - }p.cl(A)) \subseteq p$ - $cl(f(A))$. Consider p- $cl(A)$ it is a p-closed set in X and also p- $cl(f(A))$ is a p-closed set in Y. Since f is gp-continuous $f^{-1}(p\text{-}cl(f(A)))$ is a gp-closed set in X . Clearly $A \subseteq f^{-1}(p\text{ - }cl(f(A)))$ which implies g- $p.cl(A) \subseteq f^{-1}(p\text{ - }cl(f(A)))$ which in turn implies $f(g\text{ - }p.cl(A)) \subseteq p\text{-}cl(f(A))$.

Remark 3.25. Converse of above proposition need not be true ; for example let $X = Y = \{1,2,3\}$ also let $T = \{X,\phi,\{1\}\}$, $T' = \{Y,\phi,\{1,3\}\}$ be topologies on X and Y respectively. Define $f:(X,T) \to (Y,T')$ by $f(1) = 2, f(2) = 1$ and $f(3) = 3$. Condition of above theorem is satisfied here but the function is not gp-continuous.

Theorem 3.26. Let (X,T) , (Y,T') be any two topological spaces and $f:(X,T) \to (Y,T')$ be a mapping between the two topological spaces. Then the following conditions are equivalent :

1. Corresponding to each point $x \in X$ and each p-open set V containing $f(x)$ there exists a g-p.open set U containing 'x' such that $f(U) \subseteq V$
2. For every $A \subseteq X$; $f(g\text{ - }p.cl(A)) \subseteq p\text{-}cl(f(A))$ holds.

Proof: First we will prove (1) implies (2). Let $y \in f(g\text{ - }p.cl(A))$. We have to prove that $y \in p\text{-}cl(f(A))$. Let V be a p-open set containing 'y' then there exists a point $x \in X$ and a g-p.open set U containing 'x' such that $f(x) = y$ and $f(U) \subseteq V$ by assumption.

$y \in f(g\text{ - }p.cl(A))$
$\Rightarrow f^{-1}(y) \in g\text{ - }p.cl(A)$
$\Rightarrow x \in g\text{ - }p.cl(A)$.

Since U is a p-open set containing ‘x’ ; $U \cap A \neq \phi \Rightarrow f(U) \cap f(A) \neq \phi$ which in turn implies $V \cap f(A) \neq \phi$ since $f(U) \subseteq V$. Thus $V \cap f(A) \neq \phi$ for every p-open set containing ‘y’. Hence $y \in p\text{-}cl(f(A))$ by Proposition 2.7 and thus $f(g\text{-}p.cl(A)) \subseteq p\text{-}cl(f(A))$.

Next to prove $(2) \Rightarrow (1)$. Assume that $\forall A \subseteq X$; $f(g\text{-}p.cl(A)) \subseteq p\text{-}cl(f(A))$. Also let $x \in X$ and V be a p-open set containing $f(x)$. Take $A = f^{-1}(V^c)$ then if $x \in A$, $f(x) \in f(A) = V^c$ which is not possible since V is a p-open set containing $f(x)$. Hence the only possibility is that $x \notin A$.

Now consider

$$\begin{aligned} \text{g-}p.cl(A) &\subseteq f^{-1}(f(g\text{-}p.cl(A))) \\ &\subseteq f^{-1}(p\text{-}cl(f(A))) \\ &= f^{-1}(p\text{-}cl(V^c)) \\ &= f^{-1}(V^c) = A \end{aligned}$$

and then the only possibility is that g-$p.cl(A) = A$. Since $x \notin A$, $x \notin g\text{-}p.cl(A)$ which implies there exists a g-p.open set U containing ‘x’ such that $U \cap A = \phi$ which implies $U \subseteq A^c$ and hence $f(U) \subseteq f(A^c) \subseteq V$ that is $f(U) \subseteq V$. Hence (1) is proved.

Theorem 3.27. Let X, Y, Z be any three topological spaces, moreover Y be a p-$T_{1/2}$ space. Also let $f : X \to Y$ and $g : Y \to Z$ be gp-continuous. Then $gof : X \to Z$ is also gp-continuous.

Proof: We have to prove that $gof : X \to Z$ is gp-continuous ; that is to prove that inverse image of p-closed set in Z is g-p.closed in X. Let H be a p-closed set in Z then $g^{-1}(H)$ is g-p.closed in Y and since Y is p-$T_{1/2}$, $g^{-1}(H)$ is p-closed in Y which implies $f^{-1}(g^{-1}(H))$ is g-p.closed in X provided H is p-closed in Z. Hence gof is gp-continuous.

Example 3.28. Let $X = Y = Z = \{1,2,3\}$ also $T = \{X, \phi, \{1,2\}\}$, $T' = \{Y, \phi, \{1\}, \{2,3\}\}$ and $T'' = \{Z, \phi, \{1,3\}\}$. Define $f : (X,T) \to (Y,T')$ by $f(1) = 3, f(2) = 2, f(3) = 3$. Clearly both f and g are gp-continuous but gof is not gp-continuous.

4.More on p-$T_{1/2}$ spaces

Theorem 4.1. A topological space (X,T) is p-$T_{1/2}$ if and only if each singleton subset is either p-open or p-closed .

Proof: Suppose X is p-$T_{1/2}$ and let $x \in X$. To prove that $\{x\}$ is p-open or p-closed. Assume that $\{x\}$ is not p-closed , then $X - \{x\}$ is not p-open and the only p-open set containing it is X which implies $X - \{x\}$ is g-p.closed and since X is p-$T_{1/2}$,

$X-\{x\}$ is p-closed which implies $\{x\}$ is p-open. Hence $\{x\}$ is either p-open or p-closed. For sufficiency we assume that $\{x\}$ is either p-open or p-closed. To prove that X is p-$T_{1/2}$. Let A be any g-p.closed subset of X we have to show that A is p-closed that is to prove that p-$cl(A)=A$ itself. Let $x\in p\text{-}cl(A)$. Given that $\{x\}$ is p-open or p-closed. If $\{x\}$ is p-open, $\{x\}\cap A\neq\phi$ since $x\in p\text{-}cl(A)$. Hence $x\in A$ and since 'x' is arbitrary p-$cl(A)\subseteq A$. Now assume $\{x\}$ is p-closed. Since $x\in p\text{-}cl(A)$ and A is g-p.closed ; p-$cl(\{x\})\cap A\neq\phi$ by Theorem 3.1 which implies $\{x\}\cap A\neq\phi$ which in turn implies $x\in A$. Thus p-$cl(A)\subseteq A$. Hence in both cases p-$cl(A)\subseteq A$ implies A is p-closed. Thus X is p-$T_{1/2}$ since A is arbitrary.

Cocollary 4.2. Any p-$T_{1/2}$ topological space is also $T_{1/2}$.
Proof: Since p-closed sets are always closed, the proof is trivial by last theorem.

Theorem 4.3. If (X,T) is a p-$T_{1/2}$ topological space and $Y\subseteq X$, then (Y,T_Y) is also p-$T_{1/2}$.
Proof: Let $y\in Y\subseteq X$. Consider $\{y\}$, it is p-open or p-closed in X since X is p-$T_{1/2}$ then $\{y\}$ is p-open or p-closed in Y by Theorem 4.1 .

Theorem 4.4. Let (X,T) be a p- $T_{1/2}$ topological space and $f:X\rightarrow Y$ is p-continuous, p-closed and onto. Then Y is p-$T_{1/2}$.
Proof: Let $B\subseteq Y$ be a g-p.closed set then by Theorem : 3.13 $f^{-1}(B)$ is g-p.closed and since X is p-$T_{1/2}$, $f^{-1}(B)$ is p-closed. Hence $B=f(f^{-1}(B))$ is p-closed in Y and thus Y is p-$T_{1/2}$.

Corollary 4.5. p-homeomorphic image of p-$T_{1/2}$ space is p-$T_{1/2}$.
Proof: Proof is trivial by last theorem.

Definition 4.6. [16] Let $\{(X_i,T_i)/i\in I\}$ be a collection of topological spaces and let $(X=\prod X_i,T)$ be their product space. Then the p-open sets in T are sets of the form $\prod U_i$; where $U_i=X_i$ for infinitely many i's and other U_i's are all prime open in T_i.

Theorem 4.7. Let $\{(X_\alpha,T_\alpha):\alpha\in I\}$ be a collection of topological spaces and let $X=\prod X_\alpha$ be their product topological space. If X is p-$T_{1/2}$ then X_α is p-$T_{1/2}$ for every $\alpha\in I$.
Proof: X contains a subspace p-homeomorphic to X_α and by using Theorem 4.4 and

Corollary 4.5 X_α is p-$T_{1/2}$.

Theorem 4.8. Let $\{(X_\alpha, T_\alpha) : \alpha \in I\}$ be a collection of topological spaces and let $X = \prod X_\alpha$ be their product topological space. Then X is p-$T_{1/2}$ if and only if X is p-T_1.

Proof: Sufficiency part is trivial since p-$T_1 \Leftrightarrow T_1 \Rightarrow p\text{-}T_{1/2}$ by Theorem 2.13 and Theorem 2.15. For necessary part, consider $\{x\}$, it is not open in product space and hence not p-open in product topology and since X is p-$T_{1/2}$, $\{x\}$ is p-closed always for every 'x' which implies X is p-T_1.

Corollary 4.9. Let $X = \prod X_\alpha$. Then X is p-$T_{1/2}$ if and only if X_α is p-T_1 for every $\alpha \in I$.

Proof: Proof is trivial by last result and by p-$T_1 \Leftrightarrow T_1$.

Remark 4.10. p-$T_{1/2}$ is not an expansive property as the following example illustrates : Let $X = \{a,b,c\}$ and $T = \{X, \phi, \{a,b\}\}$, $U = \{X, \phi, \{a\}, \{a,b\}, \{a,c\}\}$ be two topologies on X . Clearly $T \subset U$ and (X,T) is p-$T_{1/2}$ but (X,U) is not.

Remark 4.11. p-$T_{1/2}$ is not a contractive property for example let $X = \{a,b\}$ and $T = \{X, \phi, \{a\}\}$, $U = \{X, \phi, \{b\}\}$ be topologies on X . Then both T, U is p-$T_{1/2}$ but $T \cap U$ is not p-$T_{1/2}$.

Theorem 4.12. Let (X_α, T_α) be a collection of p-$T_{1/2}$ topological spaces and if $\{T_\alpha / \alpha \in I\}$ forms a chain with inclusion as the order, then $(X, \cap\{T_\alpha / \alpha \in I\})$ is p-$T_{1/2}$.

Proof: We have to prove that $(X, \cap\{T_\alpha / \alpha \in I\})$ is p-$T_{1/2}$. Let $x \in X$ it is enough to prove that $\{x\}$ is either p-open or p-closed in $\cap\{T_\alpha / \alpha \in I\}$. Assume that $\{x\}$ not p-open in $\cap\{T_\alpha / \alpha \in I\}$. Then two cases arise :

1. $\{x\}$ not open in $\cap\{T_\alpha / \alpha \in I\}$.
2. $\{x\}$ open in $\cap\{T_\alpha / \alpha \in I\}$ but not prime in $\cap\{T_\alpha / \alpha \in I\}$.

If case 1 occurs, then there exists $\beta \in I$ such that $\{x\} \notin T_\beta$. Since T_β is p-$T_{1/2}$, $X - \{x\} \in T_\beta$ and is always prime. Now if $T_\beta \subseteq T_\alpha$, then $X - \{x\} \in T_\alpha$ and is always prime in T_α. And if $T_\beta \supseteq T_\alpha$, then if $X - \{x\}$ not p-open in T_α, then $\{x\} \in T_\alpha \subseteq T_\beta$ which implies $\{x\} \in T_\beta$ which is a contradiction. Hence in both cases , that is if $T_\beta \subseteq T_\alpha$ and $T_\beta \supseteq T_\alpha$; $X - \{x\}$ is p-open in T_α for all $\alpha \in I$. Thus $\{x\}$ becomes p-closed in $\cap\{T_\alpha / \alpha \in I\}$ and that implies $\cap\{T_\alpha / \alpha \in I\}$ is p-$T_{1/2}$.

If case 2 occurs, that is if $\{x\}$ open in $\cap\{T_\alpha / \alpha \in I\}$ but not prime in $\cap\{T_\alpha / \alpha \in I\}$. Then there exists U , $V \in \cap\{T_\alpha / \alpha \in I\}$ such that $U \cap V \subseteq \{x\}$ and $\{x\} \subset U$, $\{x\} \subset V$ which implies $\{x\}$, U , V open in T_α for every $\alpha \in I$ implies $\{x\}$ not prime in T_α for every $\alpha \in I$. But since each T_α is p-$T_{1/2}$ by Theorem 4.1 $X - \{x\}$ p-open in T_α for every $\alpha \in I$ and thus $\{x\}$ p-closed in $\cap\{T_\alpha / \alpha \in I\}$. Hence $(X, \cap\{T_\alpha / \alpha \in I\})$ is p-$T_{1/2}$.

Therefore in both cases $\{x\}$ is either p-open or p-closed in $\cap\{T_\alpha / \alpha \in I\}$ for each $x \in X$ and that implies the result.

Theorem 4.13. Let (X, τ) be any topology on X , then there exists a topology U on X such that

1. $\tau \subseteq U$.
2. (X, U) is p-$T_{1/2}$.
3. If (X, γ) is p-$T_{1/2}$ where (X, γ) is such that $\tau \subseteq \gamma \subseteq U$, then $\gamma = U$

Proof: Let $G = \{\tau_\alpha / \alpha \in I\}$ be the indexed family of p-$T_{1/2}$ topologies on X finer than τ . $G \neq \phi$ since G contains atleast the discrete topology. Consider a chain of subsets of G say $\{\tau_\alpha / \alpha \in J\}$ then $\cap\{\tau_\alpha / \alpha \in J\}$ is p-$T_{1/2}$ and $\tau \subseteq \cap\{\tau_\alpha / \alpha \in J\}$. But then $\cap\{\tau_\alpha / \alpha \in J\}$ belongs to G and by applying dual statement of Zorn's lemma it contains a minimal element U such that $\tau \subseteq U$ and U is p-$T_{1/2}$ and by minimality condition 3 is also satisfied. Hence the theorem is proved.

5. Conclusion

We have studied the behavior of p-open, g-p.open, g-p.closed sets etc under various mappings involving p-open , p-closed and g-p.closed sets. Also proved some equivalent conditions for p-$T_{1/2}$ spaces, gp-continuous functions, g-p.closed sets and obtained that being p-$T_{1/2}$ is preserved under p-homeomorphisms. Some more weaker separation axioms are yet to be analysed using p-open sets and it is our proposed future work.

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