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On Strong Topological Aspects in Uryson Spaces

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Abstract. This paper is multifold in structure. At first, we further study closure continuity of maps, a concept which was introduced in [2]. In specific, it is shown that restrictions of such maps to subspaces maintain closure continuity, but not conversely. Then, strongly open set topologies are defined. We found that, in general, this gives a weaker topology but not in regular spaces where, the topologies turn out to be the same. It is also evident that many traditional theorems hold for closure continuous maps acting on or into Uryson Spaces. Just to mention, we prove that if $f, g : X \to Y$ are closure continuous functions and Y is a Uryson Space, then: (i) the set $\{x \in X : f(x) = g(x)\}$ is strongly closed in X, (ii) The graph $G_f = \{(x, f(x)) : x \in X\}$ of f is a strongly closed set in $X \times Y$. We also prove that , When the space X is Uryson, and $f : X \to Y$ is closure continuous, then the set $E = \{x \in X : f(x) = x\}$ of all fixed points under f is strongly closed in X. Finally, we show that equivalent are: (i) X is a Uryson space (ii) Weak limits of filters in X ,when exist are unique (iii) Weak limits of nets in X ,when exist are unique. (iv)The diagonal $\Delta_X = \{(x, x) : x \in X\}$is a strongly closed set in the product space $X \times X$.

Keywords: Closure Continuous Map, strongly open set.

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1. Introduction

If one deals only with first countable spaces, it is possible to describe all topological occurrences in terms of sequences, instead of open sets or neighborhoods. For example, $x \in \overline{S}$ if and only if every neighborhood of x meets S, and (in first countable spaces) if and only if S contains a sequence which converges to x . In general, however, such descriptions are impossible. For an example, we can take any set S whose complement is countable in an uncountable co-countable space X , where any point in $X \setminus S$ will serve for x .

There is a sense in which these theories (of nets and filters) are equivalent. Each is adequate for topology in the same sense that sequences are not [1].

In [2], the authors were able to show that in regular spaces, weak convergence of filters is equivalent to their ordinary convergence, and in [3] the same thing was done for nets. Also in [3], equivalent characterizations of closure points and weak closure points of open sets were given in terms of weak convergence of both nets and filters. In here, we will mainly use the results of [2] and [3] as our main tools to get our results.

Remains to mention that the concept of closure continuous maps will be a major playing figure in this article, and just to facilitate reading, we through in a few terms and notes.

A function $f : X \to Y$ is said to be closure continuous at $x_\circ \in X$ if for every neighborhood V of $f(x_\circ)$ there is a neighborhood U of $x_\circ$ with $f(\overline{U}) \subseteq \overline{V}$. If this condition is satisfied at each point of X then f is called closure continuous on X [4]. It is not hard to show that continuous functions are closure continuous, but not conversely [2].

Finally, a space X is called a Uryson space if whenever $x_1 \neq x_2 \; in \; X$, there are open sets U and V in X containing x_1 and x_2, respectively, such that $\overline{U} \cap \overline{V} = \phi$ [5]. In the end, we would like to draw the reader's attention that in the present time, most people tend to embed their applications in generalized settings. The reason for that tendency could be due to the deep specifications and tight tracks of applications. For this we refer interested readers to compare with [6,7]. In specific, and for the general setting of Rough Set Theory, One may consult [8] .

In section (1), we give two expected theorems on closure continuous maps. In section (2) we introduce and characterize strongly open(and strongly closed) sets, whereas in section (3), we implant all this in Uryson space. In section (4), fixed point theory is slightly touched in Uryson spaces, One may want to compare [10]. In specific, the set of all fixed points in Uryson space under a closure continuous map is proved to be strongly closed. The conclusion of our article will be a characterizations of Uryson spaces.

2. Theorems on closure continuous maps

The proofs of the theorems of this section are straight forward. We begin with :

Theorem 2.1. If $f : X \to Y$ and $g : Y \to Z$ are closure continuous, then so is the composition $g \circ f : X \to Z$.

Proof: Let $x \in X$ and let V be a neighborhood of $g(f(x))$. By closure continuity of g choose a neighborhood W of $f(x)$ such that $g(\overline{w}) \subseteq \overline{V}$. Since f is closure continuous, there is a neighborhood U of x such that $f(\overline{U}) \subseteq \overline{W}$.

Thus, $g \circ f(\overline{U}) \subseteq g(\overline{W}) \subseteq \overline{V}$.This proves that $g \circ f$ is closure continuous.

The next proposition usually gets mixed up with its (untrue) converse.

Proposition 2.2. If $f : X \to Y$ is closure continuous and $A \subseteq X$, then $f_A : A \to Y$ is closure continuous, where f_A is the restriction of the function f to the set A.

Proof: Suppose f is closure continuous . Let $x \in X$ and let V be a neighborhood of $f(x)$. Since f is closure continuous, choose neighborhood U of x such that $f(\overline{U}) \subseteq \overline{V}$. Now, $A \cap U$ is neighborhood of x in A and

$$\begin{aligned}\overline{A \cap U}^{A} &= \overline{A \cap U} \cap A \\ &= \overline{A} \cap \overline{U} \cap A \\ &\subseteq \overline{U}.\end{aligned}$$

Therefore, $f_A\left(\overline{A \cap U}^{A}\right) = f\left(\overline{A \cap U}^{A}\right) = f\left(A \cap \overline{U}\right) \subseteq f\left(\overline{U}\right) \subseteq \overline{V}$.

Thus, f_A is closure continuous.

As we pointed out earlier, the converse of the foregoing proposition is false. The following example shows why.

Example 2.3. Take $X = Y = \Re\,(\text{the reals})$, $A = Q$ (*the rational numbers*) and for f , take the characteristic function of the rational numbers,

$$\chi_Q(x) = \begin{cases} 1 & \text{if } x \in Q \\ 0 & \text{otherwise} \end{cases}$$

f_A is continuous on A (hence is closure continuous), being the constant function. But as a function on R, f is not closure continuous anywhere on Q. To see this, let $x \in Q$ be arbitrary, and let $V = \left\{ r \in R : \frac{1}{2} < r < \frac{3}{2} \right\}$. Then, for any neighborhood U of x , $f(\overline{U}) = \{0,1\} \not\subset \overline{V} = \left\{ r \in R : \frac{1}{2} \le r \le \frac{3}{2} \right\}$.

3. Strongly open sets

A set A in a topological space X is said to be strongly open in X if for each $x \in A$ there is a neighborhood U of x such that $\overline{U} \subseteq A$. It is clear that strongly open sets are open but not conversely. For instance, proper open sets in infinite cofinite spaces cannot be strongly open.

A set B in a topological space X is said to be strongly closed in X if its complement is strongly open in X . We characterize strongly open sets as follows:

Proposition 3.1. A set A in a topological space X is strongly open in X if and only if $A \in \Im$ for every filter $\Im$ in X which converges weakly to $x \in A$.

Proof : Suppose A is strongly open in X and that $\Im$ is a filter in X such that $\Im \xrightarrow{w} x$, with $x \in A$. Since A is strongly open, there is a neighborhood U of x such that $\overline{U} \subseteq A$. This means that $\overline{U_x}$ (the system of closures of neighborhoods of x) contains A as an element. Since $\Im \ge \overline{U_x}$, $A \in \Im$.

Conversely, suppose A is not strongly open in X. Then there is an $x \in A$ such that, for every neighborhood U of x, $\overline{U} \not\subset A$. This implies that $A \notin \overline{U_x}$. But $\overline{U_x} \xrightarrow{w} x$. Thus, take $\overline{U_x}$ for $\Im$.

Corollary 3.2. A set A in a topological space X is strongly open in X if and only if whenever (x_λ) is a net in A which converges weakly to $x \in A$, then $x_\lambda \in A$ eventually.

Proof: Suppose A is strongly open in X. Let $x \in A$ and suppose that (x_λ) is a net in A which converges weakly to x. Then, for any neighborhood U of x, $x_\lambda \in \overline{U}$ eventually. Since A is strongly open, $\overline{U} \subseteq A$, and so $x_\lambda \in A$ eventually.

Conversely, Suppose that, whenever (x_λ) is a net in A which converges weakly to $x \in A$, then $x_\lambda \in A$ eventually. Now let $x \in A$ and suppose that $\Im$ is a filter in A which converges weakly to x. Let (x_λ) be the net in A generated by $\Im$. By [3], $x_\lambda \xrightarrow{w} x$.

This implies that $x_\lambda \in A$ eventually. Therefore, $A \in \Im$. By proposition (3.1), A is strongly open.

Just as anticipated, the collection of all strongly open sets makes up a coarser topology than the original one. Details will be next.

Theorem 3.3 : Let (X,T) be a topological space, and let T_S be the set of all strongly open sets in X. Then T_S is a topology on X and $T_S \subseteq T$.

Proof: Clearly, $\phi, X \in T_S$.

Let $\{A_\gamma : \gamma \in \Gamma\}$ be a collection of members of T_S, and let $x \in \bigcup_{\gamma\in\Gamma} A_\gamma$, then $x \in A_\beta$ for some $\beta \in \Gamma$. Since A_β is strongly open, there is a neighborhood U of x such that $\overline{U} \subseteq A_\beta \subseteq \bigcup_{\lambda\in\Gamma} A_\gamma$. Hence, $\bigcup_{\lambda\in\Gamma} A_\gamma \in T_S$.

Finally, Let $A, B \in T_S$, and let $x \in A \bigcap B$ be arbitrary. By strong openness assumption of A, B, choose neighborhoods U and V of x such that $\overline{U} \subseteq A$ and $\overline{V} \subseteq B$.

Now, $U \bigcap V$ is a neighborhood of x. But then $\overline{U \bigcap V} \subseteq \overline{U} \bigcap \overline{V} \subseteq A \bigcap B$. Thus, $A \bigcap B \in T_S$. The fact that $T_S \subseteq T$ is obvious, clearly because $T_S -$ neighborhoods of any point in a set are $T-$ neighborhoods of it.

Corollary 3.4. Let (X,T) be a regular topological space, then $T_s = T$.

Proof: In regular spaces, the set of all closed neighborhoods of any point x makes up a local base at x [1]. Thus, any neighborhood of x contains a closed neighborhood of x.

Therefore, members of T are necessarily members of T_S. Hence, invoking theorem (3.3), we have $T_s = T$.

Since discrete spaces are regular, Corollary (3.4) implies that :

Corollary 3.5. If (X,T) is discrete. Then, $T_s = T$.

We close this section with the concept of strongly closed sets.

A set E is called strongly closed in a topological space X if the complement $E^c(= X - E)$ of E is strongly open in X. Of course, strongly closed sets are closed, but not conversely. Take, for example, $X = \{a,b,c\}, T = \{X, \phi, \{a\}\}$. Let $E = \{b,c\}$. Now, $E^c = \{a\}$ and so, $\overline{E^c} = \overline{\{a\}} = X \not\subset \{a\}$.

The next section will be devoted to fixed point theory of closure continuous functions on Uryson spaces.

4. Closure continuous functions on and into uryson spaces

Fixed point theory has always been a major field of research. Even when mathematicians went through new trends of generalizations, they were aware of fixed points. Thousands and thousands of articles can be recovered on fixed point theory, and so, just to mention, One can see [9]. We begin with a couple of results on strongly closed sets in connection with closure continuous functions.

Theorem 4.1. Let $f : X \to Y$ be closure continuous. Then $f^{-1}(E)$ is a strongly closed set in X whenever E is strongly closed in Y.

Proof: Let E be a strongly closed set in Y. We will show that the complement $\left(f^{-1}(E)\right)^c = f^{-1}(E^c)$ is strongly open.

Let $x \in f^{-1}(E^c)$ be arbitrary. So $f(x) \in E^c$. Since E is strongly closed, E^c is strongly open, hence there is a neighborhood V *of* $f(x)$ such that $\overline{V} \subseteq E^c$, and since f is closure continuous at x there is a neighborhood U *of* x such that $f(\overline{U}) \subseteq \overline{V}$.

Clearly, $\overline{U} \subseteq \left(f^{-1}(E)\right)^c$ and so $f^{-1}(E)$ is strongly closed in X.

Theorem 4.2. Let X be a Uryson space. Then the diagonal $\Delta_X = \{(x,x) : x \in X\}$ is a strongly closed set in the product space $X \times X$.

Proof: Let X be a Uryson space and let $(x,y) \in \Delta^c$ be arbitrary (So $x \neq y$). Pick neighborhoods U *of* x and V *of* y such that $\overline{U} \cap \overline{V} = \phi$. Now, $U \times V$ is a neighborhood of (x,y). We claim that $\overline{U \times V} \subseteq (\Delta_X)^c$. To this end, let $(z_1, z_2) \in \overline{U \times V}$ be arbitrary. Since $\overline{U \times V} = \overline{U} \times \overline{V}$, $z_1 \in \overline{U}$ *and* $z_2 \in \overline{V}$. But $\overline{U} \cap \overline{V} = \phi$, thus $z_1 \neq z_2$ which implies that $\overline{U \times V} \subseteq (\Delta_X)^c$. therefore, Δ_X is strongly closed in $X \times X$.

However, The result of the foregoing theorem becomes a little different if we consider the diagonal Δ_X in $(X,T_1)\times(X,T_2)$ for arbitrary topologies T_1 and T_2 being defined on X . In specific, we get :

Theorem 4.3. Let T_1 and T_2 be two topologies defined on a set X such that the space $(X,T_1\cap T_2)$ is Uryson. Then the diagonal $\Delta_X=\{(x,x): x\in X\}$is a strongly closed set in the product space $(X,T_1)\times(X,T_2)$.

Proof: Let$(x,y)\in\Delta^c$be arbitrary (So $x\neq y$). Since $(X,T_1\cap T_2)$ is Uryson, there are neighborhoods $U\ of\ x$ and $V\ of\ y$in $(X,T_1\cap T_2)$ such that: $\overline{U}^{T_1\cap T_2}\cap\overline{V}^{T_1\cap T_2}=\phi$.

Note that $U\times V$ is a neighborhood of (x,y)in $(X,T_1)\times(X,T_2)$ since $U\in T_1$ and $V\in T_2$. Now, we show that $\overline{U\times V}=\overline{U}^{T_1}\times\overline{V}^{T_2}\subseteq(\Delta_X)^c$.

For this, let $(z_1,z_2)\in\overline{U\times V}=\overline{U}^{T_1}\times\overline{V}^{T_2}$ be arbitrary. (So, $z_1\in\overline{U}^{T_1}$, $z_2\in\overline{V}^{T_2}$).
Since
$\overline{U}^{T_1}\subseteq\overline{U}^{T_1\cap T_2}$ and $\overline{V}^{T_2}\subseteq\overline{V}^{T_1\cap T_2}$ (by [1]), $z_1\in\overline{U}^{T_1\cap T_2}$ and $z_2\in\overline{V}^{T_1\cap T_2}$.
Since $\overline{U}^{T_1\cap T_2}\cap\overline{V}^{T_1\cap T_2}=\phi$, $z_1\neq z_2$. Therefore, $(z_1,z_2)\notin\Delta_X$, hence Δ_X is strongly closed in $(X,T_1)\times(X,T_2)$.

For the next proposition, we need the following :

Lemma 4.4. Let $f,g:X\rightarrow Y$ be closure continuous maps. Then, the function $h:X\rightarrow Y\times Y$ defined by $h(x)=(f(x),g(x))$ is closure continuous.

Proof: Let $x\in X$ and let W be a neighborhood of $(f(x),g(y))\ in\ Y\times Y$. This means that $W=V_1\times V_2$ where V_1 is a neighborhood of $f(x)$and V_2 is a neighborhood of $g(x)$. Since f and g are closure continuous there are neighborhoods $U_1\ and\ U_2$ of x such that $f(\overline{U_1})\subseteq\overline{V_1}$ and $g(\overline{U_2})\subseteq\overline{V_2}$. Of course, then $f(\overline{U_1})\times g(\overline{U_2})\subseteq\overline{V_1}\times\overline{V_2}=\overline{V_1\times V_2}$. Now, $U_1\cap U_2$ is a neighborhood of x and $\overline{U_1\cap U_2}=\overline{U_1}\cap\overline{U_2}$, hence, we have : $h(\overline{U_1\cap U_2})\subseteq f(\overline{U_1})\times g(\overline{U_2})\subseteq\overline{V_1\times V_2}=\overline{W}$. Therefore h is closure continuous.

Theorem 4.5. Let $f,g:X\rightarrow Y$ be closure continuous maps where Y is a Uryson space . Then the set $E=\{x\in X: f(x)=g(x)\}$ is strongly closed in X.

Proof: By lemma (4.4), the function $h:X\rightarrow Y\times Y$ defined by $h(x)=(f(x),g(x))$ is closure continuous. Since Y is a Uryson space, the diagonal $\Delta_Y=\{(y,y): y\in Y\}$is a strongly closed set in the product space $Y\times Y$ [Theorem 4.3]. Now, by invoking Theorem (4.1) we get that $E=h^{-1}(\Delta_Y)$ is strongly closed.

The foregoing theorem enables us to draw the following anticipated result for closure continuous maps when the image space is Uryson. However, the continuity of functions onto a Hausdorff space is known to maintain the density of the set E of theorem (4.5), [1]. So, we somehow get a similar result here.

Corollary 4.6. Let $f, g : X \to Y$ be closure continuous maps where Y is a Uryson space. Suppose that the set $E = \{x \in X : f(x) = g(x)\}$ is dense in X. Then $f = g$.
Proof: By theorem (4.5), the set E is strongly closed in X hence is the whole space X.

Theorem 4.7. Let $f : X \to Y$ be a closure continuous map where Y is a Uryson space. Then the graph $G_f = \{(x, f(x)) : x \in X\}$ of f is a strongly closed set in $X \times Y$.
Proof: Let $(x_1, y_1) \in G_f{}^c = X \times Y - G_f$. So $f(x_1) \neq y_1$. Since Y is a Uryson space, pick a neighborhood V of $f(x_1)$ and a neighborhood W of y_1 such that $\overline{V} \cap \overline{W} = \phi$. Since f is closure continuous, choose a neighborhood U of x_1 such that $f(\overline{U}) \subseteq \overline{V}$. Thus, $U \times W$ is a neighborhood of (x_1, y_1) in $X \times Y$.

To show that $\overline{U \times W} \subseteq G_f{}^c$, let $(z_1, z_2) \in \overline{U \times W}$ $(= \overline{U} \times \overline{W})$, so $z_1 \in \overline{U}$ *and* $z_2 \in \overline{W}$. Thus, $f(z_1) \in \overline{V}$. But $f(z_1) \neq z_2$, so $(z_1, z_2) \in G_f{}^c$. Therefore, $U \times W$ is a neighborhood of $(x_1, y_1) \in G_f{}^c$ such that $\overline{U \times W} \subseteq G_f{}^c$. Thus, $G_f{}^c$ is strongly open which implies that G_f is strongly closed.

Theorem 4.8. Let $f : X \to Y$ be a closure continuous map where Y is a Uryson space. Then the set $E = \{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is strongly closed in $X \times X$.
Proof: Let $(z_1, z_2) \in E^c (= (X \times X - E))$, so $f(z_1) \neq f(z_2)$. Since Y is a Uryson space, choose a neighborhood V_1 of z_1 and a neighborhood V_2 of z_2 such that $\overline{V_1} \cap \overline{V_2} = \phi$. Since f is closure continuous, pick a neighborhood U_1 of z_1 and a neighborhood U_2 of z_2 such that $f(\overline{U_1}) \subseteq \overline{V_1}$ and $f(\overline{U_2}) \subseteq \overline{V_2}$.
Now, $U_1 \times U_2$ is a neighborhood of (z_1, z_2) *in* $X \times X$. We will be done if we show that $\overline{U_1 \times U_2} \subseteq E^c$. For this, $(x, x^*) \in \overline{U_1 \times U_2} (= \overline{U_1} \cap \overline{U_2})$ be arbitrary. It follows that $x \in \overline{U_1}$ *and* $x^* \in \overline{U_2}$. But then $f(x) \in \overline{V_1}$ *and* $f(x^*) \in \overline{V_2}$ which implies that $f(x) \neq f(x*)$. Hence, $(x, x^*) \in E^c$ and the proof is complete.

We close this section with a proposition on fixed point theory.

Theorem 4.9. Let X be a Uryson space and let $f : X \to X$ be a closure continuous map. Then the set $E = \{x \in X : f(x) = x\}$ of all fixed points under f is strongly closed in X.

Proof: First we note that the identity function $i: X \to X$ is closure continuous. Now by lemma (4.4), the function $h: X \to X \times X$ defined as $h(x) = (f(x), i(x)) = (f(x), x)$ is closure continuous. By theorem (4.2), the diagonal Δ_X is a closed set in $X \times X$, and so by theorem (4.1), the set $h^{-1}(\Delta_X) = E = \{x \in X : f(x) = x\}$ is strongly closed in X.

5. Uryson spaces characterized

We give the following major characterization of Uryson spaces, but first, we need the following lemma.

Lemma 5.1. Let $\Im_1$ and $\Im_2$ be two filters in a topological space X. Then, $\Im_1$ and $\Im_2$ converge weakly to respectively, x and y if and only if $\Im_1 \times \Im_2 \xrightarrow{w} (x, y)$ in $X \times X$.

Proof: Suppose first, that $\Im_1 \xrightarrow{w} x$ and $\Im_2 \xrightarrow{w} y$ and let $V = U_x \times U_y$ be the neighborhood filter of (x, y) in $X \times X$ (U_x and U_y are the neighborhood filters of x and y, respectively). Now, since $\Im_1 \xrightarrow{w} x$ and $\Im_2 \xrightarrow{w} y$, we have: $\Im_1 \geq \overline{U_x}$ and $\Im_2 \geq \overline{U_y}$. Thus, $\Im_1 \times \Im_2 \geq \overline{U_x} \times \overline{U_y}$.

Let $V_{(x,y)} \in V$, say $V_{(x,y)} = U_1 \times U_2$ where $U_1 \in U_x$ and $U_2 \in U_y$. So, $\overline{V}_{(x,y)} = \overline{U_1 \times U_2} = \overline{U_1} \times \overline{U_2}$. Hence, $\Im_2 \times \Im_2 \geq \overline{V} = \overline{U_x \times U_y}$, thus $\Im_2 \times \Im_2 \xrightarrow{w} (x, y)$.

Conversely, suppose that $\Im_2 \times \Im_2 \xrightarrow{w} (x, y)$. Since the projection mapps π_1 and π_2 defined on $X \times X$ as $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$ are closure continuous, being continuous, $\Im_1 = \pi_1(\Im_1 \times \Im_2) \xrightarrow{w} x$ and $\Im_2 = \pi_2(\Im_1 \times \Im_2) \xrightarrow{w} y$.

Now, we give our main characterization .

Theorem 5.2. For a space X , the following are equivalent:

(i) X is a Uryson space.

(ii) The diagonal $\Delta_X = \{(x, x) : x \in X\}$ is a strongly a closed set in the product space $X \times X$.

(iii) Weak limits of filters in X when exist are unique.

(iv) Weak limits of nets in X when exist are unique.

Proof: $(i) \Rightarrow (ii)$: This is theorem(4.2).

$(ii) \Rightarrow (iii)$: Suppose that The diagonal $\Delta_X = \{(x, x) : x \in X\}$ is a strongly closed set in the product space $X \times X$ and let $\Im$ be a filter in X which converges to x and to y with $x \neq y$. Then, by lemma (5.1), $\Im \times \Im \xrightarrow{w} (x, y)$ in $X \times X$, with $(x, y) \in {\Delta_X}^c$. By Proposition(3.1), ${\Delta_X}^c \in \Im \times \Im$. Therefore, there is $F \in \Im$ such that $F \times F \subseteq {\Delta_X}^c$. Now, let $z \in F$, then $(z, z) \in {\Delta_X}^c$, which is a contradiction.

$(iii) \Leftrightarrow (iv)$: This follows from theorem (2.2) of [3].

$(iii) \Rightarrow (i)$: Suppose that Weak limits of filters in X when exist are unique, but X is not a Uryson space. Pick $x, y \in X$ with $x \neq y$ so that for all neighborhoods U of x and V of y, we have $\overline{U} \cap \overline{V} \neq \phi$. Let U_x and V_y be the neighborhood filters of x and y, respectively. Clearly, $\overline{U_x} \xrightarrow{w} x$ and $\overline{V_y} \xrightarrow{w} y$. Let $\overline{U_x} \vee \overline{V_y}$ be the filter generated by the collection $\left\{\overline{U} \cap \overline{V} : U \in U_x , V \in V_y\right\}$.

Since $\overline{U_x} \vee \overline{V_y} \geq \overline{U_x}$, $\overline{U_x} \vee \overline{V_y} \xrightarrow{w} x$. and since $\overline{U_x} \vee \overline{V_y} \geq \overline{V_y}$, $\overline{U_x} \vee \overline{V_y} \xrightarrow{w} y$. But by the uniqueness assumption of weak limits of filters, $x = y$. This, of course contradicts our supposition that $x \neq y$.

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